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By Same Author

GATE Engineering Mathematics
MRP 400.00
For Branch EC, EE, IN, ME, CE and PI

Multiple Choice Questions
GATE
Electronics & Communication Engineering
Fifth Edition June 2009
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Preface

This book doesn’t make promise but provides complete satisfaction to the readers. The market scenario is confusing and readers don’t find the optimum quality books. This book provides complete set of problems appeared in competition exams as well as fresh set of problems.

The book is categorized into units which are then sub-divided into chapters and the concepts of the problems are addressed in the relevant chapters. The aim of the book is to avoid the unnecessary elaboration and highlights only those concepts and techniques which are absolutely necessary. Again time is a critical factor both from the point of view of preparation duration and time taken for solving each problem in the examination. So the problems solving methods in the book are those which take the least distance to the solution.

But however to make a comment that this book is absolute for GATE preparation will be an inappropriate one. The theory for the preparation of the examination should be followed from the standard books. But for a wide collection of problems, for a variety of problems and the efficient way of solving them, what one needs to go through is there in the book. Each unit (e.g. Networks) is subdivided into average seven number of chapters on an average each of which contains 40 problems which are selected so as to avoid unnecessary redundancy and highly needed completeness.

I shall appreciate and greatly acknowledge the comments and suggestion from the users of this book.

R. K. Kanodia
rajkumar.kanodia@nodia.co.in
Acknowledgments

My primary debt remains to my friends and all those wonderful people who provided real inspiration and encouragement in taking up this challenging project.

No amount of thanks can ever repay the great debt which I owe to my classmates Dinesh Bisht, Biju Deka, Nitin yadav, Bhupesh Kumar, Sushant Ranjan and Mayank Thakur who have supported me during the publishing of this book. A special word of thanks to Miss Shalvi Agarwal who have written the article for back page.

I am also indebted to my grandparents, mother and my sister for their emotional support they have provided while preparing this book.

R. K. Kanodia
SYLLABUS

ENGINEERING MATHEMATICS

Linear Algebra: Matrix Algebra, Systems of linear equations, Eigen values and eigen vectors.


Differential equations: First order equation (linear and nonlinear), Higher order linear differential equations with constant coefficients, Method of variation of parameters, Cauchy's and Euler's equations, Initial and boundary value problems, Partial Differential Equations and variable separable method.

Complex variables: Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, Residue theorem, solution integrals.

Probability and Statistics: Sampling theorems, Conditional probability, Mean, median, mode and standard deviation, Random variables, Discrete and continuous distributions, Poisson, Normal and Binomial distribution, Correlation and regression analysis.


Transform Theory: Fourier transform, Laplace transform, Z-transform.

ELECTRONICS AND COMMUNICATION ENGINEERING


**Digital circuits:** Boolean algebra, minimization of Boolean functions; logic gates; digital IC families (DTL, TTL, ECL, MOS, CMOS). Combinatorial circuits: arithmetic circuits, code converters, multiplexers, decoders, PROMs and PLAs. Sequential circuits: latches and flip-flops, counters and shift-registers. Sample and hold circuits, ADCs, DACs. Semiconductor memories. Microprocessor(8085): architecture, programming, memory and I/O interfacing.

**Signals and Systems:** Definitions and properties of Laplace transform, continuous-time and discrete-time Fourier series, continuous-time and discrete-time Fourier Transform, DFT and FFT, z-transform. Sampling theorem. Linear Time-Invariant (LTI) Systems: definitions and properties; causality, stability, impulse response, convolution, poles and zeros, parallel and cascade structure, frequency response, group delay, phase delay. Signal transmission through LTI systems.

**Control Systems:** Basic control system components; block diagrammatic description, reduction of block diagrams. Open loop and closed loop (feedback) systems and stability analysis of these systems. Signal flow graphs and their use in determining transfer functions of systems; transient and steady state analysis of LTI control systems and frequency response. Tools and techniques for LTI control system analysis: root loci, Routh-Hurwitz criterion, Bode and Nyquist plots. Control system compensators: elements of lead and lag compensation, elements of Proportional-Integral-Derivative (PID) control. State variable representation and solution of state equation of LTI control systems.

**Communications:** Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

**Electromagnetics:** Elements of vector calculus: divergence and curl; Gauss and Stokes theorems, Maxwell's equations: differential and integral forms. Wave equation, Poynting vector. Plane waves: propagation through various media; reflection and refraction; phase and group velocity; skin depth. Transmission lines: characteristic impedance; impedance transformation; Smith chart; impedance matching; S parameters, pulse excitation. Waveguides: modes in rectangular waveguides; boundary conditions; cut-off frequencies; dispersion relations. Basics of propagation in dielectric waveguide and optical fibers. Basics of Antennas: Dipole antennas; radiation pattern; antenna gain.

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CHAPTER 1.1
BASIC CONCEPTS

1. A solid copper sphere, 10 cm in diameter is deprived of $10^{20}$ electrons by a charging scheme. The charge on the sphere is
(A) 160.2 C (B) $-160.2$ C
(C) 16.02 C (D) $-16.02$ C

2. A lightning bolt carrying 15,000 A lasts for 100 $\mu$s. If the lightning strikes an airplane flying at 2 km, the charge deposited on the plane is
(A) 13.33 $\mu$C (B) 75 C
(C) 1500 $\mu$C (D) 1.5 C

3. If 120 C of charge passes through an electric conductor in 60 sec, the current in the conductor is
(A) 0.5 A (B) 2 A
(C) 3.33 mA (D) 0.3 mA

4. The energy required to move 120 coulomb through 3 V is
(A) 25 mJ (B) 360 J
(C) 40 J (D) 2.78 mJ

5. $i = \, ?$

6. In the circuit of fig P1.1.6 a charge of 600 C is delivered to the 100 V source in a 1 minute. The value of $v_1$ must be
(A) 1 A (B) 2 A
(C) 3 A (D) 4 A

7. In the circuit of the fig P1.1.7, the value of the voltage source $E$ is
(A) 240 V (B) 120 V
(C) 60 V (D) 30 V

8. The energy required to move 120 coulomb through 3 V is
(A) 25 mJ (B) 360 J
(C) 40 J (D) 2.78 mJ

\[i = \, ?\]
8. Consider the circuit graph shown in fig. P1.1.8. Each branch of circuit graph represents a circuit element. The value of voltage $v_1$ is

(A) $-30 \text{ V}$
(B) $25 \text{ V}$
(C) $-20 \text{ V}$
(D) $15 \text{ V}$

9. For the circuit shown in fig P1.1.9 the value of voltage $v_o$ is

(A) $10 \text{ V}$
(B) $15 \text{ V}$
(C) $20 \text{ V}$
(D) None of the above

10. $R_1 = ?$

(A) $25 \Omega$
(B) $50 \Omega$
(C) $100 \Omega$
(D) $2000 \Omega$

11. Twelve 6 Ω resistor are used as edge to form a cube. The resistance between two diagonally opposite corner of the cube is

(A) $\frac{5}{6} \Omega$
(B) $\frac{6}{5} \Omega$
(C) $5 \Omega$
(D) $6 \Omega$

12. $v_1 = ?$

(A) $-11 \text{ V}$
(B) $5 \text{ V}$
(C) $8 \text{ V}$
(D) $18 \text{ V}$

13. The voltage $v_o$ in fig. P1.1.11 is always equal to

(A) $1 \text{ V}$
(B) $5 \text{ V}$
(C) $9 \text{ V}$
(D) None of the above

14. $R_{eq} = ?$

(A) $11.86 \Omega$
(B) $10 \Omega$
(C) $25 \Omega$
(D) $11.18 \Omega$

15. $v_s = ?$

(A) $320 \text{ V}$
(B) $280 \text{ V}$
(C) $240 \text{ V}$
(D) $200 \text{ V}$
24. Let \( i(t) = 3te^{-100t} \) A and \( v(t) = 0.6(0.01 - t)e^{-100t} \) V for the network of fig. P.1.1.24. The power being absorbed by the network element at \( t = 5 \) ms is

(A) 18.4 \( \mu \)W  
(B) 9.2 \( \mu \)W  
(C) 16.6 \( \mu \)W  
(D) 8.3 \( \mu \)W

25. In the circuit of fig. P.1.1.25 bulb A uses 36 W when lit, bulb B uses 24 W when lit, and bulb C uses 14.4 W when lit. The additional A bulbs in parallel to this circuit, that would be required to blow the fuse is

(A) 4  
(B) 5  
(C) 6  
(D) 7

26. In the circuit of fig. P.1.1.26, the power absorbed by the load \( R_L \) is

(A) 2 W  
(B) 4 W  
(C) 6 W  
(D) 8 W

27. \( v_{ab} = ? \)

(A) 15.4 V  
(B) 2.6 V  
(C) -2.6 V  
(D) 15.4 V

28. The dependent source in fig. P.1.1.30

(A) delivers 80 W  
(B) delivers 40 W  
(C) absorbs 40 W  
(D) absorbs 80 W

29. The dependent source in fig. P.1.1.31 dependent source

(A) supplies 16 W  
(B) absorbs 16 W  
(C) supplies 32 W  
(D) absorbs 32 W
32. A capacitor is charged by a constant current of 2 mA and results in a voltage increase of 12 V in a 10 sec interval. The value of capacitance is
(A) 0.75 mF  
(B) 1.33 mF  
(C) 0.6 mF  
(D) 1.67 mF

33. The energy required to charge a 10 μF capacitor to 100 V is
(A) 0.10 J  
(B) 0.05 J  
(C) 5 × 10⁻⁹ J  
(D) 10 × 10⁻⁹ J

34. The current in a 100 μF capacitor is shown in fig. P.1.1.34. If capacitor is initially uncharged, then the waveform for the voltage across it is

36. The waveform for the current in a 200 μF capacitor is shown in fig. P.1.1.36. The waveform for the capacitor voltage is

37. \( C_{eq} = ? \)

38. In the circuit shown in fig. P.1.1.38

\[ i_{in}(t) = 300 \sin 20t \text{ mA, for } t \geq 0. \]

Let \( C_1 = 40 \mu \text{F} \) and \( C_2 = 30 \mu \text{F} \). All capacitors are initially uncharged. The \( v_{in}(t) \) would be

(A) \(-0.25\cos 20t \text{ V}\)  
(B) \(0.25\cos 20t \text{ V}\)  
(C) \(-36\cos 20t \text{ mV}\)  
(D) \(36\cos 20t \text{ mV}\)
SOLUTIONS

1. (C) \( n = 10^{20}, Q = ne = e10^{20} = 16.02 \text{ C} \)
Charge on sphere will be positive.

2. (D) \( \Delta Q = i \times \Delta t = 15000 \times 100\mu = 15 \text{ C} \)

3. (B) \( i = \frac{dQ}{dt} = \frac{120}{60} = 2 \text{ A} \)

4. (B) \( W = Qv = 360 \text{ J} \)

6. (A) In order for 600 C charge to be delivered to the 100 V source, the current must be anticlockwise.
\[ i = \frac{dQ}{dt} = \frac{600}{60} = 10 \text{ A} \]

Applying KVL we get
\[ v_1 + 60 - 100 = 10 \times 20 \text{ or } v_1 = 240 \text{ V} \]

7. (A) Going from 10 V to 0 V

8. (D) \( 100 = 65 + v_2 \) \( \Rightarrow v_2 = 35 \text{ V} \)
\[ + 105 \text{ V} + 10 \text{ V} + \]

9. (B) Voltage is constant because of 15 V source.

10. (C) Voltage across 60 \( \Omega \) resistor = 30 V
Current \( = \frac{30}{60} = 0.5 \text{ A} \)
Voltage across \( R_1 \) is = 70 - 20 = 50 V
\[ R_1 = \frac{50}{0.5} = 100 \Omega \]

11. (C) The current \( i \) will be distributed in the cube branches symmetrically

12. (C) If we go from +side of 1 k\( \Omega \) through 7 V, 6 V and 5 V, we get \( v_1 = 7 + 6 - 5 = 8 \text{ V} \)

13. (D) It is not possible to determine the voltage across 1 A source.

14. (D) \[ R_{eq} = 5 + \frac{10 (R_{eq} + 5)}{10 + 5 + R_{eq}} \]
\[ \Rightarrow R_{eq}^2 + 15R_{eq} = 5R_{eq} + 75 + 10R_{eq} + 50 \]
\[ \Rightarrow R_{eq} = \sqrt{125} = 11.18 \Omega \]
v_1 - 20 \over 5 + v_2 = 20 \over 5 \Rightarrow v_2 = 20 \text{ V}

Power is \( P = v_i \times i = 20 \times 2 \Rightarrow 80 \text{ W} \)

31. (D) Power \( P = vi = 2i_1 \times i_2 = 2i_1^2 \)
\( i_2 = 4 \text{ A, } P = 32 \text{ W (absorb)} \)

32. (D) \( v_{12} - v_n = \frac{1}{C} \int idt \Rightarrow 12 = \frac{1}{C} \times 2m (t_2 - t_1) \)
\( \Rightarrow 12C = 2m \times 10 \Rightarrow C = 1.67 \text{ mF} \)

33. (B) \( E = \frac{1}{2} C v_n^2 = 5 \times 10^{-6} \times 100^2 = 0.05 \text{ J} \)

34. (D) \( v_1 = \frac{1}{C} \int idt = \frac{10 \times 10^{-3}}{100 \times 10^{-12}} (2 \times 10^{-2}) = 0.2 \text{ V} \)
This 0.2 V increases linearly from 0 to 0.2 V. Then current is zero. So capacitor hold this voltage.

35. (D) \( i = C \frac{dv}{dt} \)

For 0 < t < 1, \( \frac{dv}{dt} = 100 \times 10^{-6} \times \frac{6 - 0}{10^{-12}} = 600 \text{ mA} \)

For 1 ms < t < 2 ms,
\( \frac{dv}{dt} = 100 \times 10^{-6} \times \frac{0 - 6}{(3 - 2)m} = -600 \text{ mA} \)

36. (B) For 0 < t < 4,
\( v_c = \frac{1}{C} \int idt = \frac{1}{200 \times 10^{-12}} \int 5 \text{ m} \times 4 \text{ m} t dt = 3125t^2 \)

At t = 4 ms, \( v_c = 0.05 \text{ V} \)
It will be parabolic path. at t = 0 t-axis will be tangent.

37. (A) 2 \text{ mF} \text{ is in parallel with 1 \text{ mF} and this combination is in series with 1.5 \text{ mF}.
\( C_1 = \frac{1}{\frac{1}{2} + \frac{1}{15}} = \frac{5}{3} \text{ mF}, \text{ } \text{ } \text{ } \text{ } C_1 \text{ is in parallel with 2.5 \text{ mF}} \)
\( C_{eq} = 1 + 2.5 = 3.5 \text{ mF} \)

38. (A) \( C_a = \frac{30 \times 60}{30 + 60} = 20 \text{ mF, } \text{ } \text{ } \text{ } \text{ } C_b = \frac{30(20 + 40)}{30 + 20 + 0} = 20 \text{ mF} \)

We can say \( C_d = 20 \text{ mF, } C_{eq} = 20 + 40 = 60 \text{ mF} \)
\( v_c = \frac{1}{C} \int idt = \frac{1}{60m} \left( - \frac{300}{20} \cos 20t \right) \times 10^{-3} = -0.25 \cos 20t \text{ V} \)

39. (C) \( i_{c1} = \frac{i_{in} C_1}{C_1 + C_2} = 0.8 \sin 600t \text{ mA} \)
At t = 2 ms, \( i_{c1} = 0.75 \text{ mA} \)

40. (B) \( v_{c1} = \frac{v_{in} C_2}{C_1 + C_2} = \frac{4v_{in}}{6 + 4} = \frac{v_{in}}{0.4} \)

41. (D) \( V = 2 + 3 + 5 = 10, Q = 1 \text{ C, } C = \frac{Q}{V} = 0.1 \text{ F} \)

42. (A) \( v_L = \frac{L di}{dt} \Rightarrow 100m = L \frac{200m}{4m} \Rightarrow L = 2 \text{ mH} \)

43. (B) \( v_L = \frac{L di}{dt} = 0.01 \times 2(377 \cos 377t) \text{ V} \)
= 754 \cos 377t \text{ V}

44. (A) \( i = \frac{1}{L} \int vdt = \frac{1}{0.01} \int 120 \cos 3t dt = \frac{12000}{377} \sin 377t \)
\( P = \int \frac{12000 \times 120}{377} \sin 377t \cos 377t \text{ (sin 377t)(cos 377t)} \)
= 1910 \sin 754t \text{ W}

45. (D) \( v_L = \frac{L di}{dt}, \text{ } i_c = C \frac{dv_c}{dt} \)
\( v_c = 3v_{in} \Rightarrow i_c = 3LC \frac{di_L}{dt} = -9.6 \sin 4t \text{ A} \)

46. (B) \( v_L = \frac{L di}{dt} \)

For 2 < t < 4, \( v_L = (0.05) \left( \frac{-100 - 0}{2} \right) = -25 \text{ V} \)

For 4 < t < 8, \( v_L = (0.05) \left( \frac{100 + 100}{4} \right) = 25 \text{ V} \)

For 8 < t < 10, \( v_L = (0.05) \left( \frac{0 - 100}{2} \right) = -25 \text{ V} \)

Thus (B) is correct option.

47. (C) Algebraic sum of the current entering or leaving a cutset is equal to 0.
\( i_2 + i_4 + i_3 = 0 \Rightarrow \frac{6 + 16}{2} + i_3 = 0 \)
\( i_3 = -7 \text{ A, } \text{ } \text{ } \text{ } \text{ } v_3 = -7 \times 3 = -21 \text{ V} \)

**********
CHAPTER 1.2

GRAPH THEORY

1. Consider the following circuits:

(a) 1 and 2  
(b) 2 and 3  
(c) 3 and 4  
(d) 4 and 1

The planner circuits are

(A) 1 and 2  
(B) 2 and 3  
(C) 3 and 4  
(D) 4 and 1

2. Consider the following graphs:

Non-planner graphs are

(A) 1 and 3  
(B) 4 only  
(C) 3 only  
(D) 3 and 4

3. A graph of an electrical network has 4 nodes and 7 branches. The number of links \( l \), with respect to the chosen tree, would be

(A) 2  
(B) 3  
(C) 4  
(D) 5

4. For the graph shown in fig. P.1.1.4 correct set is

\[
\begin{array}{|c|}
\hline
\text{Node} & \text{Branch} & \text{Twigs} & \text{Link} \\
\hline
\text{(A)} & 4 & 6 & 4 & 2 \\
\text{(B)} & 4 & 6 & 3 & 3 \\
\text{(C)} & 5 & 6 & 4 & 2 \\
\text{(D)} & 5 & 5 & 4 & 1 \\
\hline
\end{array}
\]

Fig. P.1.1.4

5. A tree of the graph shown in fig. P.1.2.5 is

(A) a d e h  
(B) a c f h  
(C) a f h g  
(D) a e f g

Fig. P.1.2.5
13. The incidence matrix of a graph is as given below

\[
A = \begin{bmatrix}
-1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 \\
0 & -1 & 0 & -1 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 & -1 \\
\end{bmatrix}
\]

The graph is

(A) ![Graph A](image1)
(B) ![Graph B](image2)
(C) ![Graph C](image3)
(D) ![Graph D](image4)

15. The incidence matrix of a graph is as given below

\[
A = \begin{bmatrix}
-1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 \\
0 & -1 & 0 & -1 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 & -1 \\
\end{bmatrix}
\]

The graph is

(A) ![Graph A](image5)
(B) ![Graph B](image6)
(C) ![Graph C](image7)
(D) ![Graph D](image8)

16. The graph of a network is shown in fig. P.1.1.16. The number of possible tree are

(A) 8
(B) 12
(C) 16
(D) 20
22. The fundamental cut-set matrix of a graph is

\[ Q_F = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \]

The oriented graph of the network is

(A) (B) (C) (D)

23. A graph is shown in fig. P.1.2.23 in which twigs are solid line and links are dotted line. For this chosen tree fundamental set matrix is given below.

\[ B_F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]

The oriented graph will be

(A) (B) (C) (D)

24. A graph is shown in fig. P.1.2.24 in which twigs are solid line and links are dotted line. For this tree fundamental loop matrix is given as below

\[ B_F = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \]

Fig. P.1.2.24

The oriented graph will be

(A) (B) (C) (D)

25. Consider the graph shown in fig. P.1.2.25 in which twigs are solid line and links are dotted line.

Fig. P.1.2.25
A fundamental loop matrix for this tree is given as

\[ B_f = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & -1 & -1 & 0 \\
0 & 0 & 1 & 0 & 1 & -1
\end{bmatrix} \]

The oriented graph will be:

(A) ![Graph Image]

(B) ![Graph Image]

(C) ![Graph Image]

(D) ![Graph Image]

26. In the graph shown in fig. P.1.2.26 solid lines are twigs and dotted line are link. The fundamental loop matrix is

\[ \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{bmatrix} \]

27. Branch current and loop current relation are expressed in matrix form as

\[
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
i_6 \\
i_7 \\
i_8
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & -1 & 0 \\
0 & 0 & -1 & 1 \\
1 & 0 & 0 & -1 & I_1 \\
-1 & 1 & 0 & 0 & I_2 \\
1 & 0 & 0 & 0 & I_3 \\
0 & 1 & 0 & 0 & I_4 \\
0 & 0 & 1 & 0 & I_5 \\
0 & 0 & 0 & 1 & I_6
\end{bmatrix}
\]

where \( i_j \) represent branch current and \( I_k \) loop current. The number of independent node equation are

(A) 4

(B) 5

(C) 6

(D) 7

28. If the number of branch in a network is \( b \), the number of nodes is \( n \) and the number of dependent loop is \( l \), then the number of independent node equations will be

(A) \( n + l - 1 \)

(B) \( b - 1 \)

(C) \( b - n + 1 \)

(D) \( n - 1 \)

**Statement for Q.29–30:**

Branch current and loop current relation are expressed in matrix form as

\[
\begin{bmatrix}
i_1 \\
i_2 \\
i_3 \\
i_4 \\
i_5 \\
i_6 \\
i_7 \\
i_8
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
-1 & -1 & -1 & 0 \\
0 & 1 & 0 & 0 & I_1 \\
1 & 0 & 0 & 0 & I_2 \\
0 & 0 & -1 & -1 & I_3 \\
1 & 1 & 0 & -1 & I_4 \\
1 & 0 & 0 & 0 & I_5 \\
0 & 0 & 0 & 1 & I_6
\end{bmatrix}
\]

where \( i_j \) represent branch current and \( I_k \) loop current.

29. The rank of incidence matrix is

(A) 4

(B) 5

(C) 6

(D) 8
30. The directed graph will be

![Directed Graph](image)

31. A network has 8 nodes and 5 independent loops. The number of branches in the network is
(A) 11  
(B) 12  
(C) 8  
(D) 6

32. A branch has 6 node and 9 branch. The independent loops are
(A) 3  
(B) 4  
(C) 5  
(D) 6

**Statement for Q.33–34:**

For a network branch voltage and node voltage relation are expressed in matrix form as follows:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
V_7 \\
V_8 \\
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 \\
1 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6 \\
V_7 \\
V_8 \\
\end{bmatrix}
\]

where \(v_i\) is the branch voltage and \(V_k\) is the node voltage with respect to datum node.

33. The independent mesh equation for this network are
(A) 4  
(B) 5  
(C) 6  
(D) 7
1. (A) The circuit 1 and 2 are redrawn as below. 3 and 4 cannot be redrawn on a plane without crossing other branches.

![Fig. S1.2.1](image)

2. (B) Other three circuits can be drawn on plane without crossing.

![Fig. S 1.2.7](image)

3. (C) \( l = b - (n - 1) = 4 \).

4. (B) There are 4 nodes and 6 branches.
\[ t = n - 1 = 3, \quad l = b - n + 1 = 3 \]

5. (C) From fig. it can be seen that \( a f h g \) is a tree of given graph.

![Fig. S 1.2.5](image)

6. (B) From fig. it can be seen that \( a d f \) is a tree.

![Fig. S. 1.2.6](image)

7. (D) D is not a tree

![Fig. S 1.2.7](image)

8. (D) It is obvious from the following figure that 1, 3, and 4 are tree.
This is similar to matrix in (A). Only place of rows has been changed.

27. (A) Number of branch =8
Number of link =4
Number of twigs =8 - 4 = 4
Number of twigs = number of independent node equation.

28. (D) The number of independent node equation are \( n - 1 \).

29. (A) Number of branch \( b = 8 \)
Number of link \( l = 4 \)
Number of twigs \( t = b - l = 4 \)
rank of matrix \( = n - 1 = t = 4 \)

30. (B) We know the branch current and loop current are related as
\[ [i_b] = [B^T] [V_n] \]
So fundamental loop matrix is
\[ B_f = \begin{bmatrix}
0 & -1 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 1
\end{bmatrix} \]
f-loop 1 include branch (2, 4, 6, 7) and direction of branch–2 is opposite to other (B only).

31. (B) Independent loops = link
\( l = b - (n - 1) \)
\[ \Rightarrow 5 = b - 7, b = 12 \]

32. (B) Independent loop = link
\( l = b - (n - 1) = 4 \)

33. (A) There are 8 branches and \( 4 + 1 = 5 \) node
Number of link \( = 8 - 5 + 1 = 4 \)

So independent mesh equation = Number of link.

34. (D) We know that \( [v_b] = A_r^T [V_n] \)
So reduced incidence matrix is
\[ A_r = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 1 & -1 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0
\end{bmatrix} \]
At node-1, three branch leaves so the only option is (D).
CHAPTER 1.3

METHODS OF ANALYSIS

1. $v_1 = ?$

(A) $0.4v_s$  
(B) $1.5v_s$  
(C) $0.67v_s$  
(D) $2.5v_s$

Fig. P1.3.1

2. $v_a = ?$

(A) $-11$ V  
(B) $11$ V  
(C) $3$ V  
(D) $-3$ V

Fig. P1.3.2

3. $v_2 = ?$

(A) $0.5$ V  
(B) $1.0$ V  
(C) $1.5$ V  
(D) $2.0$ V

Fig. P1.3.5

4. $v_2 = ?$

(A) $120$ V  
(B) $-120$ V  
(C) $90$ V  
(D) $-90$ V

Fig. P1.3.4

5. $i_b = ?$

(A) $0.6$ A  
(B) $0.5$ A  
(C) $0.4$ A  
(D) $0.3$ A

Fig. P1.3.6
7. \[ i_1 = ? \]

(A) 3.3 A  
(B) 2.1 A  
(C) 1.7 A  
(D) 1.1 A

Fig. P1.3.7

8. \[ i_1 = ? \]

(A) 1 mA  
(B) 1.5 mA  
(C) 2 mA  
(D) 2.5 mA

Fig. P1.3.8

9. \[ i_1 = ? \]

(A) 4 A  
(B) 3 A  
(C) 6 A  
(D) 5 A

Fig. P1.3.9

10. \[ i_1 = ? \]

(A) 20 mA  
(B) 15 mA  
(C) 10 mA  
(D) 5 mA

Fig. P1.3.10

11. \[ i_1 = ? \]

(A) 0.01 A  
(B) –0.01 A  
(C) 0.03 A  
(D) 0.02 A

Fig. P1.3.11

12. The value of the current measured by the ammeter in Fig. P1.3.12 is

(A) \( \frac{2}{3} \) A  
(B) \( \frac{5}{3} \) A  
(C) \( \frac{5}{6} \) A  
(D) \( \frac{2}{9} \) A

Fig. P1.3.12

13. \[ i_1 = ? \]

(A) 10 mA  
(B) –10 mA  
(C) 0.4 mA  
(D) –0.4 mA

Fig. 1.3.13
14. The values of node voltage are $v_a = 12$ V, $v_b = 9.88$ V and $v_c = 5.29$ V. The power supplied by the voltage source is

![Fig. 1.3.14](image)

(A) 19.8 W  
(B) 27.3 W  
(C) 46.9 W  
(D) 54.6 W

15. $i_1, i_2, i_3 =$ ?

![Fig. P1.3.15](image)

(A) 3 A, 2 A, and 4 A  
(B) 3 A, 3 A, and 8 A  
(C) 1 A, 3 A, and 4 A  
(D) 1 A, 2 A, and 8 A

16. $v_o =$ ?

![Fig. P1.3.16](image)

(A) $\frac{6}{5}$ V  
(B) $\frac{8}{5}$ V  
(C) $\frac{6}{7}$ V  
(D) $\frac{5}{7}$ V

17. The mesh current equation for the circuit in Fig. P1.3.17 are

![Fig. 1.3.17](image)

(A) $\begin{bmatrix} 4 & -2 & 0 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$  
(B) $\begin{bmatrix} 6 & -2 & 0 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$  
(C) $\begin{bmatrix} 6 & -2 & 0 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$  
(D) $\begin{bmatrix} 4 & -2 & 0 \\ 0 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \end{bmatrix}$

18. For the circuit shown in Fig. P1.3.18 the mesh equation are

![Fig. 1.3.18](image)

(A) $\begin{bmatrix} 6k & -12k & -12k \\ -6k & 6k & -18k \\ -1k & -1k & 0k \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$  
(B) $\begin{bmatrix} 6k & -12k & -12k \\ -6k & 6k & 18k \\ -1k & 1k & 0k \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$  
(C) $\begin{bmatrix} 6k & -12k & -12k \\ -6k & 6k & 18k \\ 1k & 1k & 0k \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$  
(D) $\begin{bmatrix} 6k & -12k & -12k \\ -6k & 6k & 18k \\ 1k & 1k & 0k \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 5 \end{bmatrix}$
The value of $R_4$ is

\begin{align*}
(A) \ 40 & \quad (B) \ 15 \\
(C) \ 5 & \quad (D) \ 20
\end{align*}

26. $v_a = ?$

27. $v = ?$

28. $i_1 = ?$

29. $v_a = ?$

30. $i_a = ?$

31. $v_2 = ?$

32. $i_1 = ?$
(A) -1.636 A  
(C) -2.314 A  
(B) -3.273 A  
(D) -4.628 A

33. \( v_x = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.33} \\
  1.6 A & 100 \Omega & \frac{0.02}{v} & 50 \Omega & v_x
\end{align*} \]

(A) 32 V  
(B) -32 V  
(C) 12 V  
(D) -12 V

34. \( i_b = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.34} \\
  6 V & 1 \Omega & v_x & 3 \Omega & v_x & 2 \Omega & 4 v_x
\end{align*} \]

(A) 4 mA  
(B) -4 mA  
(C) 12 mA  
(D) -12 mA

35. \( v_b = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.35} \\
  2 V & 4 \Omega & v_x & 2 \Omega & 5 i_x
\end{align*} \]

(A) 1 V  
(B) 1.5 V  
(C) 4 V  
(D) 6 V

36. \( v_x = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.36} \\
  2 A & 100 \Omega & 25i_y & 50 \Omega & v_x & 0.2v_x
\end{align*} \]

(A) -3 V  
(B) 3 V  
(C) 10 V  
(D) -10 V

37. \( v_a = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.37} \\
  16 A & v_x & 2 \Omega & 2.5 \Omega & 5 \Omega & 10 A
\end{align*} \]

(A) 25.91 V  
(B) -25.91 V  
(C) 51.82 V  
(D) -51.82 V

38. For the circuit of Fig. P1.3.38 the value of \( v_x \), that will result in \( v_1 = 0 \), is

\[ \begin{align*}
  \text{Fig. P1.3.38} \\
  3 A & 2 A & 20 \Omega & 48 V & 10 \Omega & 40 \Omega & 0.1 v_x
\end{align*} \]

(A) 28 V  
(B) -28 V  
(C) 14 V  
(D) -14 V

39. \( i_1, i_2 = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.39} \\
  15 V & i_1 & 6 \Omega & i_2 & 18 V & 4 \Omega & 2 i_x & 2 i
\end{align*} \]

(A) 2.6 A, 1.4 A  
(B) 2.6 A, -1.4 A  
(C) 1.6 A, 1.35 A  
(D) 1.2 A, -1.35 A

40. \( v_1 = ? \)

\[ \begin{align*}
  \text{Fig. P1.3.40} \\
  14 V & v_1 & 2 \Omega & 2 A & 7 A & 2 \Omega
\end{align*} \]
(A) 10 V  (B) −10 V
(C) 7 V  (D) −7 V

41. \( v_x = ? \)

\[ \begin{align*}
\text{Fig. P1.3.41} & \quad 0.5v_x + v_y = 0.6 \quad 500 \Omega \\
& \quad 0.6 \uparrow \quad 3 \uparrow \quad 2 \uparrow \\
& \quad 0.3 \downarrow \\
& \quad v_x = 0.3 \ A
\end{align*} \]

42. The power being dissipated in the 2Ω resistor in the circuit of Fig. P1.3.42 is

\[ \begin{align*}
\text{Fig. P1.3.42} & \quad 0.6 \ A \\
& \quad 2.5 \ A \\
& \quad 5 \Omega \\
& \quad 2 \ A \\
& \quad 4 \Omega \\
& \quad 3 \Omega \\
& \quad 25 \ V \\
& \quad 30 \ V
\end{align*} \]

(A) 76.4 W  (B) 305.6 W
(C) 52.5 W  (D) 210.0 W

43. \( i_1 = ? \)

\[ \begin{align*}
\text{Fig. P1.3.43} & \quad 180 \ V \\
& \quad 0.6 \ A \\
& \quad 500 \Omega \\
& \quad 400 \Omega \\
& \quad 100 \Omega \\
& \quad 500 \Omega \\
& \quad 0.001v_y \\
& \quad v_y = 0.005v_y
\end{align*} \]

(A) 0.12 A  (B) 0.24 A
(C) 0.36 A  (D) 0.48 A

***************
38. (D) If \( v_1 = 0 \), the dependent source is a short circuit

\[
\frac{v_1}{40} + \frac{v_1 - v_2}{10} + \frac{v_1 - 48}{20} = 2 - 3 \implies v_1 = 0
\]

\[
-\frac{v_2}{10} - \frac{48}{20} = -1 \implies v_2 = -14 \text{ V}
\]

![Fig. S1.3.38](image)

\[i_s = \frac{30 + 7.5 + 2}{12} = 3.29 \text{ A} \implies i_s = 19.75 \text{ V}
\]

voltage across 2 \( \Omega \) resistor

\[30 - 19.75 = 10.25 \text{ V}, \quad P = \frac{(10.25)^2}{2} = 52.53 \text{ W}
\]

40. (B) \[14 = 3i_1 + v_y + 6(i_1 - 2) - 7 + 2v_y + 2(i_1 - 7)
\]

\[v_y = 3(i_1 - 2)
\]

\[14 = 3i_1 + 9(i_1 - 2) + 6(i_1 - 9) + 2(i_1 - 7) \implies i_1 = 5 \text{ A}
\]

\[v_1 = 6(5 - 2 - 7) + 2 \times 3(5 - 2) + 2(5 - 7) = -10 \text{ V}
\]

![Fig. S1.3.40](image)

41. (D) Let \( i_1 \) and \( i_2 \) be two loop current

\[0.5v_x = 500i_1 + 500(i_1 - i_2), \quad v_x = -500i_1
\]

\[\implies 5i_1 - 2i_2 = 0 \quad \text{...}(i)
\]

\[500(i_2 - i_1) + 900(i_2 + 0.3) + 600(i_2 - 0.6) = 0 \]

\[-5i_1 + 20i_2 = 0.9 \quad \text{...}(ii)
\]

\[i_1 = 20 \text{ mA}, \quad v_x = -500 \times 20 \text{ m} = -10 \text{ V}
\]

42. (C) \[30 = 5i_s + 3(i_s - 2.5) + 4(i_s - 2.5 + 2)
\]
CHAPTER

1.4

NETWORKS THEOREM

1. \(v_{TH}, R_{TH} = ?\)

- (A) 2 V, 4 \(\Omega\)
- (B) 4 V, 4 \(\Omega\)
- (C) 4 V, 5 \(\Omega\)
- (D) 2 V, 5 \(\Omega\)

2. \(i_N, R_N = ?\)

- (A) 3 A, \(\frac{10}{3}\) \(\Omega\)
- (B) 10 A, 4 \(\Omega\)
- (C) 1.5 A, 6 \(\Omega\)
- (D) 1.5 A, 4 \(\Omega\)

3. \(v_{TH}, R_{TH} = ?\)

- (A) −2 V, \(\frac{6}{5}\) \(\Omega\)
- (B) 2 V, \(\frac{5}{6}\) \(\Omega\)
- (C) 1 V, \(\frac{5}{6}\) \(\Omega\)
- (D) −1 V, \(\frac{6}{5}\) \(\Omega\)

4. A simple equivalent circuit of the 2 terminal network shown in fig. P.1.4.4 is

5. \(i_N, R_N = ?\)

- (A) 4 A, 3 \(\Omega\)
- (B) 2 A, 6 \(\Omega\)
- (C) 2 A, 9 \(\Omega\)
- (D) 4 A, 2 \(\Omega\)
6. \( v_{TH}, R_{TH} = ? \)

- (A) \(-100 \text{ V}, 75 \Omega\)
- (B) \(155 \text{ V}, 55 \Omega\)
- (C) \(155 \text{ V}, 37 \Omega\)
- (D) \(145 \text{ V}, 75 \Omega\)

7. \( R_{TH} = ? \)

- (A) \(3 \Omega\)
- (B) \(12 \Omega\)
- (C) \(6 \Omega\)
- (D) \(\infty\)

8. The Thevenin impedance across the terminals \(ab\) of the network shown in fig. P.1.4.8 is

- (A) \(2 \Omega\)
- (B) \(6 \Omega\)
- (C) \(6.16 \Omega\)
- (D) \(\frac{4}{3} \Omega\)

9. For the circuit shown in fig. P.1.4.9 a network and its Thevenin and Norton equivalent are given

- (A) \(8 \text{ V}, 6 \Omega\)
- (B) \(5 \text{ V}, 6 \Omega\)
- (C) \(5 \text{ V}, 32 \Omega\)
- (D) \(8 \text{ V}, 32 \Omega\)

10. \( v_1 = ? \)

- (A) \(6 \text{ V}\)
- (B) \(7 \text{ V}\)
- (C) \(8 \text{ V}\)
- (D) \(10 \text{ V}\)

11. \( i_1 = ? \)

- (A) \(3 \text{ A}\)
- (B) \(0.75 \text{ mA}\)
- (C) \(2 \text{ mA}\)
- (D) \(1.75 \text{ mA}\)

**Statement for Q.12–13:**

A circuit is given in fig. P.1.4.12–13. Find the Thevenin equivalent as given in question..
13. As viewed from terminal \( y \) and \( y' \) is
(A) 8 V, 32 \( \Omega \)   
(B) 4 V, 32 \( \Omega \)   
(C) 5 V, 6 \( \Omega \)   
(D) 7 V, 6 \( \Omega \)

14. A practical DC current source provide 20 kW to a 50\( \Omega \) load and 20 kW to a 200\( \Omega \) load. The maximum power, that can drawn from it, is
(A) 22.5 kW   
(B) 45 kW   
(C) 30.3 kW   
(D) 40 kW

Statement for Q.15–16:

In the circuit of fig. P.1.4.15–16 when \( R = 0 \) \( \Omega \), the current \( i_R \) equals 10 A.

15. The value of \( R \), for which it absorbs maximum power, is
(A) 4 \( \Omega \)   
(B) 3 \( \Omega \)   
(C) 2 \( \Omega \)   
(D) None of the above

16. The maximum power will be
(A) 50 W   
(B) 100 W   
(C) 200 W   
(D) value of E is required

17. Consider a 24 V battery of internal resistance \( r = 4 \) \( \Omega \) connected to a variable resistance \( R_L \). The rate of heat dissipated in the resistor is maximum when the current drawn from the battery is \( i \). The current drawn form the battery will be \( i/2 \) when \( R_L \) is equal to
(A) 2 \( \Omega \)   
(B) 4 \( \Omega \)   
(C) 8 \( \Omega \)   
(D) 12 \( \Omega \)

18. \( i_N, R_N = \) ?

19. \( v_{TH}, R_{TH} = \) ?

20. \( v_{TH}, R_{TH} = \) ?

21. \( R_{TH} = \) ?

22. In the circuit shown in fig. P.1.4.22 the effective resistance faced by the voltage source is
23. In the circuit of fig. P.1.4.23 the value of $R_{TH}$ at terminal $ab$ is

(A) $-3 \, \Omega$

(B) $\frac{9}{8} \, \Omega$

(C) $\frac{-8}{3} \, \Omega$

(D) None of the above

24. $R_{TH} = ?$

(A) $\infty$

(B) 0

(C) $\frac{3}{125} \, \Omega$

(D) $\frac{125}{3} \, \Omega$

25. In the circuit of fig. P.1.4.25, the $R_L$ will absorb maximum power if $R_L$ is equal to

(A) $\frac{400}{3} \, \Omega$

(B) $\frac{2}{9} \, k\Omega$

(C) $\frac{800}{3} \, \Omega$

(D) $\frac{4}{9} \, k\Omega$

Statement for Q.26–27:

In the circuit shown in fig. P.1.4.26–27 the maximum power transfer condition is met for the load $R_L$. 

26. The value of $R_L$ will be

(A) 2 $\Omega$

(B) 3 $\Omega$

(C) 1 $\Omega$

(D) None of the above

27. The maximum power is

(A) 0.75 W

(B) 1.5 W

(C) 2.25 W

(D) 1.125 W

28. $R_{TH} = ?$

(A) 100 $\, \Omega$

(B) 136.4 $\, \Omega$

(C) 200 $\, \Omega$

(D) 272.8 $\, \Omega$

29. Consider the circuits shown in fig. P.1.4.29a & b
The relation between \( i_a \) and \( i_b \) is
(A) \( i_b = i_a + 6 \)  
(B) \( i_b = \frac{i_a}{2} \)  
(C) \( i_b = 1.5i_a \)  
(D) \( i_b = i_a \)

\[ R_{eq} = ? \]

30. \( R_{eq} = 18 \Omega \)

31. In the lattice network the value of \( R_L \) for the maximum power transfer to it is
(A) 6.67 \( \Omega \)  
(B) 9 \( \Omega \)  
(C) \( \frac{36}{13} \) \( \Omega \)  
(D) 8 \( \Omega \)

\[ R_{eq} \]

32. If \( v_{s1} = v_{s2} = 6 \) V then the value of \( v_a \) is
(A) 3 V  
(B) 4 V  
(C) 6 V  
(D) 5 V

\[ v_{s1} \quad v_{s2} \]

33. If \( v_{s1} = 6 \) V and \( v_{s2} = -6 \) V then the value of \( v_a \) is
(A) 4 V  
(B) -4 V  
(C) 6 V  
(D) -6 V

34. A network \( N \) feeds a resistance \( R \) as shown in fig. P1.4.34. Let the power consumed by \( R \) be \( P \). If an identical network is added as shown in figure, the power consumed by \( R \) will be

\[ R \]

35. A certain network consists of a large number of ideal linear resistors, one of which is \( R \) and two constant ideal source. The power consumed by \( R \) is \( P_1 \) when only the first source is active, and \( P_2 \) when only the second source is active. If both sources are active simultaneously, then the power consumed by \( R \) is
(A) \( P_1 \pm P_2 \)  
(B) \( \sqrt{P_1} \pm \sqrt{P_2} \)  
(C) \( (\sqrt{P_1} \pm \sqrt{P_2})^2 \)  
(D) \( (P_1 \pm P_2)^2 \)

36. A battery has a short-circuit current of 30 A and an open circuit voltage of 24 V. If the battery is connected to an electric bulb of resistance \( 2 \Omega \), the power dissipated by the bulb is
(A) 80 W  
(B) 1800 W  
(C) 112.5 W  
(D) 228 W

37. The following results were obtained from measurements taken between the two terminal of a resistive network

<table>
<thead>
<tr>
<th>Terminal voltage</th>
<th>12 V</th>
<th>0 V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal current</td>
<td>0 A</td>
<td>1.5 A</td>
</tr>
</tbody>
</table>

The Thévenin resistance of the network is
(A) 16 \( \Omega \)  
(B) 8 \( \Omega \)  
(C) 0  
(D) \( \infty \)
38. A DC voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Reading on two scales are as follows
   (a) 0 – 10 V scale : 4 V
   (b) 0 – 15 V scale : 5 V

   The Thevenin voltage and the Thevenin resistance of the network is
   (A) \( \frac{16}{3} \) V, \( \frac{200}{3} \) kΩ
   (B) \( \frac{32}{3} \) V, \( \frac{1}{15} \) MΩ
   (C) 18 V, \( \frac{2}{15} \) MΩ
   (D) 36 V, \( \frac{200}{3} \) kΩ

39. Consider the network shown in fig. P.1.4.39.

   \[ R_L \]
   \[ + \]
   \[ \overset{v_{ab}}{\text{Linear }} \]
   \[ \overset{R_L}{\text{Network}} \]
   \[ - \]

   The power absorbed by load resistance \( R_L \) is shown in table:

<table>
<thead>
<tr>
<th>( R_L )</th>
<th>10 kΩ</th>
<th>30 kΩ</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>3.6 MW</td>
<td>4.8 MW</td>
</tr>
</tbody>
</table>

   The value of \( R_L \), that would absorb maximum power, is
   (A) 60 kΩ
   (B) 100 Ω
   (C) 300 Ω
   (D) 30 kΩ

40. Measurement made on terminal \( ab \) of a circuit of fig.P.1.4.40 yield the current-voltage characteristics shown in fig. P.1.4.40. The Thevenin resistance is

   \[ i(mA) \]
   \[ 30 \]
   \[ 20 \]
   \[ 10 \]
   \[ 0 \]
   \[ -1 \]
   \[ -2 \]
   \[ -3 \]
   \[ -4 \]

   (A) 300 Ω
   (B) –300 Ω
   (C) 100 Ω
   (D) –100 Ω

   ***********
7. (C) After killing the source, \( R_{TH} = 6 \Omega \)

![Fig. S.1.4.7](image)

8. (B) After killing all source, \( R_{TH} = 3 \| 6 + 8 \| 8 = 6 \Omega \)

9. (D) \( v_{xv} = 2 \times 2 + 4 = 8 \, V = v_{TH} \)
\[
R_{TH} = 2 + 3 = 5 \, \Omega = R_N, \quad i_N = \frac{v_{TH}}{R_{TH}} = \frac{8}{5} \, A
\]

10. (A) If we solve this circuit direct, we have to deal with three variable. But by simple manipulation variable can be reduced to one. By changing the LHS and RHS in Thevenin equivalent

![Fig. S1.4.10](image)

\[
v_1 = \frac{4}{1+1} + \frac{12}{1 + \frac{1}{6}} + \frac{1}{1+2} = 6 \, V
\]

11. (B) If we solve this circuit direct, we have to deal with three variable. But by simple manipulation variable can be reduced to one. By changing the LHS and RHS in Thevenin equivalent

![Fig. S1.4.11](image)

\[
i_t = \frac{20 - 6 - 8}{2k + 4k + 2k} = 0.75 \, mA
\]

12. (B) We Thevenized the left side of \( xx' \) and source transformed right side of \( yy' \)

![Fig. S1.4.12](image)

\[
v_{xx'} = v_{TH} = \frac{4}{1} + \frac{8}{24} = 5 \, V,
\]
\[
R_{TH} = 8 \| (16 + 8) = 6 \Omega
\]

13. (D) Thevenin equivalent seen from terminal \( yy' \) is
\[
v_{yy'} = v_{TH} = \frac{4}{1} + \frac{8}{24} = 7 \, V,
\]
\[
R_{TH} = (8 + 16) \| 8 = 6 \Omega
\]

14. (A)

![Fig. S1.4.14](image)

\[
\left( \frac{ir}{r + 50} \right)^2 50 = 20k, \quad \left( \frac{ir}{r + 200} \right)^2 200 = 20k
\]
\[\Rightarrow r = 100 \, \Omega\]
\[i = 30 \, A, \quad P_{\text{max}} = \frac{(30)^2 \times 100}{4} = 22.5 \, kW\]

15. (C) Thevenized the circuit across \( R, R_{TH} = 2 \Omega \)

![Fig. S1.4.15](image)

16. (A) \( i_c = 10 \, A, \, R_{TH} = 2 \Omega, \)
\[
P_{\text{max}} = \left( \frac{10}{2} \right)^2 \times 2 = 50 \, W
\]
Now in this circuit all straight-through connection have been cut as shown in fig. S1.4.32b

\[ v_a = \frac{6 \times (2 + 3)}{2 + 3 + 1} = 5 \text{ V} \]

33. (B) Since both source have opposite polarity, hence short circuit the all straight-through connection as shown in fig. S1.4.33

\[ v_a = -\frac{6 \times (6 + 3)}{2 + 1} = -4 \text{ V} \]

34. (C) Let Thevenin equivalent of both network

\[
P = \left( \frac{V_{TH}}{R_{TH} + R} \right)^2 \frac{1}{R}
\]

\[
P' = \left( \frac{V_{TH}}{R + \frac{R_{TH}}{2}} \right)^2 \frac{1}{R} = 4 \left( \frac{V_{TH}}{2R + R_{TH}} \right)^2 \frac{1}{R}
\]

Thus \( P < P' < 4P \)

35. (C) \( i_1 = \sqrt{P_1} \) and \( i_2 = \sqrt{P_2} \)

using superposition \( i = i_1 + i_2 = \sqrt{P_1} \pm \sqrt{P_2} \)

36. (C) \( r = \frac{v_{oc}}{i_{ac}} = 1.2 \Omega \)

\[ P = \frac{24^2}{(1.2 + 2)^2} \times 2 = 112.5 \text{ W} \]

37. (B) \( R_{TH} = \frac{v_{oc}}{i_{ac}} = \frac{12}{15} = 8 \Omega \)

38. (A) Let \( \frac{1}{\text{sensitivity}} = \frac{1}{20k} = 50 \mu\text{A} \)

For 0 – 10 V scale \( R_m = 10 \times 20k = 200 \text{ k}\Omega \)

For 0 – 50 V scale \( R_m = 50 \times 20k = 1 \text{ M}\Omega \)

For 4 V reading \( i = \frac{4}{10} \times 50 = 20 \mu\text{A} \)

For 5 V reading \( i = \frac{5}{50} \times 50\mu = 5 \mu\text{A} \)

39. (D) \( v_{10k} = \sqrt{10k \times 3.6m} = 6 \)

\[ v_{30k} = \sqrt{30k \times 4.8m} = 12 \text{ V} \]

\[ 6 = \frac{10}{10 + R_{TH}} v_{TH} \Rightarrow 10v_{TH} = 6R_{TH} + 60 \]

\[ 12 = \frac{30}{30 + R_{TH}} \Rightarrow 5v_{TH} = 2R_{TH} + 60 \]

\( R_{TH} = 30 \text{ k}\Omega \)

40. (D) At \( v = 0 \), \( i_{ac} = 30 \text{ mA} \)

At \( i = 0 \), \( v_{oc} = -3 \text{ V} \)

\( R_{TH} = \frac{v_{oc}}{i_{ac}} = \frac{-3}{30m} = -100 \text{ \Omega} \)

************
1. The natural response of an RLC circuit is described by the differential equation

\[ \frac{d^2v}{dt^2} + 2 \frac{dv}{dt} + v = 0, \quad v(0) = 10, \quad \frac{dv(0)}{dt} = 0. \]

The \( v(t) \) is

(A) \( 10(1 + t)e^{-t} \) V  
(B) \( 10(1 - t)e^{-t} \) V  
(C) \( 10e^{-t} \) V  
(D) \( 10te^{-t} \) V

2. The differential equation for the circuit shown in fig. P1.6.2. is

\[ \frac{d^2i}{dt^2} + 1000 \frac{di}{dt} + 10^8i = 10^8i(t) \]

(A) \( v''(t) + 3000v'(t) + 102 \times 10^8v(t) = 10^8v(t) \)

(B) \( v''(t) + 1000v'(t) + 102 \times 10^8v(t) = 10^8v(t) \)

(C) \( \frac{v''(t)}{10^8} + \frac{2v'(t)}{10^8} + 102v(t) = v(t) \)

(D) \( \frac{v''(t)}{10^8} + \frac{2v'(t)}{10^8} + 198v(t) = v(t) \)

3. The differential equation for the circuit shown in fig. P1.6.3 is

\[ \frac{d^2i}{dt^2} + \frac{100}{3} \frac{di}{dt} + \frac{1}{3}i = -2e^{-t} - 7e^{-3t} \]

(A) \( i''(t) + 1100i'(t) + 11 \times 10^6i = 10^6i(t) \)

(B) \( i''(t) + 1100i'(t) + 11 \times 10^5i = 10^5i(t) \)

(C) \( \frac{i''(t)}{10^6} + \frac{11i'(t)}{10^4} + 1i = i(t) \)

(D) \( \frac{i''(t)}{10^8} + \frac{11i'(t)}{10^4} + 1i = i(t) \)

4. In the circuit of fig. P.1.6.4 \( v_s = 0 \) for \( t > 0 \). The initial condition are \( v(0) = 6 \) V and \( \frac{dv(0)}{dt} = -3000 \) V/s. The \( v(t) \) for \( t > 0 \) is

5. The circuit shown in fig. P1.6.5 has been open for a long time before closing at \( t = 0 \). The initial condition is \( v(0) = 2 \) V. The \( v(t) \) for \( t > 0 \) is

(A) \( 2e^{-100t} + 8e^{-400t} \) V  
(B) \( 6e^{-100t} + 8e^{-400t} \) V  
(C) \( 6e^{-100t} - 8e^{-400t} \) V  
(D) None of the above
Statement for Q.6–7:

Circuit is shown in fig. P.1.6. Initial conditions are $i_1(0) = i_1(0) = 11 \text{ A}$

5. $i_1(1 \text{s}) = ?$
   (A) 0.78 A  (B) 1.46 A  (C) 2.56 A  (D) 3.62 A

7. $i_2(1 \text{s}) = ?$
   (A) 0.78 A  (B) 1.46 A  (C) 2.56 A  (D) 3.62 A

9. $v_c(t) = ?$ for $t > 0$
   (A) $4e^{-1000t} - e^{-2000t} \text{ V}$
   (B) $(3 + 6000t)e^{-2000t} \text{ V}$
   (C) $2e^{-1000t} + e^{-2000t} \text{ V}$
   (D) $(3 - 6000t)e^{-2000t} \text{ V}$

10. The switch of the circuit shown in fig. P.1.6.10 is opened at $t = 0$ after long time. The $v(t)$ for $t > 0$ is

   (A) $4e^{-2t} \sin 2t \text{ V}$  
   (B) $-4e^{-2t} \sin 2t \text{ V}$  
   (C) $4e^{-2t} \cos 2t \text{ V}$  
   (D) $-4e^{-2t} \cos 2t \text{ V}$

11. In the circuit of fig. P.1.6.11 the switch is opened at $t = 0$ after long time. The current $i_L(t)$ for $t > 0$ is

   (A) $e^{-2t}(2 \cos t + 4 \sin t) \text{ A}$  
   (B) $e^{-2t}(3 \sin t - 4 \cos t) \text{ A}$
   (C) $e^{-2t}(-4 \sin t + 2 \cos t) \text{ A}$  
   (D) $e^{-2t}(2 \sin t - 4 \cos t) \text{ A}$

Statement for Q.12–14:

In the circuit shown in fig. P.1.6.12–14 all initial condition are zero.

12. If $i_1(t) = 1 \text{ A}$, then the inductor current $i_L(t)$ is
    (A) $1 \text{ A}$  
    (B) $t \text{ A}$  
    (C) $t + 1 \text{ A}$  
    (D) $0 \text{ A}$

13. If $i_1(t) = 0.5t \text{ A}$, then $i_L(t)$ is
    (A) $0.5t + 3.25 \times 10^{-3} \text{ A}$  
    (B) $2t - 3250 \text{ A}$  
    (C) $0.5t - 0.25 \times 10^{-3} \text{ A}$  
    (D) $2t + 3250 \text{ A}$

14. If $i_1(t) = 2e^{-250t} \text{ A}$ then $i_L(t)$ is
    (A) $\frac{4000}{3}te^{-250t} \text{ A}$  
    (B) $\frac{4000}{3} e^{-250t} \text{ A}$
    (C) $\frac{200}{7} e^{-250t} \text{ A}$  
    (D) $\frac{200}{7} te^{-250t} \text{ A}$
15. The forced response for the capacitor voltage \( v(t) \) is

\[
100 \Omega \quad - v_c + 50 \Omega \quad 20 \text{ mH}
\]

Fig. P1.6.15

(A) \( 0.2t + 1.17 \times 10^{-3} \text{ V} \)  
(B) \( 0.2t - 1.17 \times 10^{-3} \text{ V} \)  
(C) \( 1.17 \times 10^{-3}t - 0.2 \text{ V} \)  
(D) \( 1.17 \times 10^{-3}t + 0.2 \text{ V} \)

16. For a RLC series circuit \( R = 20 \Omega, \ L = 0.6 \text{ H} \), the value of \( C \) will be

[CD =critically damped, OD =over damped, UD =under damped].

<table>
<thead>
<tr>
<th>CD</th>
<th>OD</th>
<th>UD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( C = 6 \text{ mF} )</td>
<td>( C &gt; 6 \text{ mF} )</td>
<td>( C &lt; 6 \text{ mF} )</td>
</tr>
<tr>
<td>(B) ( C = 6 \text{ mF} )</td>
<td>( C &lt; 6 \text{ mF} )</td>
<td>( C &gt; 6 \text{ mF} )</td>
</tr>
<tr>
<td>(C) ( C &gt; 6 \text{ mF} )</td>
<td>( C = 6 \text{ mF} )</td>
<td>( C &lt; 6 \text{ mF} )</td>
</tr>
<tr>
<td>(D) ( C &lt; 6 \text{ mF} )</td>
<td>( C = 6 \text{ mF} )</td>
<td>( C &gt; 6 \text{ mF} )</td>
</tr>
</tbody>
</table>

17. The circuit shown in fig. P1.6.17 is critically damped. The value of \( R \) is

\[
120 \Omega \quad 10 \text{ mF} \quad 4 \text{ H}
\]

Fig. P1.6.17

(A) \( 40 \text{ \Omega} \)  
(B) \( 60 \text{ \Omega} \)  
(C) \( 120 \text{ \Omega} \)  
(D) \( 180 \text{ \Omega} \)

18. The step response of an RLC series circuit is given by

\[
\frac{d^2i(t)}{dt^2} + \frac{2di(t)}{dt} + 5i(t) = 10, \quad i(0^-) = 0, \quad \frac{di(0^-)}{dt} = 4.
\]

The \( i(t) \) is

(A) \( 1 + e^{-t} \cos 4t \text{ A} \)  
(B) \( 4 - 2e^{-t} \cos 4t \text{ A} \)  
(C) \( 2 + e^{-t} \sin 4t \text{ A} \)  
(D) \( 10 + e^{-t} \sin 4t \text{ A} \)

19. In the circuit shown in fig. P 1.5.19 \( v(t) \) for \( t > 0 \) is

\[
2(\text{sin} 2t + 2) \text{ A}
\]

Fig. P1.6.19

(A) \( 50 - (46.5 \sin 3t + 62 \cos 3t)e^{-4t} \text{ V} \)  
(B) \( 50 + (46.5 \sin 3t + 62 \cos 3t)e^{-4t} \text{ V} \)  
(C) \( 50 + (62 \cos 4t + 46.5 \sin 4t)e^{-3t} \text{ V} \)  
(D) \( 50 - (62 \cos 4t + 46.5 \sin 4t)e^{-3t} \text{ V} \)

20. In the circuit of fig. P1.6.20 the switch is closed at \( t=0 \) after long time. The current \( i(t) \) for \( t > 0 \) is

\[
\frac{1}{10} \text{ F}
\]

Fig. P1.6.20

(A) \(-10 \sin 8t \text{ A} \)  
(B) \(10 \sin 8t \text{ A} \)  
(C) \(-10 \cos 8t \text{ A} \)  
(D) \(10 \cos 8t \text{ A} \)

21. In the circuit of fig. P1.6.21 switch is moved from 8 V to 12 V at \( t=0 \). The voltage \( v(t) \) for \( t > 0 \) is

\[
8 \text{ V} \quad 12 \text{ V} \quad 1/6 \text{ F}
\]

Fig. P1.6.21

(A) \( 12 - (4 \cos 2t + 2 \sin 2t)e^{-t} \text{ V} \)  
(B) \( 12 - (4 \cos 2t + 8 \sin 2t)e^{-t} \text{ V} \)  
(C) \( 12 + (4 \cos 2t + 8 \sin 2t)e^{-t} \text{ V} \)  
(D) \( 12 + (4 \cos 2t + 2 \sin 2t)e^{-t} \text{ V} \)
22. In the circuit of fig. P1.5.22 the voltage \( v(t) \) is

\[
A) \ 40 - (20 \cos 0.6t + 15 \sin 0.6t)e^{-0.8t} \ V \\
B) \ 35 + (15 \cos 0.6t + 20 \sin 0.6t)e^{-0.8t} \ V \\
C) \ 35 - (15 \cos 0.6t + 20 \sin 0.6t) e^{-0.8t} \ V \\
D) \ 35 - 15 \cos 0.6t e^{-0.8t} \ V
\]

23. In the circuit of fig. P1.6.23 the switch is opened at \( t = 0 \) after long time. The current \( i(t) \) for \( t > 0 \) is

\[
A) e^{-2.306t} + e^{-0.8693t} \ A \\
B) -e^{-2.306t} + 2e^{-0.8693t} \ A \\
C) e^{-4.431t} + e^{-0.9032t} \ A \\
D) 2e^{-4.431t} - e^{-0.9032t} \ A
\]

24. In the circuit of fig. P1.6.24 switch is moved from position \( a \) to \( b \) at \( t = 0 \). The \( i_L(t) \) for \( t > 0 \) is

\[
A) (4 - 6t)e^{4t} \ A \\
B) (3 - 6t)e^{-4t} \ A \\
C) (3 - 9t)e^{-3t} \ A \\
D) (3 - 8t)e^{-3t} \ A
\]

25. In the circuit shown in fig. P1.6.25 a steady state has been established before switch closed. The \( i(t) \) for \( t > 0 \) is

\[
A) 0.73e^{-2t} \sin 4.58t \ A \\
B) 0.89e^{-2t} \sin 6.38t \ A \\
C) 0.73e^{-4t} \sin 4.58t \ A \\
D) 0.89e^{-4t} \sin 6.38t \ A
\]

26. The switch is closed after long time in the circuit of fig. P1.6.26. The \( v(t) \) for \( t > 0 \) is

\[
A) -8 + 6e^{-3t} \sin 4t \ V \\
B) -12 + 4e^{-3t} \cos 4t \ V \\
C) -12 + (4 \cos 4t + 3 \sin 4t)e^{-3t} \ V \\
D) -12 + (4 \cos 4t + 6 \sin 4t)e^{-3t} \ V
\]

27. \( i(t) = \)
28. In the circuit of fig. P1.6.28 \( i(0) = 1 \text{ A} \) and \( v(0) = 0 \). The current \( i(t) \) for \( t > 0 \) is

\[
\begin{align*}
4\omega(t) \text{ A} & \\
1 \text{ H} & \\
2 \Omega & \\
0.5 \mu \text{ F} & \\
\end{align*}
\]

(A) \( 4 + 6.38 e^{-0.5t} \text{ A} \)  
(B) \( 4 - 6.38 e^{-0.5t} \text{ A} \)  
(C) \( 4 + (3\cos 1.32t + 113\sin 1.32t)e^{-0.5t} \text{ A} \)  
(D) \( 4 - (3\cos 1.32t + 113\sin 1.32t)e^{-0.5t} \text{ A} \)

29. In the circuit of fig. P1.6.29 a steady state has been established before switch closed. The \( v_\text{s}(t) \) for \( t > 0 \) is

\[
\begin{align*}
3 \text{ A} & \\
5 \Omega & \\
10 \text{ mF} & \\
1 \text{ H} & \\
\end{align*}
\]

(A) \( 100te^{-10t} \text{ V} \)  
(B) \( 200te^{-10t} \text{ V} \)  
(C) \( 400te^{-50t} \text{ V} \)  
(D) \( 800te^{-50t} \text{ V} \)

30. In the circuit of fig. P1.6.30 a steady state has been established before switch closed. The \( i(t) \) for \( t > 0 \) is

\[
\begin{align*}
1 \Omega & \\
1 \text{ H} & \\
2 \Omega & \\
6 \text{ V} & \\
\frac{1}{2} \text{ F} & \\
\end{align*}
\]

(A) \( 2e^{-2t}\sin 2t \text{ A} \)  
(B) \( -e^{-2t}\sin 2t \text{ A} \)  
(C) \( -2(1-t)e^{-2t} \text{ A} \)  
(D) \( 2(1-t)e^{-2t} \text{ A} \)

31. In the circuit of fig. P1.6.31 a steady state has been established. The \( i(t) \) for \( t > 0 \) is

\[
\begin{align*}
3 \text{ A} & \\
10 \Omega & \\
6\omega(t) \text{ A} & \\
10 \text{ mF} & \\
40 \Omega & \\
\end{align*}
\]

(A) \( 9 + 2e^{-10t} - 8e^{-2.5t} \text{ A} \)  
(B) \( 9 - 8e^{10t} + 2e^{-2.5t} \text{ A} \)  
(C) \( 9 + (2\cos 10t + \sin 10t)e^{-2.5t} \text{ A} \)  
(D) \( 9 + (\cos 10t + 2\sin 10t)e^{-2.5t} \text{ A} \)

***************

### SOLUTIONS

1. (A) \( s^2 + 2s + 1 = 0 \) \( \Rightarrow \) \( s = -1, -1 \)  
   \( v(t) = (A_1 + A_2t)e^{-t} \)  
   \( v(0) = 10 \text{ V}, \quad \frac{dv(0)}{dt} = 0 = -A_1 + A_2 \)  
   \( A_1 = A_2 = 10 \)

2. (A) \( i_L = \frac{v}{100} + 10 \times 10^{-6} \frac{dv}{dt} \)

\[
\begin{align*}
v_i &= 2i_L + 10^{-3} \frac{di_L}{dt} + v \\
&= 2\left( \frac{v}{100} + 10^{-6} \frac{dv}{dt} \right) + 10^{-3} \left( \frac{1}{100} \frac{dv}{dt} + 10^{-6} \frac{d^2v}{dt^2} \right) + v \\
10^8 v_i(t) &= v(t) + 3000v'(t) + 102v(t)\) 

3. (C) \( i_\text{s} = \frac{v_\text{C}}{100} + i_L + 10\mu \frac{dv_\text{C}}{dt} \)

\[
\begin{align*}
v_\text{C} &= 10i_L + 10^{-8} \frac{di_L}{dt} \\
i_\text{s} &= 0.1i_L + 10^{-8} \frac{di_L}{dt} + i_L + 10^{-8} \frac{d^2i_L}{dt^2} (10i_L + 10^{-8} \frac{di_L}{dt}) \\
&= 0.1i_L + 10^{-8} \frac{di_L}{dt} + i_L + 10^{-8} \frac{di_L}{dt} + 10^{-8} \frac{d^2i_L}{dt^2} \\
&= \frac{i_L(t)}{10^8} + \frac{11}{10^4} i_L(t) + 1.1i_L(t) = i_\text{s}(t) \)

4. (A) \( \frac{v}{80} + 25\mu \frac{dv}{dt} + \int(v - v_\text{s})dt = 0 \)

\[
\begin{align*}
&= \frac{d^2v}{dt^2} + 500 \frac{dv}{dt} + 40000 = 0 \\
s^2 + 500s + 40000 &= 0 \\
&\Rightarrow s = -100, \ -400 \)  
   \( v(t) = Ae^{-100t} + Be^{-400t} \)

A + B = 6, \ -100A - 400B = -3000 \) \( \Rightarrow \) B = 8, \ A = -2

5. (C) The characteristic equation is \( s^2 + \frac{1}{RC} + \frac{1}{LC} = 0 \)

After putting the values, \( s^2 + 4s + 3 = 0 \)

\( \Rightarrow v(t) = Ae^{-t} + Be^{-3t} \)
9. (B) $v_c(0^+) = 3\, \text{V}, \quad i_L(0^+)= -12\, \text{mA}$

$$\frac{v_c}{250} + i_L + 5 \times 10^{-6} \frac{dv_c}{dt} = 0$$

$$\frac{3}{250} - 12m + 5 \times 10^{-6} \frac{dv_c(0^+)}{dt} = 0 \quad \Rightarrow \quad \frac{dv_c(0^+)}{dt} = 0$$

$$s^2 + \frac{s}{250 \times 5 \times 10^{-6}} + \frac{1}{0.8 \times 5 \times 10^{-6}} = 0$$

$$\Rightarrow \quad s = -400 \pm j300$$

$$v_c(t) = e^{-400t} (A_1 \cos 300t + A_2 \sin 300t)$$

$$A_1 = 3, \quad \frac{dv_c(0)}{dt} = -400A_1 + 300A_2, \quad A_2 = 4$$

10. (B) $v(0^+) = 0, \quad i_L(0^+)= 2\, \text{A}, \quad \frac{1}{4} \frac{dv_c(0^+)}{dx} = -2$

$$s^2 + 4s + 8 = 0 \quad \Rightarrow \quad s = -2 \pm j2$$

$$v_c(t) = e^{-2t} (A_1 \cos 2t + A_2 \sin 2t)$$

$$A_1 = 4, \quad \frac{dv_c(0^+)}{dt} = -8 = -2 (0 + 0) + (0 + 2A_2), \quad A_2 = -4$$

11. (D) $i_L(0^+)= -4, \quad v_c(0^+) = 8\, \text{V}$

7. (A) $i_c = Ce^{-\frac{t}{\tau}} + De^{-2t}$

$$i_L(0^+) = 11 \Rightarrow C + D, \quad \frac{di_L(0^+)}{dt} = \frac{-143}{6} = \frac{-C}{6} - 2D$$

$C = -1$ and $D = 12$

$$i_2 = -e^{-\frac{t}{\tau}} + 12e^{-2t} \, \text{A}, \quad i_2(1s) = e^{-\frac{1}{\tau}} + 12e^{-2} = 0.78 \, \text{A}$$

8. (B) $v_c(0^+) = 30\, \text{m} \times 100 = 3\, \text{V}$

$$C \frac{dv_c(0^+)}{dt} = i_L(0^+) = 0 \Rightarrow i_L(0^+) = C\frac{dv_c(0^+)}{dt}$$

$$s^2 + \frac{100}{25 \times 10^{-3}} s + \frac{1}{25 \times 10^{-3} \times 10 \times 10^{-6}}$$

$$\Rightarrow \quad s = -2000, \quad -2000$$

$$v_c(t) = (A_1 + A_2 t)e^{-2000t}$$

$$\frac{dv_c(t)}{dt} = A_1 e^{-2000t} + (A_1 + A_2 t)e^{-2000t}(-2000)$$

$$v_c(0^+) = A_1 = 3, \quad \frac{dv_c(0^+)}{dt} = A_2 - 2000 \times 3 = 0$$

$$\Rightarrow \quad A_2 = 6000$$

12. (A) $i_c = \frac{v}{100/65} + 10^{-3} \frac{dv}{dt} + i_L, \quad v = 10 \times 10^{-3} \frac{di_L}{dt}$

$$i_c = \frac{65}{100} (10 \times 10^{-3}) \frac{di_L}{dt} + 10^{-3} (10 \times 10^{-3}) \frac{d^2i_L}{dt} + i_L = 0$$

$$\frac{d^2i_L}{dt} + 650 \frac{di_L}{dt} + 10^5 i_L = 10^5 i_c$$

Trying $i_L(t) = B$

$$0 + 0 + 10^5 B = 10^5, \quad B = 1, \quad i_L = 1\, \text{A}$$

13. (A) $\frac{\text{Trying}}{\text{at}\:B}$

$$0 + 650 A + (A + B)10^5 = 10^5(0.5t), \quad A = 0.5$$

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24. (C) \( v_C(t) = 0 \), \( i_L(0) = \frac{4 \times 6}{6 + 2} = 3 \)
\[ \frac{d v_C(0)}{dt} = i_L(0) = 3 \Rightarrow \frac{d v_C(0)}{dt} = 150 \]
\[ \alpha = \frac{6 + 14}{2 	imes 2} = 5, \quad \omega = \frac{1}{\sqrt{2 \times 0.02}} = 5 \]
\[ \alpha = \omega \] critically damped
\[ v(t) = 12 + (A + B)t e^{-at} \]
\[ 0 = 12 + A, \quad 150 = -5A + B \Rightarrow A = -12, \quad B = 90 \]
\[ v(t) = 12 + (90t - 12)e^{-at} \]
\[ i_L(t) = 0.02(-5) e^{-5t}(90t - 12) + 0.02(90)e^{-5t} = (3 - 9t)e^{-5t} \]

25. (A) \( v(0^+) = \frac{100 \times 5}{5 + 5 + 20} = \frac{50}{3} \), \( i_L(0^+) = 0 \)
\[ i_f = 0 \quad A \]
\[ \frac{d i_L(0^+)}{dt} = 20 - \frac{50}{3} = \frac{10}{3} \]
\[ \alpha = \frac{4}{2 \times 1} = 2, \quad \omega = \frac{1}{\sqrt{1 \times 25}} = 5 \]
\[ s = -2 \pm \sqrt{4 - 25} = -2 \pm j 4.58 \]
\[ i(t) = (A \cos 4.58t + B \sin 4.58t)e^{-2t} \]

26. (A) \( i_L(0^+) = 0 \), \( v_L(0^+) = 4 - 12 = -8 \)
\[ \frac{d v_L(0^+)}{dt} = i_L(0^+) = 0 \]
\[ \alpha = \frac{6}{2} = 3, \quad \omega = \frac{1}{\sqrt{1 \times 25/2}} = 5 \]
\[ \beta = -3 \pm \sqrt{9 - 25} = -3 \pm j 4 \]
\[ v(t) = -12 + (A \cos 4t + B \sin 4t)e^{-\beta t} \]
\[ v_L(t) = -8 = 12 + A, \quad \Rightarrow A = 4 \]
\[ \frac{d v_L(0)}{dt} = -3A + 4B, \quad \Rightarrow B = 3 \]

27. (C) \[ \alpha = \frac{1}{2RC} = \frac{1}{2 \times 2 \times 254} = 50 \]
\[ W_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8m \times 3m}} = 5000 \]
\[ \alpha < W_o \] underdamped response.
\[ s = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j 5000 \]
\[ i(t) = 6 + (A \cos 5000t + B \sin 5000t)e^{-50t} \text{ mA} \]
\[ i(0) = 6 = 6 + A, \quad \Rightarrow A = -6 \]
\[ \frac{d i(0)}{dt} = -50A + 5000B = 0, \quad B = -0.06 \]

28. (D) \[ i(0^+) = 1 \text{ A}, \quad v(0^+) = \frac{Ldi(0^+)}{dt} \]
\[ \alpha = \frac{1}{2 \times 2 \times 0.5} = 0.5, \quad W_o = \frac{1}{\sqrt{1 - 0.5}} = \sqrt{2} \]
\[ s = -0.5 \pm \sqrt{0.5^2 - 2} = 0.5 \pm j 1.323 \]
\[ i(t) = 4 + (A \cos 1.32t + B \sin 1.32t)e^{-0.5t} \]
\[ 1 = 4 + A, \quad \Rightarrow A = -3 \]
\[ \frac{d i(0)}{dt} = 0 = 0.5A + 1.32B, \quad B = -1.13 \]

29. (B) \[ V_o(0^+) = 0, \quad i_L(0^+) = 1 \text{ A} \]
\[ \frac{d i(0^+)}{dt} = v_i(0) = 0 \]
\[ \alpha = \frac{1}{2 \times 5 \times 0.01} = 10, \quad W_o = \frac{1}{\sqrt{1 \times 0.01}} = 10 \]
\[ \alpha = W_o \], so critically damped response
\[ s = -10, -10 \]
\[ i(t) = 3(A + B)te^{-10t}, \quad i(0) = 1 = 3 + A \]
\[ \frac{d i(0^+)}{dt} = -10A + B \]
\[ i_L(t) = 3 - (2 + 20t)e^{-10t}, \quad v_o = \frac{Ldi_i(t)}{dt} = 200te^{-10t} \]

30. (C) \[ i(0^+) = -\frac{6}{1 + 2} = -2 \text{ A}, \quad v_r(0^+) = 2 \times 1 = 2 \]
\[ \frac{d i(0^+)}{dt} = \frac{1}{2RC} = \frac{1}{2 \times 1 \times 0.25} = 2, \quad W_o = \frac{1}{\sqrt{LC}} = 2 \]
\[ \alpha = W_o \], critically damped response
\[ s = -2, -2 \]
\[ i(t) = (A + B)te^{-2t}, \quad A = -2 \]
\[ \frac{d i(t)}{dt} = (-2 + B)te^{-2t}(-2) + (0 + B)e^{-2t} \]
\[ At = 0, \Rightarrow B = -2 \]

31. (A) \[ i(0^+) = 3 \text{ A}, \quad v_C(0^+) = 0 \text{ V} = \text{4} \frac{d i(0^+)}{dt} \]
\[ i_1 = 9 \text{ A}, \quad R = 10 \Omega, \quad 40 = 8 \Omega \]
\[ \alpha = \frac{1}{2RC} = \frac{1}{2 \times 8 \times 0.01} = 625 \]
\[ W_o = \frac{1}{\sqrt{4 \times 0.01}} = 5 \]
\[ \alpha > W_o \], so overdamped response
\[ s = -625 \pm \sqrt{625^2 - 25} = -10, -25 \]
\[ i(t) = 9 + Ae^{-10t} + Be^{-25t} \]
\[ 3 = 9 + A + B, \quad 0 = -10A - 25B \]
On solving, \( A = 2, \quad B = -8 \)

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CHAPTER 1.7

SINUSOIDAL STEADY STATE ANALYSIS

1. \( i(t) = ? \)
   - (A) \( 20 \cos (300t + 68.2^\circ) \) A
   - (B) \( 20 \cos (300t - 68.2^\circ) \) A
   - (C) \( 2.48 \cos (300t + 68.2^\circ) \) A
   - (D) \( 2.48 \cos (300t - 68.2^\circ) \) A

2. \( v_c(t) = ? \)
   - (A) \( 25 \cos 300V \) t
   - (B) \( 25 \cos 10A \) t
   - (C) \( 2 \cos 2V \) t
   - (D) \( 2 \cos 5V \) t

3. \( i(t) = ? \)
   - (A) \( \frac{1}{\sqrt{2}} \cos (2t - 45^\circ) \) V
   - (B) \( \frac{1}{\sqrt{2}} \cos (2t + 45^\circ) \) V
   - (C) \( \frac{1}{\sqrt{2}} \sin (2t - 45^\circ) \) V
   - (D) \( \frac{1}{\sqrt{2}} \sin (2t + 45^\circ) \) V

4. \( v_c(t) = ? \)
   - (A) \( 2.25 \cos (5t + 150^\circ) \) V
   - (B) \( 2.25 \cos (5t - 150^\circ) \) V
   - (C) \( 2.25 \cos (5t + 140.71^\circ) \) V
   - (D) \( 2.25 \cos (5t - 140.71^\circ) \) V

5. \( i(t) = ? \)
   - (A) \( 2 \sin (2t + 5.77^\circ) \) A
   - (B) \( \cos (2t - 84.23^\circ) \) A
   - (C) \( 2 \sin (2t - 5.77^\circ) \) A
   - (D) \( \cos (2t + 84.23^\circ) \) A
13. In the bridge shown in fig. P1.7.13, \(Z_1 = 300 \Omega\), \(Z_2 = (300 - j600) \Omega\), \(Z_3 = (200 + j100) \Omega\). The \(Z_4\) at balance is

(A) 400 + j300 \(\Omega\)  
(B) 400 - j300 \(\Omega\)  
(C) j100 \(\Omega\)  
(D) -j900 \(\Omega\)

14. In a two element series circuit, the applied voltage and the resulting current are \(v(t) = 60 + 66\sin(10^3t)\) V, \(i(t) = 2.3\sin(10^3t + 68.3^\circ)\) A. The nature of the elements would be

(A) \(R - C\)  
(B) \(L - C\)  
(C) \(R - L\)  
(D) \(R - R\)

15. \(V_o = \) ?

(A) 223 \(\angle -56^\circ\) V  
(B) 223\(\angle 56^\circ\) V  
(C) 124 \(\angle -154^\circ\) V  
(D) 124\(\angle 154^\circ\) V

16. \(v_o(t) = \) ?

(A) 315 \(\cos(t + 112^\circ)\) V  
(B) 43.2 \(\cos(t + 23^\circ)\) V  
(C) 315 \(\cos(t - 112^\circ)\) V  
(D) 43.2 \(\cos(t - 23^\circ)\) V

17. \(i_1(t) = \) ?

(A) 2.36 \(\cos(4t - 4107^\circ)\) A  
(B) 2.36 \(\cos(4t + 4107^\circ)\) A  
(C) 1.37 \(\cos(4t - 4107^\circ)\) A  
(D) 2.36 \(\cos(4t + 4107^\circ)\) A

18. \(i_2(t) = \) ?

(A) 2.04 \(\sin(4t + 92.13^\circ)\) A  
(B) -2.04 \(\sin(4t + 21.3^\circ)\) A  
(C) 2.04 \(\cos(4t + 21.3^\circ)\) A  
(D) -2.04 \(\cos(4t + 92.13^\circ)\) A

19. \(I_x = \) ?

(A) 394\(\angle 46.28^\circ\) A  
(B) 4.62\(\angle 97.38^\circ\) A  
(C) 7.42\(\angle 92.49^\circ\) A  
(D) 6.78\(\angle 49.27^\circ\) A

20. \(V_x = \) ?

(A) 29.11\(\angle 166^\circ\) V  
(B) 29.11\(\angle -166^\circ\) V  
(C) 43.24\(\angle 124^\circ\) V  
(D) 43.24\(\angle -124^\circ\) V
**Statement for Q.27–32:**

Determine the complex power for the given values in question.

27. \( P = 269 \text{ W}, Q = 150 \text{ VAR (capacitive)} \)
   (A) 150 - j269 VA  (B) 150 + j269 VA  
   (C) 269 - j150 VA  (D) 269 + j150 VA

28. \( Q = 2000 \text{ VAR, } \text{pf} = 0.9 \) (leading)
   (A) 4129.8 + j2000 VA  (B) 2000 + j4129.8 VA  
   (C) 2000 - j4129.8 VA  (D) 4129.8 - j2000 VA

29. \( S = 60 \text{ VA}, Q = 45 \text{ VAR (inductive)} \)
   (A) 39.69 + j45 VA  (B) 39.69 - j45 VA  
   (C) 45 + j39.69 VA  (D) 45 - j39.69 VA

30. \( V_{rms} = 220 \text{ V}, P = 1 \text{ kW}, lZ = 40 \Omega \) (inductive)
   (A) 1000 - j68125 VA  (B) 1000 + j68125 VA  
   (C) 68125 + j1000 VA  (D) 68125 - j1000 VA

31. \( V_{rms} = 21^\circ 20^\circ \text{ V}, V_{rms} = 21^\circ 20^\circ \text{ V, } I_{rms} = 8.5^\circ - 50^\circ \text{ A} \)
   (A) 154.6 + j89.3 VA  (B) 154.6 - j89.3 VA  
   (C) 61 + j167.7 VA  (D) 61 - j167.7 VA

32. \( V_{rms} = 120^\circ 30^\circ \text{ V}, Z = 40 + j80 \Omega \)
   (A) 72 + j144 VA  (B) 72 - j144 VA  
   (C) 144 + j72 VA  (D) 144 - j72 VA

33. \( V_o = ? \)

   ![Fig. P1.7.33](image)
   (A) 7.1^\circ 32.29^\circ \text{ kV}  (B) 42.59^\circ 32.29^\circ \text{ kV}  
   (C) 38.49^\circ 24.39^\circ \text{ kV}  (D) 38.49^\circ 32.29^\circ \text{ kV}

34. A relay coil is connected to a 210 V, 50 Hz supply. If it has resistance of 30 \( \Omega \) and an inductance of 0.5 H, the apparent power is
   (A) 30 VA  (B) 275.6 VA  
   (C) 157 VA  (D) 187 VA

35. In the circuit shown in Fig. P1.7.35 power factor is
   (A) 56.31 (leading)  (B) 56.31 (lagging)  
   (C) 0.555 (lagging)  (D) 0.555 (leading)

36. The power factor seen by the voltage source is
   (A) 0.8 (leading)  (B) 0.8 (lagging)  
   (C) 36.9 (leading)  (D) 39.6 (lagging)

37. The average power supplied by the dependent source is
   (A) 96 W  (B) -96 W  
   (C) 92 W  (D) -192 W

38. In the circuit of Fig. P1.7.38 the maximum power absorbed by \( Z_L \) is
   (A) 180 W  (B) 90 W  
   (C) 140 W  (D) 700 W
39. The value of the load impedance, that would absorb the maximum average power is

\[ Z_L = j \frac{100}{100} - j40 \Omega \]

The line impedance connecting the source to load is 0.3 + j0.2Ω. If the current in a phase of load 1 is \( I = 10 \angle 20^\circ A_{\text{rms}} \), the current in source in ab branch is

(A) 15 \angle -122^\circ A_{\text{rms}}  
(B) 8.67 \angle -122^\circ A_{\text{rms}}  
(C) 15 \angle 279^\circ A_{\text{rms}}  
(D) 8.67 \angle -579^\circ A_{\text{rms}}

40. If phase sequence is abc then phase voltage \( V_a, V_b, \) and \( V_c \) are respectively

(A) 231 \angle 0^\circ, 231 \angle 120^\circ, 231 \angle 240^\circ  
(B) 231 \angle -30^\circ, 231 \angle -150^\circ, 231 \angle 90^\circ  
(C) 231 \angle 30^\circ, 231 \angle 150^\circ, 231 \angle -90^\circ  
(D) 231 \angle 60^\circ, 231 \angle 180^\circ, 231 \angle -60^\circ

41. If phase sequence is acb then phase voltage are

(A) 231 \angle 0^\circ, 231 \angle 120^\circ, 231 \angle 240^\circ  
(B) 231 \angle -30^\circ, 231 \angle -150^\circ, 231 \angle 90^\circ  
(C) 231 \angle 30^\circ, 231 \angle 150^\circ, 231 \angle -90^\circ  
(D) 231 \angle 60^\circ, 231 \angle 180^\circ, 231 \angle -60^\circ

42. A balanced three-phase Y-connected load has one phase voltage \( V_c = 277 \angle 45^\circ V \). The phase sequence is \( abc \). The line to line voltage \( V_{ab} \) is

(A) 480 \angle 45^\circ V  
(B) 480 \angle -45^\circ V  
(C) 339 \angle 45^\circ V  
(D) 339 \angle -45^\circ V

43. A three-phase circuit has two parallel balanced \( \Delta \) loads, one of the 6\( \Omega \) resistor and one of 12\( \Omega \) resistors. The magnitude of the total line current, when the line-to-line voltage is 480 V_{\text{rms}}, is

(A) 120 A_{\text{rms}}  
(B) 360 A_{\text{rms}}  
(C) 208 A_{\text{rms}}  
(D) 470 A_{\text{rms}}

44. In a balanced three-phase system, the source has an \( abc \) phase sequence and is connected in delta. There are two parallel Y-connected load. The phase impedance of load 1 and load 2 is \( 4 + j4 \Omega \) and \( 10 + j4 \Omega \) respectively.
1. (D) \[ Z = 3 + j(25m)(300) = 3 + j7.5 \Omega = 8.08 \angle 68.2^\circ \]
\[ I = \frac{20}{8.08 \angle 68.2^\circ} = 2.48 \angle -68.2^\circ \text{ A} \]
\[ i(t) = 2.48 \cos (300t - 68.2^\circ) \text{ A} \]

2. (A) \[ Y = \frac{1}{2} + j(1m)(10^3) = 0.5 + j = 1.12 \angle 63.43^\circ \]
\[ V_c = \frac{(1 \angle 0)}{112 \angle 63.43^\circ} = 0.89 \angle -63.43^\circ \text{ V} \]
\[ v_c(t) = 0.89 \cos (10^3t - 63.43^\circ) \text{ V} \]

3. (A) \[ Z = 5 + \frac{-j}{(0.1)(2)} = 5 - j5 = 5\sqrt{2} \angle -45^\circ \]
\[ V_c = \frac{(1 \angle 0)(5 \angle -90^\circ)}{5\sqrt{2} \angle -45^\circ} = \frac{1}{\sqrt{2}} \angle -45^\circ \text{ V} \]
\[ v_c(t) = \frac{1}{\sqrt{2}} \cos (2t - 45^\circ) \text{ V} \]

4. (D) \[ Z = 9 + j(3)(5) + \frac{-j}{(50m)(5)} = 9 + j11 \]
\[ \Rightarrow Z = 14.21 \angle 50.71^\circ \Omega \]
\[ V_c = \frac{(8 \angle 0)(4 \angle -90^\circ)}{14.21 \angle 50.71^\circ} = 2.25 \angle 140.71^\circ \text{ V} \]
\[ v_c(t) = 2.25 \cos (5t - 140.71^\circ) \text{ V} \]

5. (B) \[ V_a = \frac{10 \angle 0}{1 + \frac{1}{1} + \frac{1}{4 + j8}} = \frac{10 \angle 0}{105 + j0.4} \text{ V} \]
\[ I = \frac{V_a}{4 + j8} = \frac{10 \angle 0}{1 + j10} = 1 \angle -84.23^\circ \text{ A} \]

6. (D) \[ \omega = 2\pi \times 10 \times 10^3 = 2\pi \times 10^4 \]
\[ Y = j(1\mu)(2\pi \times 10^4) + \frac{-j}{(160\mu)(2\pi \times 10^4)} + \frac{1}{36} \]
\[ = 0.0278 - j0.0366 \text{ S} \]
\[ Z = \frac{1}{Y} = 1316 + j17.33 \Omega \]

7. (C) \[ Z = \left( \frac{-j}{\omega 22\mu} \right) || (6 + j(27m)\omega) \]
\[ = \frac{-j10^6}{22\omega} (6 + j27 \times 10^3 \omega) = \frac{27 \times 10^3 - j6 \times 10^6}{22} \]
\[ = \frac{6 + j(27m\omega - 10^6)}{22\omega} + j60 \left( 27m - \frac{10^6}{22\omega^2} \right) = 0 \]
\[ \Rightarrow \omega = 1278 \]
\[ f = \frac{\omega}{2\pi} \text{ Hz} = \frac{1278}{2\pi} = 203 \text{ Hz} \]

8. (C) \[ V_s \angle 47^\circ \text{ V}, V_z \angle 35^\circ \text{ V} \]
\[ V_i = V_s - V_z = 7.68 \angle 47^\circ - 7.51 \angle 35^\circ = 159 \angle 125^\circ \]

9. (B) \[ v_{in} = \sqrt{3^2 + (14 - 10)^2} \]

10. (C) \[ I_1 = 744 \angle -118^\circ \text{ mA} \]
\[ I_2 = 540 \angle 100^\circ \text{ mA} \]
\[ I = I_1 + I_2 = 744 \angle -118^\circ + 540.5 \angle 100^\circ \]
\[ = 460 \angle -164^\circ \text{ A} \]
\[ i(t) = 460 \cos (3t - 164^\circ) \text{ mA} \]

11. (A) \[ \sqrt{2} \angle 45^\circ = \frac{V_c}{-j4} + \frac{V_c - 20 \angle 0}{j5 + 10} \]

12. (D) \[ X = X_L + X_C = 0 \]
So reactive power drawn from the source is zero.

13. (B) \[ Z_L Z_s = Z_s Z_L \]
\[ 300 Z_s = (300 - j600)(200 + j100) \]
\[ \Rightarrow Z_s = 400 - j300 \]

14. (A) \[ R - C \] causes a positive phase shift in voltage
\[ Z = |Z| \angle 0, -90^\circ < \theta < 0, \]
\[ I = \frac{V}{|Z|} \angle \theta \]

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15. (C) \[ V_s = \frac{120 \angle 15^\circ}{40 + j20} - \frac{6 \angle 30^\circ}{40 + j20} + \frac{1}{1 - j30} + \frac{1}{50} = 124 \angle 154^\circ \]

16. (C) \[ 10 \sin (t + 30^\circ) = 10 \cos (t - 60^\circ) \]
\[ V_s = \left( \frac{j}{j} \right) \frac{1}{1 + \frac{1}{j} + \frac{1}{3}} \]
\[ = 30 \angle 150^\circ + 20 \angle -45^\circ \]
\[ V_s = 315 \angle -112^\circ \text{ V} \]

17. (C) \[ 5 \angle 0^\circ = I_1 \left( j4 + 1 + \frac{j}{4} \right) - I_2 \left( 1 - \frac{j}{4} \right) \]

\[ \Rightarrow (8 + j15)I_1 - (4 - j)I_2 = 20 \angle 0^\circ \quad \ldots \text{(i)} \]
\[ -10 \angle -30^\circ = I_2 (1 + j4 + 1 - \frac{j}{4}) - I_1 (1 - \frac{j}{4}) \]
\[ \Rightarrow (4 - j)I_1 - (8 + j15)I_2 = 40 \angle -30^\circ \quad \ldots \text{(ii)} \]
\[ I_1 [8 \angle 0^\circ] - (4 - j)I_2 = 40 \angle -30^\circ \]
\[ -176 + j248 = 41.43 + j414.64 \]
\[ \Rightarrow I_1 = 103 - j0.9 = 1.37 \angle -41.07 \]

18. (B) \[ I_2 = \frac{(8 + j15)(103 - j0.9) - 20 \angle 0^\circ}{4 - j} \]
\[ = -0.076 + j2.04 \quad \Rightarrow I_2 = 2.04 \angle 92.13^\circ \]

19. (B) \[ 10 \angle 30^\circ = 4I_1 - 0.5I_3 + (-j2)I_x \]
\[ (-j2)I_x = (I_1 - I_3)j3, \quad I_1 = \frac{I_x}{3} \]
\[ 10 \angle 30^\circ = \left( \frac{4}{3} - 0.5 - j2 \right)I_x \quad \Rightarrow I_x = 10 \angle 30^\circ \]
\[ 2.17 \angle -67.38^\circ \]

20. (B) Let \( V_o \) be the voltage across current source
\[ \frac{V_o - 4V_x + V_x - V_4}{20} = 3 \]
\[ V_o(20 + j10) - (20 + j40)V_4 = j600 \]
\[ V_s = \frac{V_x(20)}{20 + j10}, \quad V_o = \frac{V_x}{2}(2 + j) \]
\[ V_s = \left( \frac{2 + j)(20 + j10)}{2} - 20(1 + j2) \right) = j600 \]
\[ V_s = \frac{-j600}{-5 - j20} = 29.22 \angle -166^\circ \]

21. (A) \[ I_1 = V_o \left( \frac{j}{2} \right) + \frac{V_s - V_x}{j4} = j0.1V_x + j0.4V_3 \]
\[ = (0.1 \angle 90^\circ)(0.757 \angle 66.7^\circ) + (0.4 \angle 90^\circ)(0.606 \angle -69.8^\circ) \]
\[ \Rightarrow I_1 = 0.196 \angle 35.6^\circ \]

22. (A) \[ \frac{V_o}{2} + \frac{V_x - 3V_s}{j4} = 4 \angle -30^\circ \]
\[ V_o(0.5 + j0.5) = 3.46 - j2 \quad \Rightarrow V_o = 5.65 \angle -75^\circ \]

23. (D) \[ I_2 = 4 \angle 90^\circ, \quad I_3 = 2 \angle 90^\circ \]

24. (D) \[ I_2 = 3 \angle 0^\circ \text{ A}, \quad I_4 - I_3 = 6 \angle 0^\circ \text{ A} \]
\[ I_1 + (I_1 - I_a)(-j4) + (I_1 + 60^\circ + 3\angle 0^\circ)(j2) + I_3 = 60^\circ = 0 \]
\[ I_2(-j2) + I_4(j4) = -18j - 6 \]
\[ I_2 = -I_1(-j2) = 3 - 9j \]
\[ \Rightarrow I_3 = I_3 + 3 - 6j \]
\[ 15 \angle 90^\circ = (I_3 + 3\angle 0^\circ)(2) + (I_3 - I_3)(-j4) \]
\[ \Rightarrow j15 = 2I_3 + 6 + (j4)(3 - 6j) \]

25. (A) \[ Z_{th} = (j10)(8 - j5) = 9 + j4.4 \]
\[ V_{th} = \frac{(32\angle 0^\circ)(j10)}{8 + j10 - j5} = 339\angle 58^\circ \text{ V} \]

26. (D) \[ (600 - j300)I_1 + j300I_2 = 9 \] ...(i)
\[ 300I_2 = 3V_1, \quad V_1 = (j300)(I_1 - I_2) \]
\[ \Rightarrow I_2 = -j3(I_1 - I_2) \quad \Rightarrow \quad 3I_1 = (3 + j)I_2 \] ...(ii)
Solving (i) and (ii)
\[ I_3 = 12.36 \angle -16^\circ \text{ mA} \]
\[ V_w = 300I_2 = 371 \angle -16^\circ \]
\[ -2V_1 - V_1 = 0 \quad \Rightarrow \quad V_1 = 0 \]
\[ V_4 = \frac{9\angle 0^\circ}{600} = 15 \angle 0^\circ \text{ mA} \]
\[ Z_{th} = \frac{V_w}{I_w} = \frac{311 \angle -16^\circ}{15 \angle 0^\circ \times 10^{-3}} = 247 \angle -16^\circ \Omega \]

27. (C) \[ S = P - jQ = 269 - j150 \text{ VA} \]

28. (D) \[ pf = \cos \theta = 0.9 \quad \Rightarrow \quad \theta = 25.84^\circ \]
\[ Q = S \sin \theta \quad \Rightarrow \quad S = \frac{Q}{\sin \theta} = \frac{2000}{\sin 25.84^\circ} = 4588.6 \text{ VA} \]
\[ P = S \cos \theta = 4129.8, \]
\[ S = 4129.8 - j2000 \]

29. (A) \[ Q = S \sin \theta \quad \Rightarrow \quad \sin \theta = \frac{Q}{S} = \frac{45}{60} \text{ or } \]
\[ \Rightarrow \quad \theta = 48.59^\circ, \]
\[ P = S \cos \theta = 39.69, \]
\[ S = 39.69 + j45 \text{ VA} \]

30. (B) \[ S = \frac{|V_w|^2}{|Z|} = \frac{(220^2)}{40} = 1210 \]
\[ \cos \theta = \frac{P}{S} = \frac{1000}{1210} = 0.8264 \text{ or } \theta = 34.26^\circ, \]
\[ Q = S \sin \theta = 68125, \]
\[ S = 1000 + j68125 \text{ VA} \]

31. (C) \[ S = V_{rms}I_{rms} = (21\angle 20^\circ)(8.5\angle 50^\circ) \]
\[ = 61 + j167.7 \text{ VA} \]

32. (A) \[ S = \frac{|V|^2}{Z} = \frac{(120)^2}{40 - j80} = 72 + j144 \text{ VA} \]

33. (A) \[ S_1 = 16 + j\frac{16}{0.9} \sin (\cos^{-1} (0.9)) = 16 + j7.75 \]
\[ S_2 = 20 + j\frac{20}{0.8} \sin (\cos^{-1} (0.8)) = 20 + j15 \]
\[ S = S_1 + S_2 = 36 + j2.75 = 42.59 \angle 32.29^\circ \]
\[ S = V_I I^* = 6V_o \quad \Rightarrow \quad V_o = 7.1 \angle 32.29^\circ \]

34. (B) \[ Z = 30 + j(0.5)(2\pi)(50) = 30 + j157, \]
\[ S = \frac{|V|^2}{Z} = \frac{(210)^2}{30 - j157} \]
\[ \text{Apparent power} = |S| = \frac{(210)^2}{\sqrt{30^2 + 152^2}} = 275.6 \text{ VA} \]

35. (D) \[ Z = 4 + \frac{(-j2)(j5 - j2)}{-j2 + j5 - j2} \]
\[ = 4 - j6 = 7.21 \angle -56.31^\circ, \]
\[ pf = \cos 56.31^\circ = 0.555 \quad \text{leading} \]

36. (A) \[ \frac{V}{4} + \frac{3}{4} \frac{V}{1 - j15} \quad \Rightarrow \quad V = 4 \angle 36.9^\circ, \]

37. (A) \[ (2 \angle -90^\circ)4.8 = -I_s(4.8 + j192) + 0.6I_s(8) \]

\[ I_s = 5 \angle 0^\circ, \]
\[ V_a = 0.6 \times 5 \times 8 = 24 \angle 0^\circ, \]
\[ P_{ave} = \frac{1}{2} \times 24 \times 1.6 \times 5 = 96 \]
38. (A) \[ Z_{TH} = \frac{(-j10)(10 + j15)}{10 + j15 - j10} = 8 - j14 \, \Omega \]
\[ V_{TH} = \frac{120(-j10)}{10 + j5} = 107.3 \angle -116.6^\circ \, V \]
\[ I_L = \frac{107.3 \angle -116.6^\circ}{16} = 6.7 \angle -116.6^\circ \]
\[ P_{L_{\text{max}}} = \frac{1}{2} (6.7)^2 \times 8 = 180 \, \text{W} \]

39. (B) \[ Z_{TH} = \frac{(-j40)(80 + j100)}{80 + j60} = 12.8 - j49.6 \, \Omega \]

40. (B) \[ V_a = \frac{400}{\sqrt{3}} \angle -30^\circ = 231 \angle -30^\circ \, V \]
\[ V_b = 231 \angle -150^\circ \, V, \quad V_c = 231 \angle -270^\circ \, V \]

41. (C) For the abc sequence
\[ V_{ab} = V_a - V_b = V_p \angle 0^\circ - V_p \angle 120^\circ \]
\[ 400 = V_p \left(1 + \frac{1}{2} - j \frac{\sqrt{3}}{2}\right) = V_p \sqrt{3} \angle -30^\circ \]
\[ \Rightarrow \quad V_p = \frac{400}{\sqrt{3}} \angle 30^\circ \]
\[ V_a = V_p \angle 0^\circ = 231 \angle 30^\circ \, V, \]
\[ V_b = V_p \angle 120^\circ = 231 \angle 150^\circ \, V \]
\[ V_c = V_p \angle 240^\circ = 231 \angle -90^\circ \, V \]

42. (B) \[ V_A = 277 \angle (45^\circ - 120^\circ) = 277 \angle -75^\circ \, V \]
\[ V_B = 277 \angle (45^\circ + 120^\circ) = 277 \angle 165^\circ \, V \]
\[ V_{AB} = V_A - V_B = 480 \angle -45^\circ \, V \]

43. (C) \[ Z_A = 6 \parallel 12 = 4, \]
\[ I_p = \frac{480}{4} = 120 \, A_{\text{rms}} \]
\[ I_L = \sqrt{3} I_p = 208 \, A_{\text{rms}} \]

44. (B) \[ I = \frac{I_{ab}(10 + j4)}{(10 + j4) + (4 + j4)} = 10 \angle 20^\circ \]

45. (D) \[ I_{AB} = \frac{I_{ab}}{\sqrt{3}} \angle (0 + 30^\circ) = 16.22 \angle 134^\circ \, A_{\text{rms}} \]
\[ V_{AB} = I_{AB} \cdot Z_A = (16.22 \angle 134^\circ)(10 + j8) = 207.8 \angle 40^\circ \, V_{\text{rms}} \]

46. (B) \[ |S| = \frac{\sqrt{3} V_L I_L}{I_L} = 10 \, V_{\text{rms}} \]
\[ Z_Y = \frac{208}{10 \sqrt{3}} \angle 25^\circ = 12 \angle 25^\circ = 10.88 + j5.07 \, \Omega \]

**********
1. Z(s) = ?

![Fig. P1.8.1](image1)

(A) \( \frac{s^2 + 15s + 1}{s(s + 1)} \)

(B) \( \frac{s^2 + 3s + 1}{s(s + 1)} \)

(C) \( \frac{2s^2 + 3s + 2}{s(s + 1)} \)

(D) \( \frac{2s^2 + 3s + 1}{2s(s + 1)} \)

2. Z(s) = ?

![Fig. P1.8.2](image2)

(A) \( \frac{s^2 + s + 1}{s(s + 1)} \)

(B) \( \frac{2s^2 + s + 1}{s(s + 1)} \)

(C) \( \frac{s(s + 1)}{2s^2 + s + 1} \)

(D) \( \frac{s(s + 1)}{s^2 + s + 1} \)

3. Z(s) = ?

![Fig. P1.8.3](image3)

(A) \( \frac{s^2 + 1}{s^2 + 2s + 1} \)

(B) \( \frac{2(s^2 + 1)}{(s + 1)^2} \)

(C) \( \frac{2s^2 + 1}{s^2 + 2s + 2} \)

(D) \( \frac{s^2 + 1}{3s + 2} \)

4. Z(s) = ?

![Fig. P1.8.4](image4)

(A) \( \frac{3s^2 + 8s + 7}{s(5s + 6)} \)

(B) \( \frac{s(5s + 6)}{3s^2 + 8s + 7} \)

(C) \( \frac{3s^2 + 7s + 6}{s(5s + 6)} \)

(D) \( \frac{3s^2 + 7s + 6}{3s^2 + 7s + 6} \)

5. The s-domain equivalent of the circuit of Fig.P1.8.5 is

![Fig. P1.8.5](image5)

(A) Both A and B

(B) None of these

(C) Both A and C
3. The $s$–domain equivalent of the circuit shown in Fig. P1.8.6 is

![Fig. P1.8.6](image)

(A) Both A and B (B) None of these

---

8. The circuit is as shown in fig. P1.8.7–8. Solve the problem and choose correct option.

![Fig. P1.8.7–8](image)

7. $H_s(s) = \frac{V_s(s)}{V_o(s)} = ?$
   (A) $s^3 + 2s^2 + 3s + 1)^{-1}$
   (B) $s^3 + 3s^2 + 2s + 1)^{-1}$
   (C) $s^3 + 2s^2 + 3s + 2)^{-1}$
   (D) $s^3 + 3s^2 + 2s^2 + 2)^{-1}$

9. $H_s(s) = \frac{I_s(s)}{V_o(s)} = ?$
   (A) $\frac{-s}{s^3 + 3s^2 + 2s + 1}$
   (B) $-(s^3 + 3s^2 + 2s + 1)^{-1}$
   (C) $\frac{-s}{s^3 + 2s^2 + 3s + 1}$
   (D) $(s^3 + 2s^2 + 3s + 2)^{-1}$

3. For the network shown in fig. P1.8.9 voltage ratio transfer function $G_{12}$ is

![Fig. P1.8.9](image)

(A) $\frac{(s^2 + 2)}{5s^4 + 5s^2 + 1}$
   (B) $\frac{s^2 + 1}{5s^4 + 5s^2 + 1}$
   (C) $\frac{(s^2 + 1)^2}{5s^4 + 5s^2 + 1}$
   (D) $\frac{(s^2 + 1)^2}{5s^4 + 5s^2 + 1}$

10. For the network shown in fig. P1.8.10, the admittance transfer function is

![Fig. P1.8.10](image)

$Y_{12} = \frac{K(s + 1)}{(s + 2)(s + 4)}$

The value of $K$ is

(A) -3 (B) 3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

11. In the circuit of fig. P1.8.11 the switch is in position 1 for a long time and thrown to position 2 at $t = 0$. The equation for the loop currents $I_1(s)$ and $I_2(s)$ are

![Fig. P1.8.11](image)

$\begin{bmatrix} 2 + \frac{3s}{s} & -3s \end{bmatrix} I_1(s) = \begin{bmatrix} \frac{12}{s} \end{bmatrix}$
12. In the circuit of fig. P1.8.12 at terminal \( ab \) Thevenin equivalent is

\[
\begin{align*}
\frac{4}{(s+1)A} & = V_T(s) \\
\frac{1}{s} & = \frac{2 \Omega}{V(s)} \\
2 & = 2V_o(s)
\end{align*}
\]

13. In the circuit of fig. P1.8.13 just before the closing of switch at \( t = 0 \), the initial conditions are known to be \( v_{c1}(0^-) = 1 \text{ V} \), \( v_{c2}(0^-) = 0 \). The voltage \( v_{c1}(t) \) is

\[
\begin{align*}
(A) & \ \ u(t) \ \text{V} \\
(B) & \ \ 0.5u(t) \ \text{V} \\
(C) & \ \ 0.5e^{-t} \ \text{V} \\
(D) & \ \ e^{-t} \ \text{V}
\end{align*}
\]

14. The initial condition at \( t = 0^- \) of a switched capacitor circuit are shown in Fig. P1.8.14. Switch \( S_1 \) and \( S_2 \) are closed at \( t = 0 \). The voltage \( v_o(t) \) for \( t > 0 \) is

\[
\begin{align*}
(A) & \ \ \frac{7}{4} \ \text{V} \\
(B) & \ \ 9e^{-t} \ \text{V} \\
(C) & \ \ 9 \ \text{V} \\
(D) & \ \ 0 \ \text{V}
\end{align*}
\]

15. A unit step current of 1 A is applied to a network whose driving point impedance is

\[
Z(s) = \frac{V(s)}{I(s)} = \frac{(s + 3)}{(s + 2)^2}
\]

The steady state and initial values of the voltage developed across the source would be respectively

\[
\begin{align*}
(A) & \ \ \frac{7}{4} \ \text{V}, \ 1 \ \text{V} \\
(B) & \ \ \frac{1}{4} \ \text{V}, \ \frac{3}{4} \ \text{V} \\
(C) & \ \ \frac{7}{4} \ \text{V}, \ 0 \ \text{V} \\
(D) & \ \ 1 \ \text{V}, \ \frac{7}{4} \ \text{V}
\end{align*}
\]

16. In the circuit of Fig. P1.8.16 \( i(0) = 1 \text{ A} \), \( v_c(0) = 8 \text{ V} \) and \( v_i = 2e^{-2\times10^{-4}}u(t) \). The \( i(t) \) is

\[
\begin{align*}
(A) & \ \frac{1}{2}[10e^{-10^{-4}t} - 3e^{-2\times10^{-4}t} - 22e^{-4\times10^{-4}t}]u(t) \ \text{A} \\
(B) & \ \frac{1}{2}[-10e^{-10^{-4}t} + 3e^{-2\times10^{-4}t} + 22e^{-4\times10^{-4}t}]u(t) \ \text{A} \\
(C) & \ \frac{1}{2}[10e^{-10^{-4}t} + 3e^{-2\times10^{-4}t} + 22e^{-4\times10^{-4}t}]u(t) \ \text{A} \\
(D) & \ \frac{1}{2}[-10e^{-10^{-4}t} + 3e^{-2\times10^{-4}t} - 22e^{-4\times10^{-4}t}]u(t) \ \text{A}
\end{align*}
\]

17. In the circuit shown in Fig. P1.8.18 \( v(0^-) = 8 \text{ V} \) and \( i_m(t) = 4 \delta(t) \). The \( v_c(t) \) for \( t \geq 0 \) is

\[
\begin{align*}
(A) & \ \ 164e^{-t} \ \text{V} \\
(B) & \ \ 208e^{-t} \ \text{V} \\
(C) & \ \ 208(1 - e^{-3}) \ \text{V} \\
(D) & \ \ 164e^{-3} \ \text{V}
\end{align*}
\]
18. The driving point impedance $Z(s)$ of a network has the pole zero location as shown in Fig. P1.8.18. If $Z(0) = 3$, the $Z(s)$ is

$$Z(s) = \frac{4(s + 3)}{s^2 + s + 1}$$

(A) $\frac{4(s + 3)}{s^2 + s + 1}$ (B) $\frac{2(s + 3)}{s^2 + 2s + 2}$
(C) $\frac{2(s + 3)}{s^2 + 2s + 2}$ (D) $\frac{4(s + 3)}{s^2 + s + 1}$

![Fig. P1.8.18](image)

23. The transformed voltage across the 60 $\mu$F capacitor is given by

$$V_c(s) = \frac{20s + 6}{(10s + 3)(s + 4)}$$

The initial current through capacitor is

(A) $0.12$ mA (B) $-0.12$ mA
(C) $0.48$ mA (D) $-0.48$ mA

24. The current through an 4 H inductor is given by

$$I_L(s) = \frac{10}{s(s + 2)}$$

The initial voltage across inductor is

(A) 40 V (B) 20 V
(C) 10 V (D) 5 V

25. The amplifier network shown in fig. P1.8.25 is stable if

![Fig. P1.8.25](image)

(A) $K \leq 3$ (B) $K \geq 3$
(C) $K \leq \frac{1}{3}$ (D) $K \geq \frac{1}{3}$

26. The network shown in fig. P1.8.26 is stable if

![Fig. P1.8.26](image)

(A) $K \geq \frac{5}{2}$ (B) $K \leq \frac{5}{2}$
(C) $K \geq \frac{2}{5}$ (D) $K \leq \frac{2}{5}$

Statement for Q.19-21:

The circuit is as shown in the fig. P1.8.19–21. All initial conditions are zero.

![Fig. P1.8.19–21](image)

19. $I_o(s)/I_{in}(s) =$?

(A) $(s + 1)/2s$ (B) $2s(s + 1)$
(C) $(s + 1)s^{-1}$ (D) $s(s + 1)$

20. If $i_o(t) = 4\delta(t)$ then $i_i(t)$ will be

(A) $4\delta(t) - e^{-t}u(t)$ A (B) $2s(s + 1)$
(C) $4e^{-t}u(t) - 4\delta(t)$ A (D) $e^{-t}u(t) - \delta(t)$ A

21. If $i_o(t) = tu(t)$ then $i_i(t)$ will be

(A) $e^{-t}u(t)$ A (B) $(1 - e^{-t})u(t)$ A
(C) $u(t)$ A (D) $(2 - e^{-t})u(t)$ A

22. The voltage across 200 $\mu$F capacitor is given by

$$V_c(s) = \frac{2s + 6}{s(s + 3)}$$

(A) 6 V (B) 0 V
(C) $\infty$ (D) 2 V
27. A circuit has a transfer function with a pole \( s = -\frac{a}{4} \) and a zero which may be adjusted in position as \( s = -a \). The response of this system to a step input has a term of form \( Ke^{-at} \). The \( K \) will be (H= scale factor)

(A) \( H \left( 1 - \frac{a}{4} \right) \)  
(B) \( H \left( 1 + \frac{a}{4} \right) \)  
(C) \( H \left( 4 - \frac{a}{4} \right) \)  
(D) \( H \left( 4 + \frac{a}{4} \right) \)

28. A circuit has input \( v_{in}(t) = \cos 2t u(t) \) V and output \( i_o(t) = 2\sin 2t u(t) \) A. The circuit had no internal stored energy at \( t = 0 \). The admittance transfer function is

(A) \( \frac{2}{s} \)  
(B) \( \frac{s}{2} \)  
(C) \( s \)  
(D) \( \frac{1}{s} \)

29. A two terminal network consists of a coil having an inductance \( L \) and resistance \( R \) shunted by a capacitor \( C \). The poles of the driving point impedance function \( Z \) of this network are at \( -\frac{1}{2} \pm j \frac{\sqrt{3}}{2} \) and zero at \( -1 \). If \( Z(0) = 1 \) the value of \( R, L, C \) are

(A) 3 \( \Omega \), 3 \( \Omega \), \( \frac{1}{3} \) F  
(B) 2 \( \Omega \), 2 \( \Omega \), \( \frac{1}{2} \) F  
(C) 1 \( \Omega \), 2 \( \Omega \), \( \frac{1}{2} \) F  
(D) 1 \( \Omega \), 1 \( \Omega \), 1 \( F \)

30. The current response of a network to a unit step input is

\[ I_o = \frac{10(s + 2)}{s^2(s + 11s + 30)} \]

The response is

(A) Under damped  
(B) Over damped  
(C) Critically damped  
(D) None of the above

Statement for Q.31-33:

The circuit is shown in fig. P1.8.31-33.

31. The current ratio transfer function \( I_o/I_s \) is

(A) \( \frac{s(s + 4)}{s^2 + 3s + 4} \)  
(B) \( \frac{s(s + 4)}{(s + 1)(s + 3)} \)  
(C) \( \frac{s^2 + 3s + 4}{s(s + 4)} \)  
(D) \( \frac{(s + 1)(s + 3)}{s(s + 4)} \)

32. The response is

(A) Over damped  
(B) Under damped  
(C) Critically damped  
(D) can't be determined

33. If input \( i_o \) is \( 2u(t) \) A, the output current \( i_o \) is

(A) \( (2e^{-t} - 3e^{-2t})u(t) \) A  
(B) \( (3te^{-t} - e^{-2t})u(t) \) A  
(C) \( (3e^{-t} - e^{-2t})u(t) \) A  
(D) \( (e^{-2t} - 3e^{-3t})u(t) \) A

34. In the network of Fig. P1.8.34, all initial condition are zero. The damping exhibited by the network is

(A) Over damped  
(B) Under damped  
(C) Critically damped  
(D) value of voltage is requires

35. The voltage response of a network to a unit step input is

\[ V_o(s) = \frac{10}{s(s^2 + 8s + 16)} \]

The response is

(A) under damped  
(B) over damped  
(C) critically damped  
(D) can't be determined

36. The response of an initially relaxed circuit to a signal \( v_i \) is \( e^{-2t}u(t) \). If the signal is changed to \( \left( v_i + \frac{2vi}{dv} \right) \), the response would be

(A) \( 5e^{-2t}u(t) \)  
(B) \( -3e^{-2t}u(t) \)  
(C) \( 4e^{-2t}u(t) \)  
(D) \( -4e^{-2t}u(t) \)
Consider the following statements in the circuit shown in fig. P1.8.37

1. It is a first order circuit with steady state value of $v_C = \frac{10}{3}$, $i = \frac{5}{3}$ A
2. It is a second order circuit with steady state of $v_C = 2$ V, $i = 2$ A
3. The network function $\frac{V(s)}{I(s)}$ has one pole.
4. The network function $\frac{V(s)}{I(s)}$ has two poles.

The true statements are
(A) 1 and 3 (B) 1 and 4 (C) 2 and 3 (D) 2 and 4

The network function $\frac{s^2 + 10s + 24}{s^2 + 8s + 15}$ represent a
(A) RC admittance (B) RL impedance (C) LC impedance (D) None of the above

The network function $\frac{s(s + 4)}{(s + 1)(s + 2)(s + 3)}$ represents an
(A) RC impedance (B) RL impedance (C) LC impedance (D) None of these

The network function $\frac{s(3s + 8)}{(s + 1)(s + 3)}$ represents an
(A) RL admittance (B) RC impedance (C) RC admittance (D) None of the above

The network function $\frac{(s + 1)(s + 4)}{s(s + 2)(s + 5)}$ is a
(A) RL impedance function (B) RC impedance function (C) LC impedance function (D) Above all

The network function $\frac{s^2 + 7s + 6}{s + 2}$ is a
(A) RL impedance function (B) RL admittance (C) LC impedance function (D) LC admittance

A valid immittance function is
(A) $\frac{(s + 4)(s + 8)}{(s + 2)(s - 5)}$ (B) $\frac{s(s + 1)}{(s + 2)(s + 5)}$ (C) $\frac{s(s + 2)(s + 3)}{(s + 1)(s + 4)}$ (D) $\frac{s(s + 2)(s + 6)}{(s + 1)(s + 4)}$

A valid admittance function is
(A) RL admittance (B) RC admittance (C) LC admittance (D) Above all

A valid impedance function is given as
$Z(s) = \frac{3(s + 2)(s + 4)}{s(s + 3)}$

The network for this function is
18. (B) \[ Z(s) = \frac{K(s + 3)}{(s - (-1 + j))(s - (-1 - j))} = \frac{K(s + 3)}{s^2 + 2s + 2} \]

\[ Z(0) = \frac{3K}{2} = 3 \Rightarrow K = 2 \]

19. (D) \[ \frac{I_o(s)}{I_{in}(s)} = \frac{s}{s + \frac{1}{s + 1} + \frac{1}{s + 1}} = \frac{s}{s + 1} \]

20. (B) \[ I_{in}(s) = 4 \]

\[ I_s(s) = \frac{4s}{s + 1} = 4 - \frac{4}{s + 1} \Rightarrow i_s(t) = 4\delta(t) - 4e^{-t}u(t) \]

21. (B) \[ I_{in}(s) = \frac{1}{s^2} \]

\[ I_s(s) = \frac{1}{s(s + 1)} = \frac{1}{s - s + 1} \]

\[ i_s(t) = u(t) - e^{-t}u(t) = (1 - e^{-t})u(t) \]

22. (D) \[ v_c(\infty) = \lim_{s \to 0} sV_c(s) = \lim_{s \to 0} \frac{2s + 6}{s + 3} = 2 \text{ V} \]

23. (D) \[ v_c(0^-) = \lim_{s \to \infty} sV_c(s) = \frac{s(20s + 6)}{(10s + 3)(s + 4)} = -2 \text{ V} \]

\[ i_c = \frac{CdV_c}{dt} = I_c(s) = C[sV_c(s) - v_c(0^-)] \]

\[ = 60 \times 10^{-6} \left( \frac{s(20s + 6)}{(10s + 3)(s + 4)} - 2 \right) = -480 \times 10^{-6} \text{ (10s + 3)(s + 4)} + 43s + 12 \]

\[ i_c(0^+) = \lim_{s \to \infty} sI_c(s) = -480 \times 10^{-6} = -0.48 \text{ mA} \]

24. (A) \[ v_L = L \frac{di_L}{dt} = V_L(s) = L \left[ sI_L(s) - i_L(0^-) \right] \]

\[ i_L(0^-) = \lim_{s \to \infty} sI_L(s) = \frac{10}{s + 2} = 0 \]

\[ V_L(s) = \frac{40s}{s(s + 2)} = \frac{40}{s + 2} \]

\[ v_L(0^-) = \lim_{s \to \infty} sV_L(s) = \frac{-s40}{s + 2} = 40 \]

25. (A) \[ V_L(s) = KV_L(s) \]

\[ \Rightarrow \frac{V_L(s)}{2} + \frac{V_L(s) - KV_L(s)}{4 + s + \frac{1}{s}} = 0 \]

\[ 4 + s + \frac{1}{s} + 2 - 2K = 0 \]

\[ \Rightarrow s^2 + (6 - 2K)s + 1 = 0 \]

\[ (6 - 2K) > 0 \Rightarrow K < 3 \]

26. (B) Let \( v_1 \) be the node voltage of middle node

\[ V_1(s) = \frac{KV_2(s) + 2sV_2(s)}{1 + 2s + s} \]

\[ \Rightarrow (3s + 1)V_1(s) = (2s + K)V_2(s) \]

\[ \Rightarrow V_2(s) = \frac{2sV_1(s)}{2s + 1} \]

\[ \Rightarrow (2s + 1)V_2(s) = 2sV_1(s) \]

\[ \Rightarrow (3s + 1)(2s + 1) = 2s(2s + K) \]

\[ 2s^2 + (5 - 2K)s + 1 = 0, \]

\[ 5 - 2K > 0, K < \frac{5}{2} \]

27. (A) \[ H(s) = \frac{H(s + a)}{s + 4} \]

\[ R(s) = \frac{H(s + a)}{s(s + 4)} = \frac{Ha}{4s} + \frac{H(1 - \frac{a}{4})}{s + 4} \]

\[ r(t) = \frac{Ha}{4} u(t) + H(1 - \frac{a}{4})e^{-4t} \]

28. (A) \[ V_1(s) = \frac{s}{s^2 + 1}, I_s(s) = \frac{2}{s^2 + 1}, I_{in}(s) = \frac{2}{s} \]

29. (D) \[ Z(s) = \frac{1}{C} \left( s + \frac{R}{L} \right) \]

\[ Z(s) = \frac{K(s + 1)}{s + \frac{1}{2} + \frac{\sqrt{3}}{2}} \]

Fig. S1.8.29

Since \( Z(0) = 1 \), thus \( K = 1 \)

\[ \frac{1}{C} = 1, \frac{R}{L} = 1, \frac{1}{LC} = 1 \]

\[ \Rightarrow C = 1, L = 1, R = 1 \]

30. (B) The characteristic equation is

\[ s^2(s^2 + 11s + 30) = 0 \Rightarrow s^2(s + 6)(s + 5) = 0 \]

\[ s = -6, -5 \], Being real and unequal, it is overdamped.
31. (B) \[ \frac{I_s}{I} = \frac{s + 4}{s + 4 + \frac{3}{s}} = \frac{s(s + 4)}{(s + 1)(s + 3)} \]

32. (A) The characteristic equation is \((s + 1)(s + 3) = 0\).
Being real and unequal root, it is overdamped response.

33. (C) \[ i_s = 2u(t) \Rightarrow I_s(s) = \frac{2}{s} \]
\[ I_o(s) = \frac{2(s + 4)}{(s + 1)(s + 3)} = \frac{3}{s + 1} - \frac{1}{s + 3} \]
\[ i_o = (3e^{-t} - e^{-3t})u(t) \]

34. (B) \[ \frac{V_o(s)}{V_s(s)} = \frac{4}{s + 2s + 2} = \frac{s^2 + s + 2}{s^2 + s + 2} \]
The roots are imaginary so network is underdamped.

35. (C) The characteristic equation is
\[ s(s^2 + 8s + 16) = 0, (s + 4)^2 = 0, s = -4, -4 \]
Being real and repeated root, it is critically damped.

36. (B) \[ v_o = e^{-2t}u(t) \Rightarrow V_o(s) = H(s)V_s(s) = \frac{1}{s + 2} \]
\[ v_i' = v_i + \frac{2v_o}{dt} \Rightarrow V_i'(s) = (1 + 2s)V_i(s) \]
\[ V_o(s) = H(s)V_i'(s) = (1 + 2s)V_i'(s)H(s) \]
\[ V_o(s) = \frac{1 + 2s}{s + 2} = 2 - \frac{3}{s} \Rightarrow v_o' = 2\delta(s) - 3e^{-2t}u(t) \]

37. (C) It is a second order circuit. In steady state
\[ i = \frac{10}{4 + 1} = 2 \text{ A}, v = 2 \times 1 = 2 \text{ V} \]
\[ I(s) = \frac{10}{s + 4 + \frac{1}{\frac{1}{2}s}} = \frac{5(s + 2)}{(s + 2)^2 + 1} \]
\[ \frac{10}{1 + \frac{1}{2}s} \]
\[ V(s) = \frac{10}{(2s + 4 + \frac{1}{\frac{1}{2}s})} = \frac{10}{(s + 2)^2 + 1} \]
\[ \frac{V(s)}{I(s)} = \frac{2}{s + 2}, \text{ It has one pole at } s = -2 \]

38. (D) \[ \frac{s^2 + 10s + 24}{s^2 + 8s + 15} = \frac{(s + 4)(s + 6)}{(s + 3)(s + 5)} \]
The singularity near to origin is pole. So it may be RC impedance or RL admittance function.

39. (D) Poles and zero does not interlace on negative real axis so it is not a immittance function.

40. (C) The singularity nearest to origin is a zero. So it may be RL impedance or RC admittance function. Because of (D) option it is required to check that it is a valid RC admittance function. The poles and zeros interlace along the negative real axis. The residues of \( \frac{Y_{ss}(s)}{s} \) are real and positive.

41. (B) The singularity nearest to origin is a pole. So it may be RC impedance or RL admittance function.

42. (A) \[ \frac{s^2 + 7s + 6}{s + 2} = \frac{(s + 1)(s + 6)}{(s + 2)} \]
The singularity nearest to origin is at zero. So it may be RC admittance or RL impedance function.

43. (D)
(A) pole lie on positive real axis
(B) poles and zero does not interlace on axis.
(C) poles and zero does not interlace on axis.
(D) is a valid immittance function.

44. (A) \[ \frac{s^2 + 8s + 15}{s^2 + 6s + 8} = \frac{(s + 3)(s + 5)}{(s + 2)(s + 4)} \]
The singularity nearest to origin is a pole. So it may be a RL admittance or RC impedance function.

45. (A) The singularity nearest to origin is a pole. So this is RC impedance function.
\[ Z(s) = 3 + \frac{8}{s} + \frac{1}{s + 3} = 3 + \frac{8}{s} + \frac{1/3}{1 + s/3} \]

***************
CHAPTER

1.9

MAGNETICALLY COUPLED CIRCUITS

Statement for Q.1-2:

In the circuit of Fig. P1.9.1-2, \( i_1 = 4 \sin 2t \) A, and \( i_2 = 0 \).

1. \( v_1 = ? \)
   (A) \(-16 \cos 2t \) V  
   (B) \(16 \cos 2t \) V  
   (C) \(4 \cos 2t \) V  
   (D) \(-4 \cos 2t \) V

2. \( v_2 = ? \)
   (A) \(2 \cos 2t \) V  
   (B) \(-2 \cos 2t \) V  
   (C) \(8 \cos 2t \) V  
   (D) \(-8 \cos 2t \) V

Statement for Q.3-4:

Consider the circuit shown in Fig. P1.9.3-4.

3. If \( i_1 = 0 \) and \( i_2 = 2 \sin 4t \) A, the voltage \( v_1 \) is
   (A) \(24 \cos 4t \) V  
   (B) \(-24 \cos 4t \) V  
   (C) \(15 \cos 4t \) V  
   (D) \(-15 \cos 4t \) V

4. If \( i_1 = e^{-2t} \) V and \( i_2 = 0 \), the voltage \( v_2 \) is
   (A) \(-6e^{-2t} \) V  
   (B) \(6e^{-2t} \) V  
   (C) \(15e^{-2t} \) V  
   (D) \(-15e^{-2t} \) V

Statement for Q.5-6:

Consider the circuit shown in Fig. P1.9.5-6.

5. If current \( i_1 = 3 \cos 4t \) A and \( i_2 = 0 \), then voltage \( v_1 \) and \( v_2 \) are
   (A) \(v_1 = -24 \sin 4t \) V, \( v_2 = -24 \sin 4t \) V 
   (B) \(v_1 = 24 \sin 4t \) V, \( v_2 = -36 \sin 4t \) V 
   (C) \(v_1 = 15 \sin 4t \) V, \( v_2 = \sin 4t \) V 
   (D) \(v_1 = -15 \sin 4t \) V, \( v_2 = -\sin 4t \) V

6. If current \( i_1 = 0 \) and \( i_2 = 4 \sin 3t \) A, then voltage \( v_1 \) and \( v_2 \) are
   (A) \(v_1 = 24 \cos 3t \) V, \( v_2 = 36 \cos 3t \) V 
   (B) \(v_1 = 24 \cos 3t \) V, \( v_2 = -36 \cos 3t \) V 
   (C) \(v_1 = -24 \cos 3t \) V, \( v_2 = 36 \cos 3t \) V 
   (D) \(v_1 = -24 \cos 3t \) V, \( v_2 = -36 \cos 3t \) V
Statement for Q.7-8:

In the circuit shown in fig. P1.9.7-8, \( i_1 = 3 \cos 3t \) A and \( i_2 = 4 \sin 3t \) A.

7. \( v_1 = ? \)
(A) \(-6(2 \cos t + 3 \sin t)\) V  
(B) \(6(2 \cos t + 3 \sin t)\) V  
(C) \(-6(2 \cos t + 3 \sin t)\) V  
(D) \(6(2 \cos t - 3 \sin t)\) V

8. \( v_2 = ? \)
(A) \(3(8 \cos 3t - 3 \sin t)\) V  
(B) \(6(2 \cos t + 3 \sin t)\) V  
(C) \(3(8 \cos 3t + 3 \sin t)\) V  
(D) \(6(2 \cos t - 3 \sin t)\) V

Statement for Q.9-10:

In the circuit shown in fig. P1.9.9-10, \( i_1 = 5 \sin 3t \) A and \( i_2 = 3 \cos 3t \) A.

9. \( v_1 = \)
(A) \(9(5 \cos 3t + 3 \sin 3t)\) V  
(B) \(9(5 \cos 3t - 3 \sin 3t)\) V  
(C) \(9(4 \cos 3t + 5 \sin 3t)\) V  
(D) \(9(5 \cos 3t - 3 \sin 3t)\) V

10. \( v_2 = \)
(A) \(9(-4 \sin 3t + 5 \cos 3t)\) V  
(B) \(9(4 \sin 3t - 5 \cos 3t)\) V  
(C) \(9(-4 \sin 3t - 5 \cos 3t)\) V  
(D) \(9(4 \sin 3t + 5 \cos 3t)\) V

11. In the circuit shown in fig. P1.9.11 if current \( i_1 = 5 \cos (500t - 20^\circ) \) mA and \( i_2 = 20 \cos (500t - 20^\circ) \) mA, the total energy stored in system at \( t = 0 \) is

   (A) 151.14 \( \mu \)J  
   (B) 45.24 \( \mu \)J  
   (C) 249.44 \( \mu \)J  
   (D) 143.46 \( \mu \)J

12. \( L_{eq} = ? \)

13. \( L_{eq} = ? \)

14. \( L_{eq} = ? \)

15. \( L_{eq} = ? \)

16. The equivalent inductance of a pair of a coupled inductor in various configuration are
   (a) 7 H after series adding connection
   (b) 1.8 H after series opposing connection
   (c) 0.5 H after parallel connection with dotted terminal connected together.
The value of $L_1$, $L_2$ and $M$ are
(A) 3 H, 1.6 H, 1.2 H  (B) 1.6 H, 3 H, 1.4 H
(C) 3.7 H, 0.7 H, 1.3 H  (D) 2 H, 3 H, 3 H

17. $L_{eq}$ = ?

![Fig. P1.9.17](image)

(A) 0.2 H  (B) 1 H  (C) 0.4 H  (D) 2 H

18. $L_{eq}$ = ?

![Fig. P1.9.18](image)

(A) 1 H  (B) 2 H  (C) 3 H  (D) 4 H

19. In the network of fig. P1.9.19 following terminal are connected together
(i) none  (ii) $A$ to $B$
(iii) $B$ to $C$  (iv) $A$ to $C$

![Fig. P1.9.19](image)

The correct match for equivalent induction seen at terminal $a - b$ is

(i)  (ii)  (iii)  (iv)
(A) 1 H  0.875 H  0.6 H  0.75 H
(B) 13 H  0.875 H  0.6 H  0.75 H
(C) 13 H  7.375 H  6.6 H  2.4375 H
(D) 1 H  7.375 H  6.6 H  2.4375 H

20. $L_{eq}$ = ?

![Fig. P1.9.20](image)

(A) 1 H  (B) 2 H  (C) 3 H  (D) 4 H

21. $L_{eq}$ = ?

![Fig. P1.9.21](image)

(A) $\frac{41}{5}$ H  (B) $\frac{49}{5}$ H  (C) $\frac{51}{5}$ H  (D) $\frac{39}{5}$ H

**Statement for Q.22-24:**
Consider the circuit shown in fig. P1.9.22–24.

![Fig. P1.9.22–24](image)

22. The voltage $V_{AD}$ of terminal $AD$ is
(A) 60 V  (B) −60 V  (C) 180 V  (D) 240 V

23. The voltage $V_{BD}$ of terminal $BD$ is
(A) 45 V  (B) 33 V  (C) 69 V  (D) 105 V

24. The voltage $V_{CD}$ of terminal $CD$ is
(A) 30 V  (B) 0 V  (C) −36 V  (D) 36 V
33. In the circuit of fig. P1.9.33, the resonance occurs when $C$ is

(A) 1 F  
(B) $\frac{1}{2}$ F  
(C) $\frac{1}{3}$ F  
(D) $\frac{1}{6}$ F

34. In the circuit of fig. P1.9.34, the voltage gain is zero at $\omega = 333.33$ rad/s. The value of $C$ is

(A) 100 $\mu$F  
(B) 75 $\mu$F  
(C) 50 $\mu$F  
(D) 25 $\mu$F

35. In the circuit of fig. P1.9.35 at $\omega = 333.33$ rad/s, the voltage gain $v_{out}/v_{in}$ is zero. The value of $C$ is

(A) 3.33 mF  
(B) 33.33 mF  
(C) 3.33 $\mu$F  
(D) 33.33 $\mu$F

36. The Thévenin equivalent at terminal $ab$ for the network shown in fig. P1.9.36 is

(A) 6 V, 10 $\Omega$  
(B) 6 V, 4 $\Omega$  
(C) 0 V, 4 $\Omega$  
(D) 0 V, 10 $\Omega$

37. In the circuit of fig. P1.9.37, the maximum power delivered to $R_L$ is

(A) 250 W  
(B) 200 W  
(C) 150 W  
(D) 100 W

38. The average power delivered to the 8 $\Omega$ load in the circuit of fig. P1.9.38 is

(A) 8 W  
(B) 1.25 kW  
(C) 625 kW  
(D) 2.50 kW

39. In the circuit of fig. P1.9.39, the ideal source supplies 1000 W, half of which is delivered to the 100 $\Omega$ load. The value of $a$ and $b$ are

(A) 6, 0.47  
(B) 5, 0.89  
(C) 0.89, 5  
(D) 0.47, 6

40. $I_2 = ?$

(A) 1.65 $A_{rms}$  
(B) 0.18 $A_{rms}$  
(C) 0.66 $A_{rms}$  
(D) 5.90 $A_{rms}$
41. \( V_2 = ? \)

![Fig. P1.9.41](image)

(A) -12.31 V  
(B) 12.31 V  
(C) -9.231 V  
(D) 9.231 V

42. The power being dissipated in 400 Ω resistor is

![Fig. P1.9.42](image)

(A) 3 W  
(B) 6 W  
(C) 9 W  
(D) 12 W

43. \( I_x = ? \)

![Fig. P1.9.43](image)

(A) 1921°57.4° Α  
(B) 2931°59.4° Α  
(C) 1.68°43.6° Α  
(D) 1.79°43.6° Α

44. \( Z_{in} = ? \)

![Fig. P1.9.44](image)

(A) 46.3 + j6.8 Ω  
(B) 432.1 + j0.96 Ω  
(C) 10.8 + j9.6 Ω  
(D) 615.4 + j0.38 Ω

SOLUTIONS

1. (B) \( v_1 = 2 \frac{di_1}{dt} + 1 \frac{di_2}{dt} = 2 \frac{di_1}{dt} = 16 \cos 2t \ V \)

2. (C) \( v_2 = (1) \frac{di_2}{dt} + (1) \frac{di_1}{dt} = \frac{di_1}{dt} = 8 \cos 2t \ V \)

3. (B) \( v_1 = 3 \frac{di_1}{dt} - 3 \frac{di_2}{dt} = -3 \frac{di_2}{dt} = -24 \cos 4t \ V \)

4. (C) \( v_2 = 4 \frac{di_2}{dt} - 3 \frac{di_1}{dt} = -3 \frac{di_1}{dt} = 6e^{-2t} \ V \)

5. (A) \( v_1 = 2 \frac{di_1}{dt} - 2 \frac{di_2}{dt} = 2 \frac{di_1}{dt} = -24 \sin 4t \ V \)

6. (D) \( v_1 = 2 \frac{di_1}{dt} - 2 \frac{di_2}{dt} = -2 \frac{di_2}{dt} = -24 \cos 3t \ V \)

7. (A) \( v_1 = 2 \frac{di_1}{dt} + \frac{di_2}{dt} \)

\[ = -18 \sin t + 12 \cos t = 6 (2 \cos t - 3 \sin t) \ V \]

8. (A) \( v_2 = 2 \frac{di_2}{dt} + \frac{di_1}{dt} \)

\[ = 24 \cos 3t - 9 \sin 3t = 3 (8 \cos 3t - 3 \sin 3t) \ V \]

9. (A) \( v_1 = 3 \frac{di_1}{dt} - 3 \frac{di_2}{dt} \)

\[ = 45 \cos 3t + 27 \sin 3t = 9 (5 \cos 3t + 3 \sin 3t) \ V \]

10. (D) \( v_2 = -4 \frac{di_1}{dt} + 3 \frac{di_2}{dt} \)

\[ = 36 \sin 3t + 45 \cos 3t = 9 (4 \sin 3t + 5 \cos 3t) \ V \]

11. (A) \( W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M_1 i_2 \)

At \( t = 0, i_1 = 4 \cos (-20°) = 4.7 \ mA \)

\( i_2 = 20 \cos (-20°) = 18.8 \ mA \),

\( M = 0.6 \sqrt{25 \times 0.4} = 0.6 \)

\( W = \frac{1}{2}(2.5)(4.7)^2 + \frac{1}{2}(0.4)(18.8)^2 + 0.6(4.7)(18.8) = 151.3 \mu J \)

12. (C) \( L_{eq} = L_1 + L_2 + 2M = 7 \ H \)

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13. (A) \( L_{eq} = L_1 + L_2 - 2M = 4 + 2 - 2 \times 2 = 2 \) H

14. (C) \( L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{24 - 16}{6 + 4 - 8} = 4 \) H

15. (A) \( L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{8 - 4}{6 + 4} = 0.4 \) H

16. (C) \( L_1 + L_2 + 2M = 7, L_1 + L_2 - 2M = 1.8 \)
\( \Rightarrow L_1 + L_2 = 4.4, M = 1.3 \)
\( \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{0.5}{L_1 L_2 - 1.3^2} = 0.5 \times 1.8 \)
\( L_1 L_2 = 2.59, (L_1 - L_2)^2 = 4.4^2 - 4 \times 2.59 = 9 \)
\( L_1 - L_2 = 3, L_3 = 37, L_2 = 0.7 \)

17. (D) \( L_{eq} = L_1 - \frac{M^2}{L_2} = 4 - \frac{4}{2} = 2 \) H

18. (B) \( L_{eq} = L_1 - \frac{M^2}{L_2} = 5 - \frac{9}{3} = 2 \) H

19. (A)

20. (D) \( V_{L_1} = 1sI + 1sI = 2sI \)
\( V_{L_2} = 2sI + 1sI - 2sI = sI, \)
\( V_{L_3} = 3sI - 2sI = sI, \)
\( V_L = V_{L_1} + V_{L_2} + V_{L_3} = 4sI \Rightarrow L_{eq} = 4 \) H

21. (B) Let \( I_1 \) be the current through 4 H inductor and \( I_2 \) and \( I_3 \) be the current through 3 H, and 2 H inductor respectively
\( I_1 = I_2 + I_3, \) \( V_2 = V_3 \)
\( 3sI_2 + 3sI_1 = 2sI_3 + 2sI_1 \)
\( \Rightarrow 3I_2 + I_1 = 2I_3 \Rightarrow 4I_2 = I_3 \)
\( \Rightarrow I_2 = \frac{I_3}{5}, I_3 = \frac{4}{5} I_1 \)
\( V = 4sI_1 + 3sI_2 + 2sI_3 + 3sI_2 + 3sI_1 \)
\( = 7sI_1 + \frac{6}{5} I_1 + \frac{2}{5} \times 4sI_1 \)
\( = 49sI_1, L_{eq} = 49 \) H

22. (C) \( v_{AB} = 20 \frac{d(6t)}{dt} + 4 \frac{d(15t)}{dt} = 180 \) V

23. (B) \( v_{BG} = 3 \frac{d(15t)}{dt} + 4 \frac{d(6t)}{dt} - 6 \frac{d(6t)}{dt} = 33 \) V

24. (C) \( v_{CG} = -6 \frac{d(6t)}{dt} = -36 \) V

25. (B) \( Z = \frac{Z_{inv} + \omega^2 M^2}{Z_{inv}} \)
\( = 4 + j(50) \left( \frac{1}{10} + \frac{(50)^2}{5} \right) \)
\( = 4.77 + j115 \) Ω

26. (B) \( V_s = j(0.8)(10(12 \angle 0) - j(0.2)(10)(2 \angle 0) \)
\( + [3 + j(0.5)(10)](12 \angle 0 + 2 \angle 0) \)
\( = 9.6 + j21.6 = 26.64 \angle 66.04^\circ \) V

27. (A) \( [j(100\pi)(2) + 10V_2 + j(100\pi)(0.4)(2 \angle 0)] = 0 \)
\( \Rightarrow I_2 = -0.4 - j0.0064, \)
\( V_s = 10I_2 = -4 - j0.064 \)
\( = 4 \angle -179.1^\circ \)
\( \Rightarrow V_s = 4 \cos (100\pi t - 179.1^\circ) \) V

28. (B) \( 30 \angle 30^\circ = I(-j6 + j8 - j4 + j12 - j4 + 10) \)
\( \Rightarrow I = \frac{30 \angle 30^\circ}{(10 + j6)} = 2.57 - j0.043 \)
\( V_s = I(j12 - j4 + 10) \)
\( = (2.57 - j0.043)(10 + 8j) \)
\( = 26.067 + j20.14 = 32.9 \angle 37.7^\circ \) V

29. (A)

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Fig. S.1.9.29

\((-j + 2 + j4)I_1 - jI_2 = -j3 \)
\((j4 + 2)I_2 - jI_1 = -12 \angle 30^\circ \) V
\(I_1 = -1.45 - j0.56, \)
\(V_s = -2 I_1 = 2.9 + j1.12 \)
\(= 3.11 \angle 21.12^\circ \) V

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30. (D)

\[ Z_{eq} = 10 + j8 + \frac{(j14)(j10 + 2 - j6)}{j14 + j10 + 2 - j6} = 112 + j112 \Omega \]

31. (C) \( Z_m = -j6 \) || \( Z_n \)

\[ Z_s = j20 + \frac{(12)^2}{(j30 + j5 - j2 + 4)} = 0.52 + j15.7 \]

\[ Z_{in} = \frac{-j6(0.52 + j15.7)}{-j6 + 0.52 + j15.7} = 0.20 - j9.7 \Omega \]

32. (D) \( M = k\sqrt{L_1L_2} \), \( M^2 = 160 \times 10^{-12} \)

\[ Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{Z_{L2} + j\omega L_2} = \frac{250 \times 10^3 \times 2 \times 10^{-6} + (250 \times 10^3)^2 \times 160 \times 10^{-12}}{2 + j10 + j \times 250 \times 10^3 \times 80 \times 10^{-6}} \]

\[ Z_{in} = 0.02 + j0.17 \Omega \]

33. (D)

\[ V_1 = \frac{jI_1}{2C} + j4I_1 + j2\sqrt{2} I_2 \]

\[ 0 = (4 + 4j)I_2 + j2\sqrt{2}I_1 \]

\[ \Rightarrow I_2 = -j\frac{\sqrt{2}I_1}{2(1 + j)} \]

\[ V_1 = \frac{-j}{{2C}} + j4 + \frac{2}{{1 + j}} = -j + j8C + 2C - j2C \]

\[ Z_{in} = \frac{-j + j8C + 2C - j2C}{2C} \]

\[ \text{Im}(Z_{in}) = 0 \Rightarrow -j + j8C + j2C = 0 \]

\[ \Rightarrow C = \frac{1}{6} \]

34. (A) \( j30 - \frac{3j}{1000C} = 0, C = 100 \mu\text{F} \)

35. (D) The \( \pi \) equivalent circuit of coupled coil is shown in fig. S1.9.35

\[ L_1 \frac{L_2 - M^2}{M} = \frac{L_1L_2 - M^2}{L_1 - M} \]

Output is zero if \( \frac{-j}{0.27\omega} + j\omega = 0 \)

\[ C = \frac{1}{0.27\omega} = 33.33 \mu\text{F} \]

36. (C) Applying 1 V test source at ab terminal,

\[ V_{ab} = 1 V, I_x = \frac{1}{20} = 0.05 \ A, V_x = 4 \ V, \]

\[ 4 = 60 I_2 + 20 \times 0.05 \Rightarrow I_2 = 0.05 \ A \]

\[ I_{in} = I_x + I_1 = I_x + 4I_2 = 0.25 \ A \]

\[ R_{TH} = \frac{1}{I_{in}} = 4 \Omega, \quad V_{TH} = 0 \]

37. (A) Impedance seen by \( R_L = 10 \times 4^2 = 160 \ \Omega \)

For maximum power \( R_L = 160 \ \Omega, Z_o = 10 \ \Omega \)

\[ P_{Lmax} = \left( \frac{100}{10 + 10} \right)^2 \times 10 = 250 \ \text{W} \]

38. (B) \( I_2 = \frac{V_2}{8}, I_1 = \frac{I_2}{5} = \frac{V_2}{40}, V_1 = 5V_2 \)
CHAPTER 1.10

TWO PORT NETWORK

Statement for Q.1-4:

The circuit is given in fig. P.1.10.1–4

1. \[ z = \begin{bmatrix} 1 & -3 \\ 2 & 2 \\ 17 & 1 \\ 6 & 2 \end{bmatrix} \]
   (A) \[ z = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 17 & 1 \\ 6 & 2 \end{bmatrix} \]
   (B) \[ z = \begin{bmatrix} 17 & 1 \\ 6 & 2 \\ 1 & 3 \\ 6 & 2 \end{bmatrix} \]
   (C) \[ z = \begin{bmatrix} 17 & -1 \\ 6 & 2 \\ 1 & 3 \\ 6 & 2 \end{bmatrix} \]
   (D) \[ z = \begin{bmatrix} 17 & 1 \\ 6 & 2 \\ 1 & 3 \\ 6 & 2 \end{bmatrix} \]

2. \[ y = \begin{bmatrix} 3 & 1 \\ 8 & 8 \\ 1 & 17 \\ 8 & 24 \end{bmatrix} \]
   (A) \[ y = \begin{bmatrix} 3 & -1 \\ 8 & 8 \\ 1 & 17 \\ 8 & 24 \end{bmatrix} \]
   (B) \[ y = \begin{bmatrix} 3 & 1 \\ 8 & 8 \\ 1 & 17 \\ 8 & 24 \end{bmatrix} \]
   (C) \[ y = \begin{bmatrix} 17 & 1 \\ 6 & 2 \\ 1 & 3 \\ 2 & 2 \end{bmatrix} \]
   (D) \[ y = \begin{bmatrix} 17 & -1 \\ 6 & 2 \\ 1 & 3 \\ 8 & 2 \end{bmatrix} \]

3. \[ T = \begin{bmatrix} 6 & -3 \\ 3 & -24 \\ 17 & -17 \end{bmatrix} \]
   (A) \[ T = \begin{bmatrix} 8 & 1 \\ 3 & 3 \\ 1 & 2 \\ 3 & 3 \end{bmatrix} \]
   (B) \[ T = \begin{bmatrix} 8 & -1 \\ 3 & 3 \\ 1 & 2 \\ 3 & 3 \end{bmatrix} \]
   (C) \[ T = \begin{bmatrix} 3 & -8 \\ 2 & -3 \end{bmatrix} \]
   (D) \[ T = \begin{bmatrix} 17 & -3 \\ 2 & -3 \end{bmatrix} \]

4. \[ T = \begin{bmatrix} 17 & 8 \\ 2 & 3 \end{bmatrix} \]
   (A) \[ T = \begin{bmatrix} 17 & 8 \\ 2 & 3 \end{bmatrix} \]
   (B) \[ T = \begin{bmatrix} 17 & 8 \\ 2 & 3 \end{bmatrix} \]
   (C) \[ T = \begin{bmatrix} 17 & 8 \\ 2 & 3 \end{bmatrix} \]
   (D) \[ T = \begin{bmatrix} 17 & 8 \\ 2 & 3 \end{bmatrix} \]

5. \[ z = \begin{bmatrix} 21 & 1 \\ 16 & 8 \\ 1 & 7 \\ 8 & 12 \end{bmatrix} \]
   (A) \[ z = \begin{bmatrix} 21 & 1 \\ 16 & 8 \\ 1 & 7 \\ 8 & 12 \end{bmatrix} \]
   (B) \[ z = \begin{bmatrix} 21 & 1 \\ 16 & 8 \\ 1 & 7 \\ 8 & 12 \end{bmatrix} \]
   (C) \[ z = \begin{bmatrix} 21 & 1 \\ 16 & 8 \\ 1 & 7 \\ 8 & 12 \end{bmatrix} \]
   (D) \[ z = \begin{bmatrix} 21 & 1 \\ 16 & 8 \\ 1 & 7 \\ 8 & 12 \end{bmatrix} \]
Statement for Q.7-10:

A two port is described by $V_1 = I_1 + 2V_2$, $I_2 = -2I_1 + 0.4V_2$

7. $[z] = ?$
   (A) $\begin{bmatrix} 11 & -5 \\ -5 & 2.5 \end{bmatrix}$
   (B) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$
   (C) $\begin{bmatrix} 1 & -2 \\ 5 & 0.4 \end{bmatrix}$
   (D) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$

8. $[y] = ?$
   (A) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$
   (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4.4 \end{bmatrix}$
   (C) $\begin{bmatrix} -2 & 4.4 \\ 4 & -2 \end{bmatrix}$
   (D) $\begin{bmatrix} 11 & -5 \\ -5 & 2.5 \end{bmatrix}$

9. $[h] = ?$
   (A) $\begin{bmatrix} 3 & -6 \\ 4 & -4 \end{bmatrix}$
   (B) $\begin{bmatrix} 4 & -2 \\ -2 & 4.4 \end{bmatrix}$
   (C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$
   (D) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$

10. $[T] = ?$
    (A) $\begin{bmatrix} 2.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$
    (B) $\begin{bmatrix} 2.2 & -0.5 \\ 0.2 & -0.5 \end{bmatrix}$
    (C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$
    (D) $\begin{bmatrix} 1 & -2 \\ -2 & -0.4 \end{bmatrix}$
14. \([z] = ?\)

\[ \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix} \]

(A) \[ \begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix} \]

(C) \[ \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix} \]

15. \([2] = ?\)

\[ \begin{bmatrix} 3 & 3 \\ 3 & 2 \end{bmatrix} \]

(C) \[ \begin{bmatrix} 2 & 2 \\ 3 & 2 \end{bmatrix} \]

(D) \[ \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \]

16. \([y] = ?\)

\[ \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix} \]

(C) \[ \begin{bmatrix} -1 & 1 \\ 3 & 1 \end{bmatrix} \]

(D) \[ \begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix} \]

17. \([z] = ?\)

\[ \begin{bmatrix} 3 & 2 \\ 6 & 1 \end{bmatrix} \]

(B) \[ \begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix} \]

18. \([T] = ?\)

\[ \begin{bmatrix} 0.35 & -1 \\ 2 & -3.33 \end{bmatrix} \]

(D) \[ \begin{bmatrix} 0.35 & 1 \\ 2 & 3.33 \end{bmatrix} \]

19. \([h] = ?\)

\[ \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \]

(B) \[ \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \]

(D) \[ \begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \]
27. \[ y = \begin{bmatrix} \frac{3}{2} & 2 \end{bmatrix} \]

The value of \( \frac{V}{V_s} \) is

(A) \( \frac{3}{32} \) \hspace{1cm} (B) \( \frac{3}{16} \)
(C) \( \frac{2}{33} \) \hspace{1cm} (D) \( \frac{1}{17} \)

30. The T-parameters of a 2-port network are

\[ [T] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \]

If such two 2-port network are cascaded, the \( z \)-parameter for the cascaded network is

(A) \( \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix} \) \hspace{1cm} (B) \( \begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix} \)
(C) \( \begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix} \) \hspace{1cm} (D) \( \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \)

31. \[ y = \begin{bmatrix} \frac{3}{2} & 2 \end{bmatrix} \]

For the 2-port of fig. P.1.10.29, \( [y_a] = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \) mS

\[ I(s) \]

\[ V_1(s) \]

\[ 2F \]

\[ I(s) \]

\[ V_1(s) \]

\[ 2F \]

\[ V_2(s) \]

\[ 100 \Omega \]

\[ 300 \Omega \]

\[ 60 \Omega \]

\[ + \]

\[ + \]

\[ - \]

\[ - \]
The circuit shown in fig. P.1.10.37 is reciprocal if $a$ is

(A) 2  
(B) $-2$  
(C) 1  
(D) $-1$

38. $Z_{in}$ is

(A) 86.4 Ω  
(B) 64.3 Ω  
(C) 153.8 Ω  
(D) 94.3 Ω

39. $V_1$, $V_2$ is

(A) $-68.6 \text{ V}$, $114.3 \text{ V}$  
(B) $68.6 \text{ V}$, $-114.3 \text{ V}$  
(C) $114.3 \text{ V}$, $-68.6 \text{ V}$  
(D) $-114.3 \text{ V}$, $68.6 \text{ V}$

40. A 2-port network is driven by a source $V_s = 100 \text{ V}$ in series with 5 Ω, and terminated in a 25Ω resistor. The impedance parameters are

$$ [z] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix} \text{ Ω} $$

The Thevenin equivalent circuit presented to the 25Ω resistor is

(A) 80 V, 2.8 Ω  
(B) 160 V, 6.8 Ω  
(C) 100 V, 2.4 Ω  
(D) 120 V, 6.4 Ω
18. (D) Let $I_3$ be the clockwise loop current in center loop
\[ I_1 = \frac{V_1}{2} + I_3, \quad V_2 = 4(I_2 + I_3) \Rightarrow I_3 = 0.25V_2 - I_2 \]
\[ \Rightarrow I_3 = 0.35V_2 - I_2 \quad \text{(i)} \]
\[ V_1 = 4I_1 - 0.2V_1 + V_2 \]
\[ 12V_1 = 4(0.35V_2 - I_2) + V_2 = 2.4V_2 - 4I_2 \]
\[ \Rightarrow V_1 = 2V_2 - 3.33I_2 \quad \text{(ii)} \]

19. (A) \[ V_2 = 4\left(I_2 + I_1 - \frac{I_2}{2}\right) \Rightarrow I_2 = -2I_1 + \frac{1}{2}V_2 \quad \text{(ii)} \]
\[ I_1 = \frac{I_2}{2} + \left(\frac{V_1 - V_2}{2}\right) - \frac{1}{2}V_1 + \frac{V_1}{4} - \frac{V_2}{2} \]
\[ \Rightarrow V_1 = 4I_1 + \frac{3}{2}V_2 \quad \text{(i)} \]

20. (B) \[ I_1 = -V_2 + \frac{V_1}{2} + \frac{V_2 - V_1}{2} = \frac{3}{2}V_1 - \frac{3}{2}V_2 \quad \text{(i)} \]
\[ I_2 = 2V_1 + \frac{V_1}{2} + \frac{V_2 - V_1}{2} = \frac{3}{2}V_1 + \frac{3}{2}V_2 \quad \text{(ii)} \]

21. (D) \[ I_1 = 2V_1 + jV_1 + j(V_1 - V_2) \]
\[ \Rightarrow I_1 = (2 + j)V_1 - jV_2 \quad \text{(i)} \]
\[ I_2 = \frac{V_2}{1} + V_1 + j(V_2 - V_1) = (1 - j)V_1 + (1 + j)V_2 \quad \text{(ii)} \]

22. (B) \[ V_1 = \frac{I_1}{s} + sI_1 + sI_2 = \left(\frac{1}{s} + s\right)I_1 + sI_2 \quad \text{(i)} \]
\[ V_2 = 2I_2 + 2sI_1 + sI_2 \Rightarrow V_2 = sI_1 + (2 + 2s)I_2 \quad \text{(ii)} \]

23. (D) \[ Z_R = \frac{9}{n^2} = \frac{9}{9} = 1 \]

24. (C) \[ V_1 = 3sI_1 + 3sI_1 - 3sI_1 + 3sI_1 + 2sI_2 \]
\[ \Rightarrow V_2 = 6sI_1 + 2sI_2 \quad \text{(i)} \]
\[ V_2 = 3sI_2 + 2sI_1 \Rightarrow V_2 = 2sI_1 + 3sI_2 \quad \text{(ii)} \]

25. (C) \[ V_1 = \frac{V_2^2}{5} + 0(-I_2), \quad I_1 = (0)V_2 + 5(-I_2) \]

26. (A) \[ V_1 = (Z_a + Z_{ab})I_1 + Z_{ab}I_2 \quad \text{(i)} \]
\[ V_2 = (Z_a + Z_{ab})I_2 + Z_{ab}I_1 = (Z_a + Z_{ab})I_2 \quad \text{(ii)} \]

27. (A) \[ I_1 = (V_1 - V_2)Y_{ab} + V_1Y_a \]
\[ \Rightarrow I_1 = V_1(Y_a + Y_{ab}) - V_2Y_{ab} \quad \text{(i)} \]
\[ I_2 = (V_2 - V_1)Y_{ab} + V_2Y_a = -V_2Y_{ab} + V_1(Y_a + Y_{ab}) \quad \text{(ii)} \]

28. (B) $y$-parameter of 1 $\Omega$ resistor network are
\[ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]

New y-parameter = $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 0 & 3 \end{bmatrix}$.

29. (A) \[ [z] = \begin{bmatrix} 2 \text{ mS} & 0 \\ 0 & 10 \text{ mS} \end{bmatrix}^{-1} = \begin{bmatrix} 5000 & 0 \\ 0 & 100 \end{bmatrix} \]
\[ [z] = \begin{bmatrix} 5000 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 200 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix} \]

30. (C) \[ [T_n] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \]
\[ V_1 = 5V_2 - 3I_2, \quad I_1 = 3V_2 - 2I_2 \]

\[ Z_R = n^24 = 36 \]

Fig. S1.10.23b
\[ V_2 = \frac{(36 \parallel 9)I_2}{72I_2} \Rightarrow z_{22} = \frac{V_2}{I_2} = 7.2, \]
\[ z_{12} = z_{21} = 2.4 \]
31. (B)

\[ \begin{align*}
3V_1 - 5I_1 &= I_2 \\
V_2 &= \frac{1}{3}I_1 + \frac{2}{3}I_2
\end{align*} \]

...(i)

...(ii)

32. (D) \[ [y_a] = \begin{bmatrix} 3s & -2s \\ -2s & 4s \end{bmatrix} \]

33. (C) \[ h_{21} = \frac{I_2}{I_1} \]

34. (C) \[ \frac{I_1}{V_1} = \frac{1}{10} = 0.1 \]

Interchanging the port \( I'_1 = 0.1 \), \( I'_2 = 100 \times 0.1 = 10 \)

35. (B) For symmetrical network \( A = D = 3 \)

For bilateral \( AD - BC = 1 \), \( 9 - C = 1 \), \( C = 8 \) S

36. (A) \[ z_{11} = \frac{A}{C} = \frac{4}{3} \]

37. (A) \[ V_1 = 0.5V_1 + I_1 + 2(I_1 + I_2) + aI_1 \]

\[ V_1 = (6 + 2a)I_1 + 4I_2 \]

...(i)

\[ V_2 = 2(I_1 + I_2) + aI_1 \]

\[ V_2 = (2 + a)I_1 + 2I_2 \]

...(ii)

For reciprocal network

\[ z_{12} = z_{21} , 4 = 2 + a \]

\[ a = 2 \]

38. (C) \[ I_1 = 4 \times 10^3V_1 - 0.1 \times 10^{-3}V_2 \]

\[ I_2 = 50 \times 10^{-3}V_1 + 10^{-3}V_2 \]

\[ -10^{-3}V_2 = 50 \times 10^{-3}V_1 + 10^{-3}V_2 \]

\[ V_2 = 25V_1 \]

\[ 10^3I_1 = 4V_1 + 25V_1 \]

\[ V_1 \frac{10^3}{I_1} = 153.8 \]

39. (B) \[ I_1 = 10 \times 10^{-3}V_1 - 5 \times 10^{-3}V_2 \]

\[ 100 - 25I_1 + V_1 \]

\[ 100 - V_1 = 0.25V_1 - 0.125V_2 \]

\[ \Rightarrow 800 = 10V_1 - V_2 \]

...(i)

\[ I_2 = 50 \times 10^{-3}V_1 + 20 \times 10^{-3}V_2 \]

\[ V_2 = 250V_1 + 10I_2 \]

\[ V_2 = 250V_1 + 10V_1 \]

\[ V_2 = 160 + 6.8I_2 \]

\[ V_{TH} = 160 \text{ V}, R_{TH} = 6.8 \Omega \]

40. (B) \[ 100 = 5I_1 + V_1 \]

\[ V_2 = 20I_1 + 2I_2 \]

\[ \Rightarrow 100 = 25I_1 + 2I_2 \]

\[ V_2 = 40I_1 + 10I_2 \]

\[ 800 - 5V_2 = -34I_2 \]

\[ \Rightarrow V_2 = 160 + 6.8I_2 \]

\[ V_{TH} = 160 \text{ V}, R_{TH} = 6.8 \Omega \]

41. (B) \[ V_1 = z_{11}I_1 \]

\[ V_2 = z_{21}I_1 \]

\[ V_2 = z_{21}I_1 \]

\[ \frac{V_2}{V_1} = \frac{z_{21}}{z_{11}} \]

42. (B) \[ I_2 = y_{21}V_1 + y_{22}V_2 \]

\[ I_2 = -V_L Y_L \]

\[ y_{21}V_1 + (y_{22} + Y_L)V_2 = 0 \]

\[ V_2 = \frac{V_1}{V_2} \]

\[ (y_{22} + Y_L) \]

43. (A) \[ V_2 = z_{21}I_1 + z_{22}I_2 \]

\[ V_2 = -Z_L I_2 \]

\[ V_2 = z_{21}I_1 + z_{22}\left(-\frac{V_2}{Z_L}\right) \]

\[ V_2 = z_{21}I_1 + z_{22}\left(-\frac{V_2}{Z_L}\right) \]

\[ V_2(Z_L + z_{22}) = z_{21}Z_L I_1 \]

\[ V_2 = \frac{Z_2Z_L}{z_{22} + Z_L} \]

*********
CHAPTER 1.11

FREQUENCY RESPONSE

**Statement for Q.1-3:**
A parallel resonant circuit has a resistance of 2 kΩ and half power frequencies of 86 kHz and 90 kHz.

1. The value of capacitor is
   (A) 6 μF  
   (B) 20 nF  
   (C) 2 nF  
   (D) 60 μF

2. The value of inductor is
   (A) 4.3 mH  
   (B) 43 mH  
   (C) 0.16 mH  
   (D) 1.6 mH

3. The quality factor is
   (A) 22  
   (B) 100  
   (C) 48  
   (D) 200

**Statement for Q.4-5:**
A parallel resonant circuit has a midband admittance of $25 \times 10^{-3}$ S, quality factor of 80 and a resonant frequency of 200 krad/s.

4. The value of $R$ is
   (A) 40 Ω  
   (B) 56.57 Ω  
   (C) 80 Ω  
   (D) 28.28 Ω

5. The value of $C$ is
   (A) 2 μF  
   (B) 28.1 μF  
   (C) 10 μF  
   (D) 14.14 μF

6. A parallel $RLC$ circuit has $R = 1$ kΩ and $C = 1$ μF. The quality factor at resonance is 200. The value of inductor is
   (A) 35.4 μH  
   (B) 25 μH  
   (C) 17.7 μH  
   (D) 50 μH

7. A parallel circuit has $R = 1$ kΩ, $C = 50 \mu$F and $L = 10$ mH. The quality factor at resonance is
   (A) 100  
   (B) 90.86  
   (C) 70.7  
   (D) None of the above

8. A series resonant circuit has an inductor $L = 10$ mH. The resonant frequency $\omega_0 = 10^6$ rad/s and bandwidth is $BW = 10^3$ rad/s. The value of $R$ and $C$ will be
   (A) 100 μF, 10 Ω  
   (B) 100 pF, 10 Ω  
   (C) 100 pF, 10 MΩ  
   (D) 100 μF, 10 MΩ

9. A series resonant circuit has $L = 1$ mH and $C = 10$ μF.
   The required $R$ for the BW 15.9 Hz is
   (A) 0.1 Ω  
   (B) 0.2 Ω  
   (C) 159 mΩ  
   (D) 500 Ω

10. For the $RLC$ parallel resonant circuit when $R = 8$ kΩ, $L = 40$ mH and $C = 0.25$ μF, the quality factor $Q$ is
    (A) 40  
    (B) 20  
    (C) 30  
    (D) 10

11. The maximum voltage across capacitor would be
    (A) 3200 V  
    (B) 3 V  
    (C) −3 V  
    (D) 1600 V

Fig. P1.11.11

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12. For the circuit shown in fig. P1.11.11 resonant frequency $f_o$ is

![Fig. P1.11.12](image)

(A) 346 kHz  
(B) 55 kHz  
(C) 196 kHz  
(D) 286 kHz

13. For the circuit shown in fig. P1.11.13 the resonant frequency $f_o$ is

![Fig. P1.11.13](image)

(A) 12.9 kHz  
(B) 12.9 MHz  
(C) 2.05 MHz  
(D) 2.05 kHz

14. The network function of circuit shown in fig.P1.11.14 is

$$H(\omega) = \frac{V_o}{V_i} = \frac{4}{1 + j0.01\omega}$$

![Fig. P1.11.14](image)

The value of the $C$ and $A$ is

(A) 10 $\mu$F, 6  
(B) 5 $\mu$F, 10  
(C) 5 $\mu$F, 6  
(D) 10 $\mu$F, 10

16. $H(\omega) = \frac{V_o}{V_i} = ?$

![Fig. P1.11.16](image)

(A) $(5 + j20\omega)^{-1}$  
(B) $(5 + j4\omega)^{-1}$  
(C) $(5 + j30\omega)^{-1}$  
(D) $5(1 + j6\omega)^{-1}$

17. The value of input frequency is required to cause a gain equal to 1.5. The value is

![Fig. P1.11.17](image)

(A) 20 rad/s  
(B) 20 Hz  
(C) 10 rad/s  
(D) No such value exists.

18. In the circuit of fig. P1.11.18 phase shift equal to $-45^\circ$ is required at frequency $\omega=20$ rad/s. The value of $R$ is

![Fig. P1.11.18](image)

(A) 200 k$\Omega$  
(B) 150 k$\Omega$  
(C) 100 k$\Omega$  
(D) 50 k$\Omega$

19. For the circuit of fig. P1.11.19 the input frequency is adjusted until the gain is equal to 0.6. The value of the frequency is

![Fig. P1.11.19](image)

(A) 20 rad/s  
(B) 20 Hz  
(C) 40 rad/s  
(D) 40 Hz
31. Bode diagram of the network function $V_o/V_s$ for the circuit of fig. P1.11.30 is

![Bode diagram](image)

**SOLUTIONS**

1. (B) $\text{BW} = \omega_2 - \omega_1 = 2\pi(90 - 86)k = 8\pi \text{ krad/s}$
   
   $$\text{BW} = \frac{1}{RC} \implies C = \frac{1}{RBW} = \frac{1}{8\pi \times 10^5 \times 2 \times 10^3} = 19.89 \text{ nF}$$

2. (C) $\omega_2 = \frac{(\omega_1 + \omega_2)}{2} = \frac{2\pi(90 + 86)}{2} = 176\pi \text{ krad/s}$
   
   $$\omega_2 = \frac{1}{\sqrt{LC}} \implies L = \frac{1}{\omega_2^2} \cdot C$$
   
   $$L = \frac{1}{(176\pi \times 10^3)^2(20 \times 10^{-3})} = 0.16 \text{ mH}$$

3. (A) $Q = \frac{\omega_2}{B} = \frac{176\pi k}{8\pi k} = 22$

4. (A) At mid-band frequency $Z = R$, $Y = \frac{1}{R}$
   
   $$R = \frac{1}{25 \times 10^{-3}} = 40 \Omega$$

5. (C) $Q = \omega_2 RC$
   
   $$C = \frac{Q}{\omega_2 R} = \frac{80}{200 \times 10^3 \times 40} = 10 \mu\text{F}$$

6. (B) $Q_0 = R \sqrt{\frac{C}{L}} \implies 200 = 10^3 \sqrt{\frac{10^{-6}}{L}}$
   
   $$L = 25 \mu\text{H}$$

7. (C) $Q_0 = R \sqrt{\frac{C}{L}} = 10^3 \sqrt{\frac{50 \times 10^{-6}}{10 \times 10^{-3}}} = 70.7$

8. (B) $\omega_0 = \frac{1}{\sqrt{LC}}$
   
   $$C = \frac{1}{10 \times 10^{-3} \times (10^6)^2} = 100 \text{ pF}$$

   $$\text{BW} = \frac{R}{L} \implies R = 10 \times 10^{-3} \times 10^3 = 10$$

9. (A) $\text{BW} = \frac{R}{L}$
   
   $$\implies \frac{R}{1 \times 10^{-3}} = 15.9 \times 2\pi = 0.1\Omega$$

10. (B) $Q = R \sqrt{\frac{C}{L}}$

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11. (A) Thevenin equivalent seen by L–C combination

\[ 3 = v_i + 10 \left( \frac{v_i - 0.105v_i}{125} \right) \quad \Rightarrow \quad v_i = 100 \]

\[ I_{vc} = \frac{1100}{125} = 0.8 \, \text{V} \]

Open Circuit: \( v_i = 0 \), \( v_{vc} = 3 \, \text{V} \)

\[ R_{TH} = \frac{3}{0.8} = 3.75 \, \Omega \]

\[ Q_v = \frac{\omega v}{R} = \frac{1000 \times 4}{375} = 1066.67 \]

\[ |v_{c,max}| = Q_v R_{TH} = 1066.67 \times 3 = 3200 \, \text{V} \]

12. (B) Applying 1 A at input port \( V_i = 10 \, \text{V} \) voltage across 1 A source

\[ V_{tot} = 10 + j0.10^{-3} - \frac{j}{\omega 50 \times 10^{-3}} (5 + 1) \]

\[ Z_{in} = V_{tot} \]

At resonance \( \text{Im} \{ Z_{in} \} = 0 \)

\[ \Rightarrow \quad \omega a 10^{-3} = 0 \]

\[ f = 55 \, \text{kHz} \]

13. (C) \( Y = j\omega 600 \times 10^{-12} + \frac{1}{2 \times 10^{12}} + \frac{1}{1.8 + j0.10^{-5}} \)

\[ = j\omega 6 \times 10^{-10} + 45.45 + \frac{1.8 - j0.10^{-5}}{3.24 + \omega^2 10^{-56}} \]

At resonance \( \text{Im} \{ Y \} = 0 \)

\[ \omega a 6 \times 10^{-10} (324 + \omega^210^{-10}) - \omega a 0^{-5} = 0 \]

\[ 3.24 + \omega^2 10^{-50} = 16.67 \times 10^5 \omega a = 129 \, \text{Mrad/s} \]

\[ f = \frac{\omega a}{2\pi} = 2.05 \, \text{MHz} \]

14. (C) \( V_C = \frac{V_i}{jC\omega} = \frac{V_i}{1 + j2 \times 10^3 C\omega} \)

\[ V_o = AV = \frac{15k}{16k + 30k} = \frac{2AV}{3} = \frac{2AV}{3(1 + j2 \times 10^3 C\omega)} \]

\[ \Rightarrow \quad \frac{V_o}{V_i} = \frac{2A}{1 + j2\pi \times 10^6 C\omega} \]

\[ \Rightarrow \quad \frac{2A}{3} = 4 \quad \Rightarrow \quad A = 6, \quad 2 \times 10^3 C = 0.01 \]

\[ C = 5 \, \mu\text{F} \]

15. (A) \( I_a = \frac{V_i}{20 + j4\omega} \quad V_o = \frac{3I_a}{0.25j\omega} \)

\[ V_o \]

16. (A) \( Z_i = \frac{10}{j\omega(0.5)} = \frac{10}{1 + j3\omega} \)

\[ V_o \]

17. (D) \( H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC} \)

\[ \text{gain} = \frac{1}{\sqrt{1 + \omega^2 RC}} \]

For any value of \( \omega R C \) gain \( \leq 1 \).

Thus (D) is correct option.

18. (D) \( H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega CR} \)

\[ \text{phase shift} = -\tan^{-1}(\omega CR) = -45^\circ \]

\( \omega CR = 1 \),

\[ 20 \times 1 \times 10^{-6} R = 1 \quad \Rightarrow \quad R = 50 \, \text{k}\Omega. \]

19. (A) \( H(\omega) = \frac{V_o}{V_i} = \frac{R}{\sqrt{1 + j\omega L}} \)

\[ \text{gain} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{30}{\sqrt{900^2 + 4\omega^2 + 0.6}} \]

\( \omega \approx \frac{\sqrt{50^2 - 30^2}}{2} = 20 \, \text{rad/s} \)

20. (A) \( H(\omega) = \frac{V_o}{V_i} = \frac{1}{1 + j\omega CR} = \frac{1}{1 + j} \)

\[ \text{Phase shift} = -\tan^{-1}(\omega CR) = -45^\circ \]

\[ \text{gain} = \frac{1}{|1 + j|} = \frac{1}{\sqrt{2}} = 0.707 \]

21. (B) \( BW = \omega a - \omega b = 2\pi(456 - 434) = 44 \pi \)

\( \omega b = 2\pi f_o = QBW = 20 \times 44 \pi \)

\( f_o = 440 \, \text{Hz} \)

22. (C) \( f_o = \frac{1}{2\pi \sqrt{LC}} \)
23. (B) \( f_o = \frac{1}{2\pi} \frac{1}{LC} - \frac{R^2}{L^2} \)

\[ R = \frac{400}{240 \times 10^{-3}} = 10^7 \text{ \Omega} \]

\[ \frac{1}{LC} = \frac{240 \times 10^{-3} \times 120 \times 10^{-15}}{288} = 10^{15} \text{ } \]

\[ R < \frac{1}{LC}, \quad \omega_o = \frac{1}{2\pi LC} = 938 \text{ kHz} \]

24. (B) \( \omega_o = \frac{1}{RC} \), \( R \) and \( C \) should be as small as possible.

\[ R = \frac{3.3}{(18) \left(\frac{3.3 + 18}{10 + 30}\right)} = 1.165 \text{ k}\Omega \]

\[ C = \frac{10}{(30) \left(\frac{10 + 30}{10 + 30}\right)} = 7.5 \text{ pF} \]

\[ \omega = \frac{1}{1.165 \times 7.5 \times 10^{-9}} = 114.5 \times 10^5 \text{ rad/s} \]

25. (D) \( R' = K_m \cdot R = 800 \times 12 \times 10^3 = 9.6 \text{ M}\Omega \)

\[ L' = \frac{K_m \cdot L}{K_m} = \frac{800 \times 40 \times 10^{-6}}{1000} = 32 \text{ \mu F} \]

\[ C' = \frac{C}{K_m} \cdot K_p = 30 \times \frac{10^{-9}}{80} \times 1000 = 0.375 \text{ pF} \]

26. (A) \( L'C'^2 = \frac{LC}{K_f^2} \Rightarrow \frac{K_f^2}{1} = 4 \times 20 \times 10^{-3} \times 10^{-6} \)

\[ \Rightarrow K_f = 2 \times 10^{-4} \]

\[ \frac{L'}{C'} = \frac{L}{C} \cdot \left(\frac{K_m^2}{K_f^2}\right) \Rightarrow K_m^2 = \frac{(1)(20 \times 10^{-3})}{(2)(4 \times 10^{-3})} \]

\[ \Rightarrow K_m = 0.05 \]

27. (D) \( \omega_e = 2\pi f_e = \frac{1}{RC} \)

\[ \Rightarrow R = \frac{1}{2\pi \times 20 \times 10^{-3} \times 0.5 \times 10^{-6}} = 15.9 \text{ \Omega} \]

28. (A) \( R_{th} \) across the capacitor is

\[ R_{th} = (1\text{k} + 4\text{k}) || 5\text{k} = 2.5 \text{ k}\Omega \]

\[ f_e = \frac{1}{2\pi \times 2.5 \times 10^3 \times 40 \times 10^{-9}} = 1.06 \text{ kHz} \]

29. (B) \( \omega_e = 2\pi f_e = \frac{1}{RC} \)

\[ \Rightarrow R = \frac{1}{2\pi \times 15 \times 10 \times 10^{-6}} = 1.06 \text{ k}\Omega \]

30. (B) \( 20 \log H = 20 \log \frac{1}{\omega^2} = -40 \log \omega \)

31. (D) \( \frac{V_o}{V_s} = \frac{2 + \frac{1}{j\omega30 \times 10^{-3}}}{6 + \frac{1}{j\omega30 \times 10^{-3}}} = \frac{1 + \frac{j\omega}{356}}{16.67} \)

-20 dB/decade line starting from \( \omega = 5.56 \text{ rad/s} \)

20 dB/decade line starting from \( \omega = 16.67 \text{ rad/s} \)

Hence -20 dB/decade line for \( 5.56 < \omega < 16.67 \) parallel to \( \omega \) axis to \( \omega > 16.67 \)
In the problems assume the parameter given in following table. Use the temperature \( T = 300 \) K unless otherwise stated.

<table>
<thead>
<tr>
<th>Property</th>
<th>Si</th>
<th>GaAs</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandgap Energy</td>
<td>1.12</td>
<td>1.42</td>
<td>0.66</td>
</tr>
<tr>
<td>Dielectric Constant</td>
<td>11.7</td>
<td>13.1</td>
<td>16.0</td>
</tr>
<tr>
<td>Effective density of states in conduction band ( N_c ) ( \text{cm}^{-3} )</td>
<td>( 2.8 \times 10^{18} )</td>
<td>( 4.7 \times 10^{17} )</td>
<td>( 1.04 \times 10^{19} )</td>
</tr>
<tr>
<td>Effective density of states in valence band ( N_v ) ( \text{cm}^{-3} )</td>
<td>( 1.04 \times 10^{18} )</td>
<td>( 7.0 \times 10^{18} )</td>
<td>( 6.0 \times 10^{18} )</td>
</tr>
<tr>
<td>Intrinsic carrier concentration ( n_i ) ( \text{cm}^{-3} )</td>
<td>( 1.5 \times 10^{10} )</td>
<td>( 1.8 \times 10^{6} )</td>
<td>( 2.4 \times 10^{13} )</td>
</tr>
<tr>
<td>Mobility</td>
<td>1350</td>
<td>480</td>
<td>3900</td>
</tr>
<tr>
<td>Electron</td>
<td>8500</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Hole</td>
<td>1040</td>
<td>1900</td>
<td></td>
</tr>
</tbody>
</table>

1. In germanium semiconductor material at \( T = 400 \) K the intrinsic concentration is
   (A) \( 26.8 \times 10^{14} \) cm\(^{-3}\)   (B) \( 18.4 \times 10^{14} \) cm\(^{-3}\)
   (C) \( 8.5 \times 10^{14} \) cm\(^{-3}\)    (D) \( 3.6 \times 10^{14} \) cm\(^{-3}\)

2. The intrinsic carrier concentration in silicon is to be no greater than \( n_i = 1 \times 10^{12} \) cm\(^{-3}\). The maximum temperature allowed for the silicon is (\( E_g = 1.12 \) eV)
   (A) 300 K    (B) 360 K
   (C) 382 K    (D) 364 K

3. Two semiconductor material have exactly the same properties except that material A has a bandgap of 1.0 eV and material B has a bandgap energy of 1.2 eV. The ratio of intrinsic concentration of material A to that of material B is
   (A) 2016    (B) 47.5
   (C) 58.23   (D) 1048

4. In silicon at \( T = 300 \) K the thermal-equilibrium concentration of electron is \( n_0 = 5 \times 10^4 \) cm\(^{-3}\). The hole concentration is
   (A) \( 4.5 \times 10^{15} \) cm\(^{-3}\)   (B) \( 4.5 \times 10^{15} \) m\(^{-3}\)
   (C) \( 0.3 \times 10^{-6} \) cm\(^{-3}\)    (D) \( 0.3 \times 10^{-6} \) m\(^{-3}\)

5. In silicon at \( T = 300 \) K if the Fermi energy is 0.22 eV above the valence band energy, the value of \( p_0 \) is
   (A) \( 2 \times 10^{15} \) cm\(^{-3}\)   (B) \( 10^{15} \) cm\(^{-3}\)
   (C) \( 3 \times 10^{15} \) cm\(^{-3}\)    (D) \( 4 \times 10^{15} \) cm\(^{-3}\)

6. The thermal-equilibrium concentration of hole \( p_0 \) in silicon at \( T = 300 \) K is \( 10^{15} \) cm\(^{-3}\). The value of \( n_o \) is
   (A) \( 3.8 \times 10^6 \) cm\(^{-3}\)   (B) \( 4.4 \times 10^4 \) cm\(^{-3}\)
   (C) \( 2.6 \times 10^4 \) cm\(^{-3}\)    (D) \( 4.3 \times 10^6 \) cm\(^{-3}\)

7. In germanium semiconductor at \( T = 300 \) K, the acceptor concentrations is \( N_a = 10^{13} \) cm\(^{-3}\) and donor concentration is \( N_d = 0 \). The thermal equilibrium concentration \( p_0 \) is
   (A) \( 2.97 \times 10^9 \) cm\(^{-3}\)   (B) \( 2.68 \times 10^{12} \) cm\(^{-3}\)
   (C) \( 2.95 \times 10^{13} \) cm\(^{-3}\)    (D) \( 2.4 \) cm\(^{-3}\)
Statement for Q.8-9:

In germanium semiconductor at \( T = 300 \) K, the impurity concentration are
\[
N_d = 5 \times 10^{15} \text{ cm}^{-3} \quad \text{and} \quad N_a = 0
\]

The thermal equilibrium electron concentration \( n_0 \) is
(A) \( 5 \times 10^{15} \text{ cm}^{-3} \) 
(B) \( 1.15 \times 10^{11} \text{ cm}^{-3} \)
(C) \( 115 \times 10^5 \text{ cm}^{-3} \) 
(D) \( 5 \times 10^6 \text{ cm}^{-3} \)

The thermal equilibrium hole concentration \( p_0 \) is
(A) \( 3.96 \times 10^{13} \text{ cm}^{-3} \)
(B) \( 19.5 \times 10^{13} \text{ cm}^{-3} \)
(C) \( 4.36 \times 10^{12} \text{ cm}^{-3} \)
(D) \( 3.96 \times 10^{13} \text{ cm}^{-3} \)

In germanium at \( T = 300 \) K is uniformly doped with an acceptor concentration of \( \times 10^{15} \text{ cm}^{-3} \) and a donor concentration of \( \times 10^{15} \text{ cm}^{-3} \). The position of fermi energy with respect to intrinsic Fermi level is
(A) \( \times 10^{20} \text{ cm}^{-3} \)
(B) \( 7 \text{ cm}^{-3} \)
(C) \( 9.86 \times 10^{19} \text{ cm}^{-3} \)
(D) \( 3 \times 10^3 \text{ cm}^{-3} \)

In a sample of gallium arsenide at \( T = 200 \) K, \( n_0 = 5 \times 10^5 \) and \( N_a = 0 \). The value of \( n_0 \) is
(A) \( 9.86 \times 10^{19} \text{ cm}^{-3} \)
(B) \( 4.86 \times 10^{19} \text{ cm}^{-3} \)
(C) \( 4.86 \times 10^8 \text{ cm}^{-3} \)
(D) \( 3 \times 10^3 \text{ cm}^{-3} \)

A sample of silicon at \( T = 300 \) K is doped with boron at a concentration of \( 2.5 \times 10^{13} \text{ cm}^{-3} \) and with arsenic at a concentration of \( 1 \times 10^{13} \text{ cm}^{-3} \). The material is
(A) \( p \)-type with \( p_0 = 1.5 \times 10^7 \text{ cm}^{-3} \)
(B) \( n \)-type with \( n_0 = 1.5 \times 10^7 \text{ cm}^{-3} \)
(C) \( n \)-type with \( n_0 = 1.5 \times 10^7 \text{ cm}^{-3} \)
(D) \( n \)-type with \( n_0 = 1.5 \times 10^7 \text{ cm}^{-3} \)

In a sample of gallium arsenide at \( T = 200 \) K, \( n_0 = 5 \times 10^5 \) and \( N_a = 0 \). The value of \( n_0 \) is
(A) \( 9.86 \times 10^{19} \text{ cm}^{-3} \)
(B) \( 4.86 \times 10^{19} \text{ cm}^{-3} \)
(C) \( 4.86 \times 10^8 \text{ cm}^{-3} \)
(D) \( 3 \times 10^3 \text{ cm}^{-3} \)

For a particular semiconductor at \( T = 300 \) K, the Fermi energy level with respect to the center of the bandgap is
(A) \( +0.045 \text{ eV} \)
(B) \( -0.046 \text{ eV} \)
(C) \( +0.039 \text{ eV} \)
(D) \( -0.039 \text{ eV} \)

A silicon sample contains acceptor atoms at a concentration of \( 5 \times 10^{15} \text{ cm}^{-3} \). Donor atoms are added forming and \( n \)-type compensated semiconductor such that the Fermi level is 0.215 eV below the conduction band edge. The concentration of donors added are
(A) \( 1.2 \times 10^{16} \text{ cm}^{-3} \)
(B) \( 4.6 \times 10^{16} \text{ cm}^{-3} \)
(C) \( 39 \times 10^{12} \text{ cm}^{-3} \)
(D) \( 2.4 \times 10^{12} \text{ cm}^{-3} \)

A silicon semiconductor sample at \( T = 300 \) K is doped with phosphorus atoms at a concentration of \( 10^{15} \text{ cm}^{-3} \). The position of the Fermi level with respect to the intrinsic Fermi level is
(A) \( 0.3 \text{ eV} \)
(B) \( 0.2 \text{ eV} \)
(C) \( 0.1 \text{ eV} \)
(D) \( 0.4 \text{ eV} \)

A silicon crystal having a cross-sectional area of 0.001 cm² and a length of 20 \( \mu m \) is connected to its ends to a 20 V battery. At \( T = 300 \) K, we want a current of 100 mA in crystal. The concentration of donor atoms to be added is
(A) \( 2.4 \times 10^{13} \text{ cm}^{-3} \)
(B) \( 4.6 \times 10^{13} \text{ cm}^{-3} \)
(C) \( 7.8 \times 10^{14} \text{ cm}^{-3} \)
(D) \( 8.4 \times 10^{14} \text{ cm}^{-3} \)

The cross sectional area of silicon bar is 100 \( \mu m^2 \). The length of bar is 1 mm. The bar is doped with arsenic atoms. The resistance of bar is
(A) \( 2.58 \Omega \)
(B) \( 11.36 \Omega \)
(C) \( 1.36 \Omega \)
(D) \( 24.8 \Omega \)

A thin film resistor is to be made from a GaAs film doped \( n \)-type. The resistor is to have a value of 2 kΩ. The resistive length is to be 200 \( \mu m \) and area is to be \( 10^{-6} \text{ cm}^2 \). The doping efficiency is known to be 90%. The mobility of electrons is 8000 cm²/V·s. The doping needed is
(A) \( 8.7 \times 10^{15} \text{ cm}^{-3} \)
(B) \( 8.7 \times 10^{21} \text{ cm}^{-3} \)
(C) \( 4.6 \times 10^{15} \text{ cm}^{-3} \)
(D) \( 4.6 \times 10^{21} \text{ cm}^{-3} \)
21. A silicon sample doped $n$-type at $10^{18} \text{cm}^{-3}$ have a resistance of $10 \Omega$. The sample has an area of $10^{-6} \text{cm}^2$ and a length of $10 \mu\text{m}$. The doping efficiency of the sample is ($\mu_n = 800 \text{cm}^2/\text{V} \cdot \text{s}$)
(A) 43.2%  
(B) 78.1%  
(C) 96.3%  
(D) 54.3%

22. Six volts is applied across a 2 cm long semiconductor bar. The average drift velocity is $10^4 \text{cm/s}$. The electron mobility is
(A) $4396 \text{cm}^2/\text{V} \cdot \text{s}$  
(B) $3 \times 10^4 \text{cm}^2/\text{V} \cdot \text{s}$  
(C) $6 \times 10^4 \text{cm}^2/\text{V} \cdot \text{s}$  
(D) $3333 \text{cm}^2/\text{V} \cdot \text{s}$

23. For a particular semiconductor material following parameters are observed:

\[
\mu_n = 1000 \text{cm}^2/\text{V} \cdot \text{s},
\mu_p = 600 \text{cm}^2/\text{V} \cdot \text{s},
N_e = N_v = 10^{19} \text{cm}^{-3}
\]

These parameters are independent of temperature. The measured conductivity of the intrinsic material is $\sigma = 10^{-6} (\Omega \cdot \text{cm})^{-1}$ at $T = 300 \text{ K}$. The conductivity at $T = 500 \text{ K}$ is
(A) $2 \times 10^{-4} (\Omega \cdot \text{cm})^{-1}$  
(B) $4 \times 10^{-5} (\Omega \cdot \text{cm})^{-1}$  
(C) $2 \times 10^{-5} (\Omega \cdot \text{cm})^{-1}$  
(D) $6 \times 10^{-3} (\Omega \cdot \text{cm})^{-1}$

24. An $n$-type silicon sample has a resistivity of 5 $\Omega \cdot \text{cm}$ at $T = 300 \text{ K}$. The mobility is $\mu_n = 1350 \text{cm}^2/\text{V} \cdot \text{s}$. The donor impurity concentration is
(A) $2.86 \times 10^{-14} \text{cm}^3$  
(B) $9.25 \times 10^{14} \text{cm}^3$  
(C) $11.46 \times 10^{15} \text{cm}^3$  
(D) $1.1 \times 10^{-15} \text{cm}^3$

25. In a silicon sample the electron concentration drops linearly from $10^{18} \text{ cm}^{-3}$ to $10^{16} \text{ cm}^{-3}$ over a length of 2.0 $\mu\text{m}$. The current density due to the electron diffusion current is ($D_n = 35 \text{ cm}^2/\text{s}$).
(A) $9.3 \times 10^4 \text{A/cm}^2$  
(B) $2.8 \times 10^4 \text{A/cm}^2$  
(C) $9.3 \times 10^9 \text{A/cm}^2$  
(D) $2.8 \times 10^9 \text{A/cm}^2$

26. In a GaAs sample the electrons are moving under an electric field of 5 $\text{kV/cm}$ and the carrier concentration is uniform at $10^{16} \text{ cm}^{-3}$. The electron velocity is the saturated velocity of $10^7 \text{ cm/s}$. The drift current density is
(A) $1.6 \times 10^4 \text{A/cm}^2$  
(B) $2.4 \times 10^4 \text{A/cm}^2$  
(C) $1.6 \times 10^9 \text{A/cm}^2$  
(D) $2.4 \times 10^8 \text{A/cm}^2$

27. For a sample of GaAs scattering time is $\tau_n = 10^{-13} \text{ s}$ and electron's effective mass is $m_e' = 0.067 m_e$. If an electric field of 1 $\text{kV/cm}$ is applied, the drift velocity produced is
(A) $2.6 \times 10^6 \text{ cm/s}$  
(B) 263 $\text{ cm/s}$  
(C) $14.8 \times 10^6 \text{ cm/s}$  
(D) 482

28. A gallium arsenide semiconductor at $T = 300 \text{ K}$ is doped with impurity concentration $N_d = 10^{16} \text{ cm}^{-3}$. The mobility $\mu_n$ is 7500 $\text{cm}^2/\text{V} \cdot \text{s}$. For an applied field of 10 $\text{V/cm}$ the drift current density is
(A) 120 $\text{A/cm}^2$  
(B) 120 $\text{A/cm}^2$  
(C) $12 \times 10^4 \text{A/cm}^2$  
(D) $12 \times 10^4 \text{A/cm}^2$

29. In a particular semiconductor the donor impurity concentration is $N_d = 10^{14} \text{ cm}^{-3}$. Assume the following parameters,

\[
\mu_n = 1000 \text{ cm}^2/\text{V} \cdot \text{s},
N_e = 2 \times 10^{18} \left( \frac{T}{300} \right)^{3/2} \text{ cm}^{-3},
N_v = 1 \times 10^{16} \left( \frac{T}{300} \right)^{3/2} \text{ cm}^{-3},
E_g = 1.1 \text{ eV}.
\]

An electric field of $E = 10 \text{ V/cm}$ is applied. The electric current density at 300 K is
(A) 2.3 $\text{A/cm}^2$  
(B) 1.6 $\text{A/cm}^2$  
(C) 9.6 $\text{A/cm}^2$  
(D) 3.4 $\text{A/cm}^2$

**Statement for Q.30-31:**

A semiconductor has following parameter

\[
\mu_n = 7500 \text{ cm}^2/\text{V} \cdot \text{s},
\mu_p = 300 \text{ cm}^2/\text{V} \cdot \text{s},
n_i = 3.6 \times 10^{12} \text{ cm}^{-3}
\]

30. When conductivity is minimum, the hole concentration is
(A) $72 \times 10^{11} \text{ cm}^{-3}$  
(B) $1.8 \times 10^{13} \text{ cm}^{-3}$  
(C) $1.44 \times 10^{11} \text{ cm}^{-3}$  
(D) $9 \times 10^{13} \text{ cm}^{-3}$

31. The minimum conductivity is
(A) $0.6 \times 10^{-3} (\Omega \cdot \text{cm})^{-1}$  
(B) $1.7 \times 10^{-3} (\Omega \cdot \text{cm})^{-1}$  
(C) $2.4 \times 10^{-3} (\Omega \cdot \text{cm})^{-1}$  
(D) $6.8 \times 10^{-3} (\Omega \cdot \text{cm})^{-1}$
32. A particular intrinsic semiconductor has a resistivity of 50 (Ω cm) at \( T = 300 \) K and 5 (Ω cm) at \( T = 330 \) K. If change in mobility with temperature is neglected, the bandgap energy of the semiconductor is
(A) 1.9 eV
(B) 1.3 eV
(C) 2.6 eV
(D) 0.64 eV

33. Three scattering mechanism exist in a semiconductor. If only the first mechanism were present, the mobility would be 500 \( \text{cm}^2/\text{V s} \). If only the second mechanism were present, the mobility would be 750 \( \text{cm}^2/\text{V s} \). If only third mechanism were present, the mobility would be 1500 \( \text{cm}^2/\text{V s} \). The net mobility is
(A) 2750 \( \text{cm}^2/\text{V s} \)
(B) 1114 \( \text{cm}^2/\text{V s} \)
(C) 818 \( \text{cm}^2/\text{V s} \)
(D) 250 \( \text{cm}^2/\text{V s} \)

34. In a sample of silicon at \( T = 300 \) K, the electron concentration varies linearly with distance, as shown in fig. P2.1.34. The diffusion current density is found to be \( J_n = 0.19 \) A/cm². If the electron diffusion coefficient is \( D_n = 25 \) cm²/s, the electron concentration at is

![Graph showing linear relationship between electron concentration and distance](Fig. P2.1.34)

(A) \( 4.86 \times 10^8 \) cm⁻³
(B) \( 2.5 \times 10^{12} \) cm⁻³
(C) \( 9.8 \times 10^{26} \) cm⁻³
(D) \( 5.4 \times 10^{15} \) cm⁻³

35. The hole concentration in \( p \)-type GaAs is given by
\[
p = 10^{16} \left( 1 - \frac{x}{L} \right) \text{cm}^{-3} \text{ for } 0 \leq x \leq L
\]
where \( L = 10 \mu\text{m} \). The hole diffusion coefficient is 10 cm²/s. The hole diffusion current density at \( x = 5 \mu\text{m} \) is
(A) 20 A/cm²
(B) 16 A/cm²
(C) 24 A/cm²
(D) 30 A/cm²

36. For a particular semiconductor sample consider following parameters:

Hole concentration \( p_0 = 10^{15} e^{-\frac{x}{L}} \) cm⁻³, \( x \geq 0 \)
Electron concentration \( n_0 = 5 \times 10^{14} e^{-\frac{x}{L}} \) cm⁻³, \( x \leq 0 \)
Hole diffusion coefficient \( D_p = 10 \) cm²/s
Electron diffusion coefficients \( D_n = 25 \) cm²/s
Hole diffusion length \( L_p = 5 \times 10^{-4} \) cm,
Electron diffusion length \( L_n = 10^{-3} \) cm

The total current density at \( x = 0 \) is
(A) 1.2 A/cm²
(B) 5.2 A/cm²
(C) 3.8 A/cm²
(D) 2 A/cm²

37. A germanium Hall device is doped with \( 5 \times 10^{15} \) donor atoms per cm³ at \( T = 300 \) K. The device has the geometry \( d = 5 \times 10^{-3} \) cm, \( W = 2 \times 10^{-2} \) cm and \( L = 0.1 \) cm. The current is \( I_s = 250 \mu\text{A} \), the applied voltage is \( V_x = 100 \) mV, and the magnetic flux is \( B_x = 5 \times 10^{-2} \) tesla. The Hall voltage is
(A) -0.31 mV
(B) 0.31 mV
(C) 3.26 mV
(D) -3.26 mV

**Statement for Q.38-39:**

A silicon Hall device at \( T = 300 \) K has the geometry \( d = 10^{-3} \) cm, \( W = 10^{-2} \) cm, \( L = 10^{-1} \) cm. The following parameters are measured: \( I_s = 0.75 \) mA, \( V_x = 15 \) V, \( V_H = +5.8 \) mV, tesla

38. The major fraction carrier concentration is
(A) \( 8 \times 10^{15} \) cm⁻³, \( n \)-type
(B) \( 8 \times 10^{15} \) cm⁻³, \( p \)-type
(C) \( 4 \times 10^{15} \) cm⁻³, \( n \)-type
(D) \( 4 \times 10^{15} \) cm⁻³, \( p \)-type

39. The majority carrier mobility is
(A) 430 cm²/V s
(B) 215 cm²/V s
(C) 390 cm²/V s
(D) 195 cm²/V s

40. In a semiconductor \( n_0 = 10^{15} \) cm⁻³ and \( n_i = 10^{10} \) cm⁻³. The excess-carrier life time is \( 10^{-6} \) s. The excess hole concentration is \( \delta p = 4 \times 10^{13} \) cm⁻³. The electron-hole recombination rate is
(A) \( 4 \times 10^{19} \) cm³s⁻¹
(B) \( 4 \times 10^{14} \) cm³s⁻¹
(C) \( 4 \times 10^{24} \) cm³s⁻¹
(D) \( 4 \times 10^{11} \) cm³s⁻¹
**SOLUTIONS**

1. (C) \( n_i^2 = N_c N_v e \left( \frac{-E_g}{kT} \right) \)

\[ V_t = 0.0259 \left( \frac{400}{300} \right) = 0.0345 \]

For Ge at 300 K, 
\[ N_c = 1.04 \times 10^{19}, N_v = 6.0 \times 10^{18}, E_g = 0.66 \text{ eV} \]
\[ n_i^2 = 1.04 \times 10^{19} \times 6.0 \times 10^{18} \times \left( \frac{400}{300} \right)^3 \times e^{\frac{0.66}{0.0345}} \]
\[ \Rightarrow n_i = 8.5 \times 10^{14} \text{ cm}^{-3} \]

2. (C) \( n_i^2 = N_c N_v e \left( \frac{-E_g}{kT} \right) \)

\[ \left( 10^{12} \right)^2 = 2.8 \times 10^{19} \times 104 \times 10^9 \left( \frac{T}{300} \right)^3 e^{-\frac{1.13eV}{kT}} \]
\[ T^3 e^{-\frac{1.13eV}{kT}} = 9.28 \times 10^{-8}, \text{ By trial } T = 382 \text{ K} \]

3. (B) \( n_d = e^{\frac{E_g - E_F}{kT}} \)

\[ \frac{n_{dA}}{n_{dB}} = e^{\left( \frac{E_g - E_F}{kT} \right)} = 2257.5 \Rightarrow \frac{n_{dA}}{n_{dB}} = 475 \]

4. (A) \( p_0 = n_0^2 = \frac{(1.5 \times 10^{15})^2}{5 \times 10^4} = 4.5 \times 10^{15} \text{ cm}^{-3} \)

5. (A) \( p_0 = N_c e^{-\frac{(E_F - E_c)}{kT}} = 104 \times 10^{19} \times e^{-0.22} = 2 \times 10^{15} \text{ cm}^{-3} \)

6. (B) \( p_0 = N_c e^{-\frac{(E_F - E_c)}{kT}} \Rightarrow E_F - E_c = kT \ln \left( \frac{N_c}{p_0} \right) \)

At 300 K, \( N_c = 10 \times 10^{19} \text{ cm}^{-3} \)
\[ E_F - E_c = 0.0259 \ln \left( \frac{104 \times 10^{19}}{10^{15}} \right) = 0.239 \text{ eV} \]
\[ n_0 = N_c e^{-\frac{(E_F - E_c)}{kT}} \]

At 300 K, \( N_c = 2.8 \times 10^{19} \text{ cm}^{-3} \)
\[ E_F - E_c = 1.12 - 0.239 = 0.881 \text{ eV} \]
\[ n_0 = 4.4 \times 10^{14} \text{ cm}^{-3} \]

7. (C) \( p_0 = \frac{N_a - N_d}{2} + \sqrt{\left( \frac{N_a - N_d}{2} \right)^2 + n_i^2} \)

For Ge \( n_i = 2.4 \times 10^3 \)
\[ p_0 = \frac{10^{13}}{2} + \sqrt{\left( \frac{10^{13}}{2} \right)^2 + (2.4 \times 10^{13})^2} = 2.95 \times 10^{13} \text{ cm}^{-3} \]
20. (A) \( R = \frac{L}{\sigma A} \), \( \sigma = e\mu_n n_0 \), \( R = \frac{L}{e\mu_n n_0 A} \)

\[ n_0 = \frac{L}{e\mu_n AR} \]

\( n_0 = 0.9 N_d \)

\[ 20 \times 10^{-4} \]

\( \frac{0.9(1.6 \times 10^{-19})(8000)(10^{-6})(2 \times 10^3)}{2 \times 10^3} = 8.7 \times 10^{15} \text{ cm}^{-3} \]

Efficiency \( \frac{n_0}{N_d} \times 100 = \frac{7.8 \times 10^{17}}{10^{16}} \times 100 = 78.1 \% \)

21. (B) \( \sigma = e\mu_n n_0 \), \( R = \frac{L}{\sigma A} \), \( n_0 = \frac{L}{e\mu_n AR} \)

\[ 10^{-4} \]

\( \frac{1(6 \times 10^{-19})(1000)(10^{-6})}{10} = 7.81 \times 10^{17} \text{ cm}^{-3} \)

22. (D) \( E = \frac{V}{L} = \frac{6}{2} = 3 \text{ V/cm} \), \( v_d = \mu_n E \),

\( \mu_n = \frac{v_d}{E} = \frac{10^4}{3} = 3333 \text{ cm}^2/\text{V} \cdot \text{s} \)

23. (D) \( \sigma = e n_i (\mu_n + \mu_p) \)

\[ 10^{-6} = (1.6 \times 10^{-19})(1000 + 600) n_i \]

At \( T = 300 \text{ K} \), \( n_i = 391 \times 10^9 \text{ cm}^{-3} \)

\[ n_i^2 = N_e N_v e^{-\frac{E_g}{kT}} \Rightarrow E_g = kT \ln \left( \frac{N_e N_v}{n_i^2} \right) \]

\[ = 2(0.0259) \ln \left( \frac{10^{19}}{3.91 \times 10^{17}} \right) = 1.122 \text{ eV} \]

At \( T = 500 \text{ K} \), \( kT = 0.0259 \left( \frac{500}{300} \right) = 0.0432 \text{ eV} \),

\[ n_i^2 = (10^{19})^2 e^{-\frac{1.122}{0.0432}} \text{ cm}^{-3} \]

\( n_i = 2.29 \times 10^{13} \text{ cm}^{-3} \)

\[ = (1.6 \times 10^{-19})(2.29 \times 10^{13})(1000 + 600) \]

\[ = 5.86 \times 10^{-3} \text{ (} \Omega \cdot \text{cm})^{-1} \]

24. (B) \( \rho = \frac{1}{\sigma} = \frac{1}{e\mu_n N_d} \)

\[ N_d = \frac{1}{\rho e\mu_n} = \frac{1}{5(1.6 \times 10^{-19})(1350)} = 9.25 \times 10^{14} \text{ cm}^{-3} \]

25. (B) \( J = eD_n \frac{dn}{dx} \)

\( = (1.6 \times 10^{-19})(35 \frac{10^{18} - 10^{16}}{2 \times 10^{-4}}) = 2.8 \times 10^4 \text{ A/cm}^2 \)

26. (A) \( J = evn = (1.6 \times 10^{-19})(10^{-5})(10^{-16}) = 1.6 \times 10^4 \text{ A/cm}^2 \)

27. (A) \( v_d = \frac{e\mu_n E}{m_v} = \frac{(1.6 \times 10^{-19})(10^{-13})(10^5)}{(0.067)(9.1 \times 10^{-31})} \)

\( = 26.2 \times 10^3 \text{ m/s} = 2.6 \times 10^6 \text{ cm/s} \)

28. (A) \( N_d >> n_i \Rightarrow n_0 = N_d \)

\( J = e\mu_n n_0 E = (1.6 \times 10^{-19})(7500)(10^{16})(10) = 120 \text{ A/cm}^2 \)

29. (D) \( n_i^2 = N_e N_v e^{-\frac{E_g}{kT}} \)

\[ = (2 \times 10^{19})(1 \times 10^{19}) e^{-\frac{1.122}{0.0259}} = 7.18 \times 10^{19} \]

\( n_i = 8.47 \times 10^9 \text{ cm}^{-3} \)

\( N_d >> n_i \Rightarrow N_d = n_0 \)

\( J = e\mu_n n_0 E \)

\( = (1.6 \times 10^{-19})(1000)(10^{14})(100) = 1.6 \text{ A/cm}^2 \)

30. (A) \( \sigma = e\mu_n n_0 + e\mu_p p_0 \) and \( n_0 = \frac{n_i^2}{p_0} \)

\[ \Rightarrow \sigma = e\mu_n \frac{n_i^2}{p_0} + e\mu_p p_0 \]

\[ \frac{d\sigma}{dp_0} = 0 = \frac{-1(e\mu_n n_i^2 + e\mu_p)}{p_0^2} \]

\[ \Rightarrow p_0 = n_i \left( \frac{\mu_n}{\mu_p} \right) = 3.6 \times 10^{12} \left( \frac{7500}{300} \right) \]

\[ = 7.2 \times 10^{11} \text{ cm}^{-3} \]

31. (B) \( \sigma_{\text{min}} = 2\sigma e\mu_n \mu_p / \mu_n + \mu_p = 2en_i \sqrt{\mu_n \mu_p} \)

\[ = 2 \times 1.6 \times 10^{-19}(3.6 \times 10^{12}) \sqrt{7500} \times 300 \]

\[ = 1.7 \times 10^{-3} \text{ (} \Omega \cdot \text{cm})^{-1} \]

32. (B) \( \sigma = \frac{1}{\rho} = e\mu_n n_i \),

\[ \frac{1}{\rho_1} = n_1 = e^{-\frac{E_g}{kT_1}} \]

\[ \frac{1}{\rho_2} = n_2 = e^{-\frac{E_g}{kT_2}} \]

\[ 0.1 = e^{-\frac{E_g}{kT_1}} \frac{1}{T_1} \frac{1}{T_2} \]

\[ E_{\text{g}} \left( \frac{330 - 300}{330 \times 300} \right) = \ln 10 \]

\[ E_{\text{g}} = 22(k 300) \ln 10 = 1.31 \text{ eV} \]

33. (D) \( \frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3} \)
In this chapter, \( N_d \) and \( N_a \) denotes the net donor and acceptor concentration in the individual \( n \) and \( p \)-region.

1. An abrupt silicon in thermal equilibrium at \( T = 300 \) K is doped such that \( E_c - E_p = 0.21 \) eV in the \( n \)-region and \( E_p - E_c = 0.18 \) eV in the \( p \)-region. The built-in potential barrier \( V_{bi} \) is

(A) 0.69 V  
(B) 0.83 V  
(C) 0.61 V  
(D) 0.88 V

2. A silicon \( pn \) junction at \( T = 300 \) K has \( N_d = 10^{14} \) cm\(^{-3} \) and \( N_a = 10^{17} \) cm\(^{-3} \). The built-in voltage is

(A) 0.63 V  
(B) 0.93 V  
(C) 0.026 V  
(D) 0.038 V

3. In a uniformly doped GaAs junction at \( T = 300 \) K, at zero bias, only 20\% of the total space charge region is to be in the \( p \)-region. The built-in potential barrier is \( V_{bi} = 1.20 \) V. The majority carrier concentration in \( n \)-region is

(A) \( 1 \times 10^{16} \) cm\(^{-3} \)  
(B) \( 4 \times 10^{16} \) cm\(^{-3} \)  
(C) \( 1 \times 10^{22} \) cm\(^{-3} \)  
(D) \( 4 \times 10^{22} \) cm\(^{-3} \)

**Statement for Q.4–5:**

An abrupt silicon \( pn \) junction at zero bias and \( T = 300 \) K has dopant concentration of \( N_a = 10^{17} \) cm\(^{-3} \) and \( N_d = 5 \times 10^{15} \) cm\(^{-3} \).

4. The Fermi level on \( n \)-side is

(A) 0.1 eV  
(B) 0.2 eV  
(C) 0.3 eV  
(D) 0.4 eV

5. The Fermi level on \( p \)-side is

(A) 0.2 eV  
(B) 0.1 eV  
(C) 0.4 eV  
(D) 0.3 eV

**Statement for Q.6–8:**

A silicon \( pn \) junction at \( T = 300 \) K with zero applied bias has doping concentrations of \( N_d = 5 \times 10^{16} \) cm\(^{-3} \) and \( N_a = 5 \times 10^{15} \) cm\(^{-3} \).

6. The width of depletion region extending into the \( n \)-region is

(A) \( 4 \times 10^{-6} \) cm  
(B) \( 3 \times 10^{-6} \) cm  
(C) \( 4 \times 10^{-5} \) cm  
(D) \( 3 \times 10^{-5} \) cm

7. The space charge width is

(A) \( 32 \times 10^{-5} \) cm  
(B) \( 4.5 \times 10^{-5} \) cm  
(C) \( 4.5 \times 10^{-4} \) cm  
(D) \( 32 \times 10^{-4} \) cm

8. In depletion region maximum electric field \( |E_{max}| \) is

(A) \( 1 \times 10^{4} \) V/cm  
(B) \( 2 \times 10^{4} \) V/cm  
(C) \( 3 \times 10^{4} \) V/cm  
(D) \( 4 \times 10^{4} \) V/cm

9. An \( n-n \) isotype doping profile is shown in fig. P2.2.9. The built-in potential barrier is \( (n_i = 1.5 \times 10^{10} \) cm\(^{-3} \) )

![Fig. P2.2.9](www.gatehelp.com)
Statement for Q.10–11:

A silicon abrupt junction has dopant concentration \( N_a = 2 \times 10^{16} \, \text{cm}^{-3} \) and \( N_d = 2 \times 10^{15} \, \text{cm}^{-3} \). The applied reverse bias voltage is \( V_R = 8 \, \text{V} \).

10. The maximum electric field \( |E_{max}| \) in depletion region is
   (A) \( 15 \times 10^4 \, \text{V/cm} \)
   (B) \( 7 \times 10^4 \, \text{V/cm} \)
   (C) \( 3.5 \times 10^4 \, \text{V/cm} \)
   (D) \( 5 \times 10^4 \, \text{V/cm} \)

11. The space charge region is
   (A) 2.5 \( \mu \)m
   (B) 25 \( \mu \)m
   (C) 50 \( \mu \)m
   (D) 100 \( \mu \)m

12. A uniformly doped silicon \( pn \) junction has \( N_a = 5 \times 10^{17} \, \text{cm}^{-3} \) and \( N_d = 10^{17} \, \text{cm}^{-3} \). The junction has a cross-sectional area of \( 10^{-4} \, \text{cm}^2 \) and has an applied reverse-bias voltage of \( V_R = 5 \, \text{V} \). The total junction capacitance is
   (A) 10 pF
   (B) 5 pF
   (C) 7 pF
   (D) 3.5 pF

Statement for Q.13–14:

An ideal one-sided silicon \( n^+ p \) junction has uniform doping on both sides of the abrupt junction. The doping relation is \( N_{n^+} = 50 N_a \). The built-in potential barrier is \( V_{bi} = 0.75 \, \text{V} \). The applied reverse bias voltage is \( V_R = 10 \, \text{V} \).

13. The space charge width is
   (A) 1.8 \( \mu \)m
   (B) 1.8 mm
   (C) 1.8 cm
   (D) 1.8 m

14. The junction capacitance is
   (A) \( 3.8 \times 10^{-9} \, \text{F/cm}^2 \)
   (B) \( 9.8 \times 10^{-9} \, \text{F/cm}^2 \)
   (C) \( 2.4 \times 10^{-9} \, \text{F/cm}^2 \)
   (D) \( 5.7 \times 10^{-9} \, \text{F/cm}^2 \)

15. Two \( p^+ n \) silicon junction is reverse biased at \( V_R = 5 \, \text{V} \). The impurity doping concentration in junction A are \( N_a = 10^{18} \, \text{cm}^{-3} \) and \( N_d = 10^{15} \, \text{cm}^{-3} \), and those in junction B are \( N_a = 10^{18} \, \text{cm}^{-3} \) and \( N_d = 10^{15} \, \text{cm}^{-3} \). The ratio of the space charge width is
   (A) 4.36
   (B) 9.8
   (C) 19
   (D) 3.13

16. The maximum electric field in reverse-biased silicon \( pn \) junction is \( |E_{max}| = 3 \times 10^5 \, \text{V/cm} \). The doping concentration are \( N_a = 4 \times 10^{16} \, \text{cm}^{-3} \) and \( N_d = 4 \times 10^{17} \, \text{cm}^{-3} \). The magnitude of the reverse bias voltage is
   (A) 3.6 V
   (B) 9.8 V
   (C) 7.2 V
   (D) 12.3 V

17. An abrupt silicon \( pn \) junction has an applied reverse bias voltage of \( V_R = 10 \, \text{V} \). It has dopant concentration of \( N_a = 10^{18} \, \text{cm}^{-3} \) and \( N_d = 10^{15} \, \text{cm}^{-3} \). The \( pn \) junction area is \( 6 \times 10^{-4} \, \text{cm}^2 \). An inductance of 2.2 mH is placed in parallel with the \( pn \) junction. The resonant frequency is
   (A) 1.7 MHz
   (B) 2.6 MHz
   (C) 3.6 MHz
   (D) 4.3 MHz

18. A uniformly doped silicon \( p^+ n \) junction is to be designed such that at a reverse bias voltage of \( V_R = 10 \, \text{V} \) the maximum electric field is limited to \( E_{max} = 10^6 \, \text{V/cm} \). The maximum doping concentration in the \( n \)-region is
   (A) \( 3 \times 10^{19} \, \text{cm}^{-3} \)
   (B) \( 3 \times 10^{17} \, \text{cm}^{-3} \)
   (C) \( 6 \times 10^{17} \, \text{cm}^{-3} \)
   (D) \( 6 \times 10^{19} \, \text{cm}^{-3} \)

19. A diode has reverse saturation current \( I_s = 10^{-10} \, \text{A} \) and non ideality factor \( \eta = 2 \). If diode voltage is 0.9 V, then diode current is
   (A) 11 mA
   (B) 35 mA
   (C) 83 mA
   (D) 143 mA

20. A diode has reverse saturation current \( I_s = 10^{-18} \, \text{A} \) and nonideal factor \( \eta = 1.05 \). If diode has current of 70 \( \mu \)A, then diode voltage is
   (A) 0.63 V
   (B) 0.87 V
   (C) 0.54 V
   (D) 0.93 V

21. An ideal \( pn \) junction diode is operating in the forward bias region. The change in diode voltage, that will cause a factor of 9 increase in current, is
   (A) 83 mV
   (B) 59 mV
   (C) 43 mV
   (D) 31 mV

22. An \( pn \) junction diode is operating in reverse bias region. The applied reverse voltage, at which the ideal reverse current reaches 90% of its reverse saturation current, is
   (A) 59.6 mV
   (B) 2.7 mV
   (C) 4.8 mV
   (D) 42.3 mV
23. For a silicon $p'-n$ junction diode the doping concentrations are $N_n = 10^{16}$ cm$^{-3}$ and $N_d = 10^{16}$ cm$^{-3}$. The minority carrier hole diffusion coefficient is $D_p = 12$ cm$^2$/s and the minority carrier hole life time is $\tau_{p0} = 10^{-7}$ s. The cross sectional area is $A = 10^{-4}$ cm$^2$. The reverse saturation current is 
(A) $4 \times 10^{-12}$ A  
(B) $4 \times 10^{-15}$ A  
(C) $4 \times 10^{-11}$ A  
(D) $4 \times 10^{-7}$ A

24. For an ideal silicon $p-n$ junction diode

\[ \tau_{ne} = \tau_{pu} = 10^{-7} \text{ s}, \]
\[ D_n = 25 \text{ cm}^2/\text{s}, \]
\[ D_p = 10 \text{ cm}^2/\text{s}. \]

The ratio of $N_n/N_d$, so that 95% of the current in the depletion region is carried by electrons, is 
(A) 0.34  
(B) 0.34  
(C) 0.83  
(D) 0.83

**Statement for Q.25–26:**

A ideal long silicon $p-n$ junction diode is shown in fig. P.2.2.25-26. The $n-$ region is doped with $10^{16}$ organic atoms per cm$^3$ and the $p-$region is doped with $5 \times 10^{16}$ boron atoms per cm$^3$. The minority carrier lifetimes are $D_n = 23$ cm$^2$/s and $D_p = 8$ cm$^2$/s. The forward-bias voltage is $V_a = 0.61$ V.

![Fig. P.2.2.25-26](image_url)

25. The excess hole concentration is 
(A) $6.8 \times 10^{12} e^{-246x}$ cm$^{-3}$, $x \geq 0$  
(B) $6.8 \times 10^{12} e^{-246x}$ cm$^{-3}$, $x \geq 0$  
(C) $3.8 \times 10^{14} e^{-355x}$ cm$^{-3}$, $x \geq 0$  
(D) $3.8 \times 10^{14} e^{-355x}$ cm$^{-3}$, $x \geq 0$

26. The hole diffusion current density at $x = 3$ µm is 
(A) 0.6 A/cm$^2$  
(B) 0.6 $\times 10^{-3}$ A/cm$^2$  
(C) 0.4 A/cm$^2$  
(D) 0.4 $\times 10^{-3}$ A/cm$^2$

27. The doping concentrations of a silicon $p-n$ junction are $N_d = 10^{16}$ cm$^{-3}$ and $N_n = 8 \times 10^{15}$ cm$^{-3}$. The cross-sectional area is $10^{-3}$ cm$^2$. The minority carrier lifetimes are $\tau_{n0} = 1$ µs and $\tau_{p0} = 0.1$ µs. The minority carrier diffusion coefficients are $D_n = 35$ cm$^2$/s and $D_p = 10$ cm$^2$/s. The total number of excess electron in the $p-$region, if applied forward bias is $V_a = 0.5$ V, is 
(A) $4 \times 10^{7}$ cm$^{-3}$  
(B) $6 \times 10^{10}$ cm$^{-3}$  
(C) $4 \times 10^{10}$ cm$^{-3}$  
(D) $6 \times 10^{7}$ cm$^{-3}$

28. Two ideal $p-n$ junction have exactly the same electrical and physical parameters except for the band gap of the semiconductor materials. The first has a band gap energy of $0.525$ eV and a forward-bias current of $10$ mA with $V_a = 0.255$ V. The second $p-n$ junction diode is to be designed such that the diode current $I = 10$ µA at a forward-bias voltage of $V_a = 0.32$ V. The bandgap energy of second diode would be 
(A) 0.77 eV  
(B) 0.67 eV  
(C) 0.57 eV  
(D) 0.47 eV

29. A $p-n$ junction biased at $V_a = 0.72$ V has DC bias current $I_{DC} = 2$ mA. The minority carrier lifetime is 1 µs is both the $n$ and $p$ regions. The diffusion capacitance is in 
(A) 49.3 nF  
(B) 38.7 nF  
(C) 77.4 nF  
(D) 98.6 nF

30. A $p'-n$ silicon diode is forward biased at a current of 1 mA. The hole life time in the $n-$region is 0.1 µs. Neglecting the depletion capacitance the diode impedance at 1 MHz is 
(A) $38.7 + j12.1 \Omega$  
(B) $235 + j7.5 \Omega$  
(C) $38.7 - j12.1 \text{ m}\Omega$  
(D) $235 - j7.5 \text{ \Omega}$

31. The slope of the diffusion capacitance versus forward-bias current of a $p'-n$ diode is $2.5 \times 10^{-4}$ F/A. The hole lifetime is 
(A) $1.3 \times 10^{-7}$ s  
(B) $1.3 \times 10^{-4}$ s  
(C) $6.5 \times 10^{-8}$ s  
(D) $6.5 \times 10^{-4}$ s

32. A silicon $p-n$ junction with doping profile of $N_n = 10^{16}$ cm$^{-3}$ and $N_d = 10^{15}$ cm$^{-3}$ has a cross sectional area of $10^{-2}$ cm$^2$. The length of the $p-$region is 2 mm and length of the $n-$region is 1 mm. The approximately series resistance of the diode is 
(A) 62 Ω  
(B) 43 Ω  
(C) 72 Ω  
(D) 81 Ω
33. A gallium arsenide $pn$ junction is operating in reverse-bias voltage $V_R = 5$ V. The doping profile are $N_a = N_d = 10^{19}$ cm$^{-3}$. The minority carrier life-time are $\tau_{n0} = \tau_{p0} = 10^{-8}$ s. The reverse-biased generation current density is $(e_c = 131, \ n = 1.8 \times 10^8)$

(A) $1.9 \times 10^{-8}$ A/cm$^2$  
(B) $1.9 \times 10^{-9}$ A/cm$^2$

(C) $1.6 \times 10^{-8}$ A/cm$^2$  
(D) $1.4 \times 10^{-9}$ A/cm$^2$

34. For silicon the critical electric field at breakdown is approximately $E_{crit} = 4 \times 10^5$ V/cm. For the breakdown voltage of 25 V, the maximum $n$-type doping concentration in an abrupt $p^*n$-junction is

(A) $2 \times 10^{16}$ cm$^{-3}$  
(B) $4 \times 10^{16}$ cm$^{-3}$

(C) $2 \times 10^{18}$ cm$^{-3}$  
(D) $4 \times 10^{18}$ cm$^{-3}$

35. A uniformly doped silicon $pn$ junction has dopant profile of $N_a = N_d = 5 \times 10^{16}$ cm$^{-3}$. If the peak electric field in the junction at breakdown is $E = 4 \times 10^5$ V/cm, the breakdown voltage of this junction is

(A) 35 V  
(B) 30 V

(C) 25 V  
(D) 20 V

36. An abrupt silicon $p^*n$ junction has an $n$-region doping concentration of $N_d = 5 \times 10^{15}$ cm$^{-3}$. The minimum $n$-region width, such that avalanche breakdown occurs before the depletion region reaches an ohmic contact, is $(V_B \approx 100$ V)

(A) 5.1 $\mu$m  
(B) 3.6 $\mu$m

(C) 7.3 $\mu$m  
(D) 6.4 $\mu$m

37. A silicon $pn$ junction diode has doping profile $N_a = N_d = 5 \times 10^{19}$ cm$^{-3}$. The space charge width at a forward bias voltage of $V_a = 0.4$ V is

(A) 102 $\AA$  
(B) 44 $\AA$

(C) 153 $\AA$  
(D) 62 $\AA$

38. A GaAs $pn^*$ junction LED has following parameters

$D_n = 25$ cm$^2$/s, $D_p = 12$ cm$^2$/s

$N_d = 5 \times 10^{17}$ cm$^{-3}$, $N_a = 10^{16}$ cm$^{-3}$

$\tau_{n0} = 10$ ns, $\tau_{p0} = 10$ ns

The injection efficiency of the LED is

(A) 0.83  
(B) 0.99

(C) 0.64  
(D) 0.46
= 3.01 \times 10^{-5} \text{ cm},
\frac{C_r}{W} = \frac{117 \times 8.85 \times 10^{-14} \times 10^{-4}}{301 \times 10^{-5}} = 3.5 \times 10^{-12} \text{ F}

13. (A) \ V_{bi} = V_i \ln \left( \frac{N_a N_d}{n_i^2} \right)
\frac{0.751}{0.0259} = \ln \left( \frac{50 N_a^2}{225 \times 10^{20}} \right)
N_a = 4.2 \times 10^{15} \text{ cm}^{-3}, \ N_d = 2.1 \times 10^{17} \text{ cm}^{-3}
W = \frac{2e(V_{bi} + V_R)\left\{ \frac{1}{N_a} + \frac{1}{N_d} \right\}^2}{2e(V_{bi} + V_R)\left\{ \frac{1}{N_a} + \frac{1}{N_d} \right\}^2}
N_d >> N_a \Rightarrow W = \frac{2e(\frac{V_{bi} + V_R}{2})^2}{\left\{ \frac{1}{N_a} + \frac{1}{N_d} \right\}^2}
\left[ \frac{2 \times (117 \times 8.85 \times 10^{-4})(1.0752)}{1.6 \times 10^{-19} \times 4.2 \times 10^{15}} \right] = 1.8 \times 10^{-4} \text{ cm}
= 1.8 \mu \text{m}

14. (D) \ C' = \left[ \frac{eeN_i N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right]^{\frac{1}{2}}
For \ N_d >> N_a, \ C' = \left[ \frac{eeN_a}{2(V_{bi} + V_R)} \right]^{\frac{1}{2}}
\left[ \frac{1.6 \times 10^{-19} \times 117 \times 8.85 \times 10^{-4} \times 4.2 \times 10^{15}}{2(10 + 0.754)} \right]^{\frac{1}{2}} = 5.7 \times 10^{-9} \text{ F/cm}^2

15. (D) \ W = \frac{2e(V_{bi} + V_R)\left\{ \frac{1}{N_a} + \frac{1}{N_d} \right\}^2}{2e(V_{bi} + V_R)\left\{ \frac{1}{N_a} + \frac{1}{N_d} \right\}^2}
W_A = \left( \frac{(V_{bi} + V_R) (N_{ab} + N_{db}) N_{ab} N_{db}}{(V_{bi} + V_R) (N_{ab} + N_{db}) (N_{ab} N_{db})} \right)^{\frac{1}{2}}
V_{bi} = V_i \ln \left( \frac{N_a N_d}{n_i^2} \right)
V_{biA} = 0.0259 \ln \left( \frac{10^{18} \times 10^{25}}{225 \times 10^{20}} \right) = 0.754 \text{ V}
V_{biB} = 0.0259 \ln \left( \frac{10^{18} \times 10^{25}}{225 \times 10^{20}} \right) = 0.814 \text{ V}
W_A = \left( \frac{5.754 \left( \frac{10^{18} \times 10^{15}}{10^{18} + 10^{16}} \right) \left( \frac{10^{16}}{10^{15}} \right)^{\frac{1}{2}}} {5.814 \left( \frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \left( \frac{10^{16}}{10^{15}} \right)^{\frac{1}{2}}} \right) = 3.13

16. (C) \ V_{bi} = V_i \ln \left( \frac{N_a N_d}{n_i^2} \right)
\frac{0.0259}{\ln \left( \frac{4 \times 10^{16} \times 4 \times 10^{17}}{2.25 \times 10^{20}} \right)} = 0.826 \text{ V}
|E_{max}| = \left[ \frac{2e(V_{bi} + V_R)}{e} \left( \frac{N_a N_d}{N_a + N_d} \right)^{\frac{1}{2}} \right]
V_{bi} + V_R = \frac{eE_{max}^2}{2e} \left( \frac{1}{N_a} + \frac{1}{N_d} \right)
\left( \frac{117 \times 8.85 \times 10^{-14})(3 \times 10^{10})^2}{2 \times 1.6 \times 10^{-18}} \right) \left( \frac{1}{4 \times 10^{16}} + \frac{1}{4 \times 10^{17}} \right) \text{ V}
= 8.008 \text{ V}
V_R = 8.008 - 0.826 = 7.18 \text{ V}

17. (B) \ V_{bi} = V_i \ln \left( \frac{N_a N_d}{n_i^2} \right)
0.0259 \ln \left( \frac{10^{18} \times 10^{15}}{225 \times 10^{20}} \right) = 0.754 \text{ V}
C' = \left[ \frac{eeN_i N_d}{2(V_{bi} + V_R)} \right]^{\frac{1}{2}}
For \ N_a >> N_d, \ C' = \left[ \frac{eeN_i}{2(V_{bi} + V_R)} \right]^{\frac{1}{2}}
\left[ \frac{1.6 \times 10^{-19} \times 117 \times 8.85 \times 10^{-4} \times 10^{15}}{2(10 + 0.754)} \right]^{\frac{1}{2}} = 2.77 \times 10^{-9} \text{ F/cm}^2
C = AC' = 6 \times 10^{-4} \times 2.77 \times 10^{-9} = 1.66 \times 10^{-12} \text{ F}
f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{2.2 \times 10^{-3} \times 1.66 \times 10^{-12}}} = 2.6 \text{ MHz}

18. (B) |E_{max}| = \frac{eN_i x_e}{e}
For a p+ n junction, \ x_e = \left[ \frac{2e(V_{bi} + V_R)}{e} \right]^{\frac{1}{2}}
So that \ |E_{max}| = \left[ \frac{2eN_i}{e} (V_{bi} + V_R) \right]^{\frac{1}{2}}
Assuming \ V_{bi} << V_R, \ N_d = \frac{eE_{max}^2}{2eV_R} \left( \frac{117 \times 8.85 \times 10^{-14})(10)^2}{2(1.6 \times 10^{-14})(10)} \right)
= 3.24 \times 10^{17} \text{ cm}^{-3}

19. (B) I_D = I_i \left( \frac{V_0}{e^{\frac{V_0}{V_i}} - 1} \right) = 10^{-10}(e^{2(0.0259)} - 1) = 35 \text{ mA}

20. (B) \ I_D = I_i \left( \frac{V_0}{e^{\frac{V_0}{V_i}} - 1} \right) \Rightarrow V_D = \eta I_i \ln \left( 1 + \frac{I_D}{I_i} \right)
21. \( I_d \approx I_e \left( \frac{V_j}{V} \right) \), \( \frac{I_{d1}}{I_{d2}} = \frac{V_j}{V} = e^{-\frac{(V_j - V)}{V_j}} \)

\( V_1 - V_2 = V_l \ln \left( \frac{I_{d1}}{I_{d1}} \right) = 0.0259 \ln 10 = 59.6 \text{ mV} \)

22. (A) \( I = I_e \left( \frac{V}{V_e} - 1 \right) \Rightarrow V = V_l \ln \left( \frac{I}{I_e} + 1 \right) \)

\( \frac{I}{I_e} = -0.90 \) ( –ive due to reverse current)

\( V = 0.0259 \ln (1 - 0.9) = 59.6 \text{ mV} \)

23. (B) \( I_s = Aen^2 \frac{1}{N_a} \sqrt{\frac{D}{\tau_{po}}} \)

\( = (10^{-4})(1.6 \times 10^{-19})(1.5 \times 10^{10}) \sqrt{\frac{12}{10^{-7}}} = 394 \times 10^{-15} \text{ A} \)

24. (D) \( \frac{J_s}{J_p} = 0.95 \), \( J_s = en^2 \frac{1}{N_a} \sqrt{\frac{D_p}{\tau_{po}}} \), \( J_p = en^2 \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{po}}} \)

\( \frac{\sqrt{D_s}}{\sqrt{D_s} + \frac{N_s}{N_d} \sqrt{D_p}} = 0.95 \), \( 5 = \frac{N_s}{N_d} \sqrt{10} = 0.95 \)

\( \Rightarrow \frac{N_s}{N_d} = 0.083 \)

25. (C) \( \rho_n = p_n - p_{n0} = p_{n0} \left[ e^\left( \frac{qV}{kT} \right) - 1 \right] \left[ e^\left( \frac{qV}{kT} \right) \right] \)

\( p_{n0} = n^2 \frac{N_d}{N_d} = (15 \times 10^{10})^2 = 2.25 \times 10^4 \text{ cm}^{-3} \)

\( L_p = \sqrt{\frac{D_{p, \tau_{po}}}{\tau_{po}}} = \sqrt{\frac{8(1 \times 10^{-8})}{2.83 \times 10^{-4}}} = 2.83 \times 10^{-4} \text{ cm} \)

\( \delta p_n = 2.25 \times 10^4 \left( e^{\frac{0.61}{0.0259}} - 1 \right) \left[ e^{\frac{0.61}{0.0259}} \right] \)

\( = 3.8 \times 10^{14} e^{-3554x} \text{ cm}^{-3} \)

26. (A) \( J_p = -eD_x \frac{\partial (\delta p_n)}{\partial x} = eD_x (3.8 \times 10^{14})(3554)e^{-3554x} \)

\( x = 3 \mu\text{m} = 3 \times 10^{-4} \text{ cm} \)

\( J_p = (1.6 \times 10^{-19})(18)(3.8 \times 10^{14})(3554)e^{-3554x(3 \times 10^{-4})} \)

\( = 0.6 \text{ A/cm}^2 \)

27. (A) \( N_p = A L_n n_{p0} \left[ e^{\left( \frac{qV}{kT} \right)} - 1 \right] \)

\( n_{p0} = \frac{n^2}{N_a} = \frac{(15 \times 10^{10})^2}{8 \times 10^{15}} = 2.8 \times 10^4 \text{ cm}^{-3} \)

\( L_n = \sqrt{D_{n, \tau_{po}}} = \sqrt{35 \times 10^{-8}} = 5.9 \times 10^{-3} \text{ Cm} \)

\( N_p = 10^{-3} \times 5.9 \times 10^{-3} \times 2.8 \times 10^4 \left[ \frac{0.5}{0.0259} - 1 \right] \)

\( = 4 \times 10^7 \text{ cm}^{-3} \)

28. (A) \( I \propto n^2 e^{\frac{V}{V_e}} \propto e^{\left( \frac{V - E_g}{V_e} \right)} \left( \frac{V}{V_e} \right) \)

\( \Rightarrow \frac{I_1}{I_2} = \frac{e^{\left( \frac{V_1 - E_g}{V_e} \right)} \left( \frac{V_1}{V_e} \right)}{e^{\left( \frac{V_2 - E_g}{V_e} \right)} \left( \frac{V_2}{V_e} \right)} = e^{\left( \frac{V_1 - V_2}{V_e} \right)} \left( \frac{V_1}{V_e} - \frac{V_2}{V_e} - E_g \right) \)

\( = 10 \times 10^{-3} = \frac{e^{(0.255 - 0.32 - 0.525 - 0.259)} - 0.6250}{0.0259} \)

\( 10^7 = e^{(0.36 - 0.59 - 0.59 - 0.6250)} \)

\( E_{g2} = 0.59 + 0.0259 \ln 10^3 = 0.769 \text{ Ev} \)

29. (B) \( C_d = \frac{I_{p0} + I_{n0} - I_{dq}}{2V_i} \)

\( \tau_{n0} = \tau_{p0} = 10^{-6} \text{ s}, \quad I_{p0} + I_{n0} - I_{dq} = 2 \text{ mA} \)

\( C_d = \frac{2 \times 10^{-3} \times 10^{-6}}{2(0.0259)} = 3.86 \times 10^{-8} \text{ F} \)

30. (D) \( g_d = \frac{I_{dq}}{V_i} = \frac{10^{-3}}{0.0259} = 3.86 \times 10^{-2} \text{ S} \)

\( C_d = \frac{I_{dq} \tau_{p0}}{2V_i} = \frac{10^{-3} \times 10^{-7}}{2 \times (0.0259)} = 193 \times 10^{-8} \text{ F} \)

\( Z = \frac{1}{Y} = \frac{1}{g_d + j0C_d} = 235 - j75.5 \text{ Ω} \)

31. (A) For a \( p^+ n \) diode \( I_{p0} >> I_{n0} \)

\( C_d = \frac{1}{2V_i} \left| I_{p0} \tau_{p0} \right|, \quad \frac{\tau_{p0}}{2V_i} = 2.5 \times 10^{-6} \)

\( \Rightarrow \tau_{p0} = 2 \times 0.0259 \times 2.5 \times 10^{-6} = 1.3 \times 10^{-7} \text{ s} \)

32. (C) \( R_p = \frac{\rho_{L}}{A} = \frac{L}{A(e\mu_p N_a)} = \frac{0.2}{(10^{-3})(1.6 \times 10^{-19})(480)(10^{18})} = 26 \text{ Ω} \)
\[ R_n = \frac{\rho_n L}{A} = \frac{L}{Ae(\mu_n N_d)} \]
\[ = 0.10 \frac{(10^{-5})(1.6 \times 10^{15})(1350)(10^{12})}{(1.6 \times 10^{15})} = 46.3 \Omega \]
\[ R = R_p + R_n = 72.3 \Omega \]

33. (B) \[ V_{bi} = V_i \ln \left( \frac{N_d N_{a,0}}{n_i^2} \right) \]
\[ = 2(0.0259) \ln \left( \frac{10^{16}}{1.8 \times 10^{19}} \right) = 1.16 \text{ V} \]
\[ W = \left[ \frac{2e_i(V_{bi} + V_R)}{e} \left( \frac{1}{N_a} + \frac{1}{N_n} \right) \right]^{\frac{1}{2}} \]
\[ = \left[ \frac{2 \times (13.1 \times 8.85 \times 10^{-14})(6.16)(2 \times 10^{16})}{16 \times 10^{-19}} \right]^{\frac{1}{2}} = 1.34 \times 10^{-4} \text{ cm} \]
\[ J_{gen} = \frac{eN_0 W}{2\tau_a} \]
\[ = \frac{1.6 \times 10^{-19} \times 1.8 \times 10^6 \times 1.34 \times 10^{-4}}{2 \times 10^{-8}} = 193 \times 10^{-9} \text{ A/cm}^2 \]

34. (A) \[ V_B = \frac{eE_{hit}}{2eN_B} \]
\[ = \frac{(117 \times 8.85 \times 10^{-6})(4 \times 10^5)^2}{2 \times 16 \times 10^{-16} \times N_B} \]
\[ N_B = N_d = 2 \times 10^{16} \text{ cm}^{-3} \]

35. (D) \[ E_{max} = \frac{eN_x x_n}{e} \Rightarrow x_n = \frac{eE_{max}}{eN_d} \]
\[ = \frac{(117 \times 8.85 \times 10^{-14})(4 \times 10^5)}{(16 \times 10^{-19})(5 \times 10^{15})} = 5.18 \times 10^{-5} \text{ cm} \]
\[ V_{bi} = V_i \ln \left( \frac{N_d N_{a,0}}{n_i^2} \right) = 2(0.0259) \ln \left( \frac{5 \times 10^{16}}{15 \times 10^{10}} \right) = 0.778 \text{ V} \]
\[ x_n = \left[ \frac{2e_iV_{bi} N_d}{e} \left( \frac{1}{N_a} + \frac{1}{N_n} \right) \right]^{\frac{1}{2}} \]
\[ = 5.18 \times 10^{-5} \Rightarrow V_{bi} + V_R = 20.7, \quad V_R = 19.9 \text{ V}, \quad V_R = V_B \]

36. (A) For a p’n diode, Neglecting \( V_i \) compared to \( V_B \),
\[ x_n = \left[ \frac{2e_iV_{bi} N_d}{eN_d} \right]^{\frac{1}{2}} = \left[ \frac{2(117 \times 8.85 \times 10^{-14})(100)}{(1.6 \times 10^{-15})(5 \times 10^{15})} \right]^{\frac{1}{2}} = 5.1 \mu \text{m} \]

37. (D) \[ V_{bi} = V_i \ln \left( \frac{N_d N_{a,0}}{n_i^2} \right) \]
\[ = 2(0.0259) \ln \left( \frac{5 \times 10^{19}}{15 \times 10^{10}} \right) = 1.14 \text{ V} \]
\[ W = \frac{2e_i(V_{bi} + V_R)}{e} \left( \frac{1}{N_a} + \frac{1}{N_n} \right) \]
\[ = \left[ 2 \times (117 \times 8.85 \times 10^{-14})(1.14 - 0.4) \left( \frac{2}{5 \times 10^{15}} \right) \right]^{\frac{1}{2}} \]
\[ = 6.19 \times 10^{-7} \text{ cm} = 62 \text{ A}^2 \]

38. (B) \[ L_n = \sqrt{D_n \tau_{a,0}}, \quad L_p = \sqrt{D_p \tau_{p,0}} \]
\[ \eta_{nj} = \frac{D_n p_{nj}}{L_n} + \frac{D_p p_{nj}}{L_p} = \frac{n_{p,0} \frac{E_n}{\tau_{a,0}}}{\eta_{nj}} \frac{D_n p_{nj}}{L_n} + \frac{D_p p_{nj}}{L_p} \]
\[ n_{p,0} = \frac{n_j^2}{N_a} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 324 \times 10^{-4} \text{ cm}^{-3} \]
\[ p_{nj} = \frac{n_j^2}{N_d} = \frac{(1.8 \times 10^6)^2}{5 \times 10^{17}} = 6.48 \times 10^6 \text{ cm}^{-3} \]
\[ \sqrt{D_n} = \frac{25}{10 \times 10^{-6}} = 5 \times 10^4, \quad \sqrt{D_p} = \frac{12}{10 \times 10^{-6}} = 3.5 \times 10^4 \]
\[ \eta_{nj} = \left( \frac{5 \times 10^4)(3.24 \times 10^{-4})}{(5 \times 10^4)(3.24 \times 10^{-4}) + (3 \times 10^4)(6.48 \times 10^{-4})} \right] = 0.986 \]

39. (B) The areal density at threshold is
\[ n_{2D} = \frac{J_d \tau_{\gamma}}{e} = \frac{(500)(1.5 \times 10^{-8})}{1.6 \times 10^{-13}} = 4.69 \times 10^{12} \text{ cm}^{-3} \]

The carrier density is
\[ n_{th} = \frac{n_{2D}}{d_{th}} = \frac{4.69 \times 10^{12}}{10^{-6}} = 4.69 \times 10^{18} \text{ cm}^{-3} \]

Once the threshold is reached, the carrier density does not change. When \( J > J_{th} \) the electron hole recombination is
\[ \tau_{\gamma}(J) = \frac{J_{th}}{J} \tau_{\gamma}(J_{th}) = \frac{15 \times 10^{-9}}{5} = 3 \times 10^{-10} \text{ s} \]

The optical power produced is \( p = \frac{J A}{h \omega} \)
\[ = \frac{(5 \times 500)(2 \times 10^{-5})(1.43 \times 1.6 \times 10^{12})}{1.6 \times 10^{-15}} = 715 \text{ MW} \]

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**CHAPTER**

**2.3**

**THE BIPOLAR JUNCTION TRANSISTOR**

**Statement for Q.1-2:**

The parameters in the base region of an npn bipolar transistor are as follows $D_n = 20 \text{ cm}^2/\text{s}, n_{B0} = 10^4 \text{ cm}^{-3}$, $x_B = 1 \mu\text{m}$, $A_{BE} = 10^{-4} \text{ cm}^2$.

1. If $V_{BE} = 0.5 \text{ V}$, then collector current $I_C$ is
   (A) 7.75 $\mu$A  
   (B) 16 $\mu$A  
   (C) 0.16 $\mu$A  
   (D) 77.5 $\mu$A

2. If $V_{BE} = 0.7 \text{ V}$, then collector current $I_C$ is
   (A) 418 $\mu$A  
   (B) 210 $\mu$A  
   (C) 175 $\mu$A  
   (D) 98 $\mu$A

3. In bipolar transistor biased in the forward-active region the base current is $I_B = 50 \mu$A and the collector currents is $I_C = 2.7$ mA. The $\alpha$ is
   (A) 0.949  
   (B) 54  
   (C) 0.982  
   (D) 0.018

4. A uniformly doped silicon npn bipolar transistor is to be biased in the forward active mode with the B-C junction reverse biased by 3 V. The transistor doping are $N_E = 10^{17} \text{ cm}^{-3}$, $N_B = 10^{16} \text{ cm}^{-3}$ and $N_C = 10^{15} \text{ cm}^{-3}$. The BE voltage, at which the minority carrier electron concentration at $x = 0$ is 10% of the majority carrier hole concentration, is
   (A) 0.94 V  
   (B) 0.64 V  
   (C) 0.48 V  
   (D) 0.24 V

5. A uniformly doped npn bipolar transistor is biased in the forward-active region. The transistor doping concentration are $N_E = 5 \times 10^{17} \text{ cm}^{-3}$, $N_B = 10^{16} \text{ cm}^{-3}$ and $N_C = 10^{15} \text{ cm}^{-3}$. The minority carrier concentration $p_{B0}$, $n_{B0}$ and $p_{C0}$ are
   (A) $4.5 \times 10^4$, $2.25 \times 10^4$, $2.25 \times 10^5 \text{ cm}^{-3}$  
   (B) $2.25 \times 10^4$, $2.25 \times 10^5$, $4.5 \times 10^2 \text{ cm}^{-3}$  
   (C) $2.25 \times 10^4$, $2.25 \times 10^5$, $4.5 \times 10^4 \text{ cm}^{-3}$  
   (D) $4.5 \times 10^4$, $2.25 \times 10^4$, $2.25 \times 10^5 \text{ cm}^{-3}$

6. A uniformly doped silicon pnp transistor is biased in the forward-active mode. The doping profile is $N_E = 10^{18} \text{ cm}^{-3}$, $N_B = 5 \times 10^{16} \text{ cm}^{-3}$ and $N_C = 10^{15} \text{ cm}^{-3}$. For $V_{BE} = 0.6 \text{ V}$, the $p_B$ at $x = 0$ is (See fig. P2.3.7-8)
   (A) $5.2 \times 10^{19} \text{ cm}^{-3}$  
   (B) $5.2 \times 10^{13} \text{ cm}^{-3}$  
   (C) $5.2 \times 10^{16} \text{ cm}^{-3}$  
   (D) $5.2 \times 10^{11} \text{ cm}^{-3}$

**Statement for Q.7-8:**

An npn bipolar transistor having uniform doping of $N_E = 10^{18} \text{ cm}^{-3}$, $N_B = 10^{16} \text{ cm}^{-3}$ and $N_C = 6 \times 10^{15} \text{ cm}^{-3}$ is operating in the inverse-active mode with $V_{BE} = -2 \text{ V}$ and $V_{BC} = 0.6 \text{ V}$. The geometry of transistor is shown in fig P2.3.7-8.

![Fig. P2.3.7-8](image-url)
The minority carrier concentration at \( x = x_B \) is
(A) \( 4.5 \times 10^{14} \) cm\(^{-3} \)  
(B) \( 2.6 \times 10^{12} \) cm\(^{-3} \)  
(C) \( 2.6 \times 10^{14} \) cm\(^{-3} \)  
(D) \( 39 \times 10^{14} \) cm\(^{-3} \)

The minority carrier concentration at \( x'' = 0 \) is
(A) \( 39 \times 10^{14} \) cm\(^{-3} \)  
(B) \( 2.7 \times 10^{12} \) cm\(^{-3} \)  
(C) \( 2.7 \times 10^{14} \) cm\(^{-3} \)  
(D) \( 4.5 \times 10^{14} \) cm\(^{-3} \)

An \( pnp \) bipolar transistor has uniform doping of \( N_E = 6 \times 10^{17} \) cm\(^{-3} \), \( N_B = 2 \times 10^{16} \) cm\(^{-3} \) and \( N_C = 5 \times 10^{14} \) cm\(^{-3} \). The transistor is operating in inverse-active mode. The maximum \( V_{CB} \) voltage, so that the low injection condition applies, is
(A) \( 0.86 \) V  
(B) \( 0.48 \) V  
(C) \( 0.32 \) V  
(D) \( 0.60 \) V

**Statement for Q.10-12:**

The following currents are measured in a uniformly doped \( npn \) bipolar transistor:
\[
I_{ES} = 120 \text{ mA}, \quad I_{CB} = 0.10 \text{ mA}, \quad I_{CE} = 118 \text{ mA}
\]
\[
I_B = 0.20 \text{ mA}, \quad I_A = 1 \mu\text{A}, \quad I_{DC} = 1 \mu\text{A}
\]

10. The \( \alpha \) is
(A) \( 0.667 \)  
(B) \( 0.733 \)  
(C) \( 0.787 \)  
(D) \( 0.8 \)

11. The \( \beta \) is
(A) \( 3.69 \)  
(B) \( 0.44 \)  
(C) \( 2.27 \)  
(D) \( 8.39 \)

12. The \( \gamma \) is
(A) \( 0.816 \)  
(B) \( 0.923 \)  
(C) \( 1.083 \)  
(D) \( 0.440 \)

13. A silicon \( npn \) bipolar transistor has doping concentration of \( N_E = 2 \times 10^{18} \) cm\(^{-3} \), \( N_B = 10^{17} \) cm\(^{-3} \) and \( N_C = 15 \times 10^{16} \) cm\(^{-3} \). The area is \( 10^{-3} \) cm\(^2\) and neutral base width is \( 1 \) \( \mu \)m. The transistor is biased in the active region at \( V_{BE} = 0.5 \) V. The collector current is
\[
(D_B = 20 \text{ cm}^2/\text{s})
\]
\[
(A) 9 \mu\text{A} \quad (B) 17 \mu\text{A} \quad (C) 22 \mu\text{A} \quad (D) 11 \mu\text{A}
\]

14. A uniformly doped \( npn \) bipolar transistor has following parameters:
\[
N_E = 10^{19} \text{ cm}^{-3}, \quad N_B = 5 \times 10^{16} \text{ cm}^{-3}, \quad N_C = 2 \times 10^{19} \text{ cm}^{-3}, \quad D_E = 8 \text{ cm}^2/\text{s}, \quad D_B = 15 \text{ cm}^2/\text{s}, \quad D_C = 14 \text{ cm}^2/\text{s}
\]
\[
x_E = 0.8 \mu\text{m}, \quad x_B = 0.7 \mu\text{m}
\]

The emitter injection efficiency \( \gamma \) is
(A) \( 0.999 \)  
(B) \( 0.977 \)  
(C) \( 0.982 \)  
(D) \( 0.934 \)

15. A uniformly doped silicon epitaxial \( npn \) bipolar transistor is fabricated with a base doping of \( N_B = 3 \times 10^{16} \) cm\(^{-3} \) and a heavily doped collector region with \( N_C = 5 \times 10^{17} \) cm\(^{-3} \). The neutral base width is \( x_E = 0.7 \mu\text{m} \) when \( V_{BE} = 0.6 \) V. The \( V_{BC} \) at punch-through is
(A) \( 26.3 \) V  
(B) \( 18.3 \) V  
(C) \( 12.2 \) V  
(D) \( 6.3 \) V

16. A silicon \( npn \) transistor has a doping concentration of \( N_B = 10^{17} \) cm\(^{-3} \) and \( N_C = 7 \times 10^{15} \) cm\(^{-3} \). The metallurgical base width is \( 0.5 \) \( \mu \)m. Let \( V_{BE} = 0.6 \) V. Neglecting the B–E junction depletion width the \( V_{CE} \) at punch-through is
(A) \( 146 \) V  
(B) \( 70 \) V  
(C) \( 295 \) V  
(D) \( 204 \) V

17. A uniformly doped silicon \( pnp \) transistor is to designed with \( N_E = 10^{19} \) cm\(^{-3} \) and \( N_C = 10^{16} \) cm\(^{-3} \). The metallurgical base width is to be \( 0.75 \) \( \mu \)m. The minimum base doping, so that the minimum punch-through voltage is \( V_{BE} = 25 \) V, is
(A) \( 4.46 \times 10^{15} \) cm\(^{-3} \)  
(B) \( 4.46 \times 10^{16} \) cm\(^{-3} \)  
(C) \( 195 \times 10^{15} \) cm\(^{-3} \)  
(D) \( 195 \times 10^{16} \) cm\(^{-3} \)

18. For a silicon \( npn \) transistor assume the following parameters:
\[
I_E = 0.5 \text{ mA}, \quad \beta = 48
\]
\[
x_E = 0.7 \mu\text{A}, \quad x_{dc} = 2 \mu\text{m}
\]
\[
C_C = C_v = 0.08 \text{ pF}, \quad C_x = 0.8 \text{ pF}
\]
\[
D_n = 25 \text{ cm}^2/\text{s}, \quad r_s = 30 \Omega
\]

The carrier cross the space charge region at a speed of \( 10^7 \) cm/s. The total delay time \( \tau_{ec} \) is
(A) \( 164.2 \) ps  
(B) \( 234.4 \) ps  
(C) \( 144.2 \) ps  
(D) \( 298.4 \) ps
19. In a bipolar transistor, the base transit time is 25% of the total delay time. The base width is 0.5 μm and base diffusion coefficient is \( D_B = 20 \text{ cm}^2/\text{s} \). The cut-off frequency is
(A) 637 MHz  
(B) 436 MHz  
(C) 12.8 GHz  
(D) 46.3 GHz

20. The base transit time of a bipolar transistor is 100 ps and carriers cross the 1.2 μm B–C space charge at a speed of \( 10^7 \text{ cm/s} \). The emitter-base junction charging time is 25 ps and the collector capacitance and resistance are 0.10 pF and 10 Ω, respectively. The cutoff frequency is
(A) 43.8 GHz  
(B) 32.6 GHz  
(C) 3.26 GHz  
(D) 1.15 GHz

**Statement for Q.21-22:**

Consider the circuit shown in fig. P2.3.21-22. If voltage \( V_s = 0.63 \text{V} \), the currents are \( I_C = 275 \mu\text{A} \) and \( I_B = 5 \mu\text{A} \).

![Fig. P2.3.21-22](image)

21. The forward common-emitter gain \( \beta_F \) is
(A) 56  
(B) 55  
(C) 0.9821  
(D) 0.9818

22. The forward current gain \( \alpha_F \) is
(A) 0.9821  
(B) 0.9818  
(C) 55  
(D) 56

23. Consider the circuit shown in fig P2.3.23. If \( V_s = 0.63 \text{V} \), \( I_1 = 275 \mu\text{A} \) and \( I_2 = 125 \mu\text{A} \), then the value of \( I_s \) is

- (A) \(-400 \mu\text{A}\)  
- (B) \(400 \mu\text{A}\)  
- (C) \(-600 \mu\text{A}\)  
- (D) \(600 \mu\text{A}\)

**Statement for Q.24-26:**

For the transistor in circuit of fig. P2.3.24-26. The parameters are \( \beta_R = 1 \), \( \beta_F = 100 \), and \( I_s = 1 \text{ fA} \).

![Fig. P2.3.24-26](image)

24. The current \( I_C \) is
(A) 1 fA  
(B) 2 fA  
(C) 1.384 fA  
(D) 0 A

25. The current \( I_R \) is
(A) 1 fA  
(B) -1 fA  
(C) 2 fA  
(D) -2 fA

26. The current \( I_B \) is
(A) 2 fA  
(B) -2 fA  
(C) 1 fA  
(D) -1 fA

27. For the transistor in fig. P2.3.27, \( I_s = 10^{-15} \text{ A} \), \( \beta_F = 100 \), \( \beta_R = 1 \). The current \( I_{CB0} \) is

![Fig. P2.3.27](image)

- (A) \(101 \times 10^{-14} \text{ A}\)  
- (B) \(2 \times 10^{-14} \text{ A}\)  
- (C) \(101 \times 10^{-15} \text{ A}\)  
- (D) \(2 \times 10^{-15} \text{ A}\)

**Statement for Q.28-31:**

Determine the region of operation for the transistor shown in circuit in question.

28. Determine the region of operation for the transistor shown in circuit in question.
(A) Forward-Active  
(B) Reverse-Active  
(C) Saturation  
(D) Cutoff

![Fig. P2.3.28](image)
29. (A) Forward-Active  (B) Reverse-Active  (C) Saturation  (D) Cutoff

30. (A) Forward-Active  (B) Reverse-Active  (C) Saturation  (D) Cutoff

31. (A) Forward-Active  (B) Reverse-Active  (C) Saturation  (D) Cutoff

Statement for Q.32-33:

For the circuit shown in fig. P2.3.32-33, let the value of $R_0 = 50 \, \text{k}\Omega$ and $V_C = 3 \, \text{V}$. The saturation current is $10^{-16} \, \text{A}$.

32. The base-emitter voltage is
   (A) 0.53 V  (B) 0.7 V  (C) 0.84 V  (D) 0.98 V

33. The current $I_1$ is
   (A) $-12.75 \, \text{mA}$  (B) $12.75 \, \text{mA}$
   (C) $12.5 \, \text{mA}$  (D) $-12.5 \, \text{mA}$

Statement for Q.34–35:

The leakage current of a transistor are $I_{CEO} = 5 \, \mu\text{A}$ and $I_{CEO} = 0.4 \, \text{mA}$, and $I_B = 30 \, \mu\text{A}$.

34. The value of $\beta$ is
   (A) 79  (B) 81  (C) 80  (D) None of the above

35. The value of $I_C$ is
   (A) 2.4 mA  (B) 2.77 mA  (C) 2.34 mA  (D) 1.97 mA

Statement for Q.36–37:

For a BJT, $I_C = 5 \, \text{mA}$, $I_B = 50 \, \mu\text{A}$ and $I_{CEO} = 0.5 \, \mu\text{A}$.

36. The value of $\beta$ is
   (A) 103  (B) 91  (C) 83  (D) 51

37. The value of $I_E$ is
   (A) 5.25 mA  (B) 5.4 mA  (C) 5.65 mA  (D) 5.1 mA

********
1. (A) \( I_C = I_e \left( \frac{V_{BE}}{V_t} \right) \)

\[
I_e = \frac{eD_nA_{BE}n_{B0}}{x_R} = \frac{(1.6 \times 10^{-9})(20)(10^{-4})(10^4)}{10^{-4}} = 32 \times 10^{-14} \text{ A}
\]

\[I_C = 3.2 \times 10^{-14} e^{\left( \frac{0.5}{0.0256} \right)} = 7.75 \mu\text{A}\]

2. (C) \( I_C = 32 \times 10^{-14} e^{\left( \frac{0.7}{0.0256} \right)} = 17.5 \text{ mA} \)

3. (C) \( \beta_p = \frac{I_C}{I_B}, \quad \alpha_p = \frac{\beta_p}{1 + \beta_p}, \quad \alpha_p = \frac{2.7m}{2.7m + 50\mu} = 0.982 \)

4. (B) \( n_{p0} = \frac{n_i^2}{N_B} = \frac{(15 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3} \)

At \( x = 0 \), \( n_p(0) = n_{p0} e^{\left( \frac{V_{BE}}{V_t} \right)} \)

\( \Rightarrow V_{BE} = V_t \ln \left( \frac{n_p(0)}{n_{p0}} \right) \)

\[n_p(0) = \frac{10}{100} \times N_B = \frac{10^{16}}{10} = 10^{15} \]

\[V_{BE} = 0.0259 \ln \left( \frac{10^{15}}{225 \times 10^{-4}} \right) = 0.635 \text{ V} \]

5. (A) \( p_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{5 \times 10^{15}} = 450 \text{ cm}^{-3} \)

\[n_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{5 \times 10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3} \]

\[p_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{5 \times 10^{15}} = 2.25 \times 10^5 \text{ cm}^{-3} \]

6. (B) \( p_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{5 \times 10^{16}} = 4.5 \times 10^3 \text{ cm}^{-3} \)

\[p_{p}(0) = p_{p0} e^{\left( \frac{V_{PE}}{V_t} \right)} = 4.5 \times 10^3 e^{\left( \frac{0.6}{0.0256} \right)} = 5.2 \times 10^{13} \text{ cm}^{-3} \]

7. (C) \( n_{p0} = \frac{n_i^2}{N_B} = \frac{(15 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3} \)

\[n_p(x = x_E) = n_{p0} e^{\left( \frac{V_{CE}}{V_t} \right)} \]

\[-2.25 \times 10^4 e^{\left( \frac{0.6}{0.0256} \right)} = 2.6 \times 10^{-14} \text{ cm}^{-3} \]

8. (D) \( p_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4 \text{ cm}^{-3} \)

\( p_{E}(x = 0) = p_{E0} e^{\left( \frac{V_{CE}}{V_t} \right)} \)

\[= 3.75 \times 10^4 e^{\left( \frac{0.5}{0.0256} \right)} = 4.31 \times 10^{14} \text{ cm}^{-3} \]

9. (B) Low injection limit is reached when \( p_{E}(0) = 0.10 \text{N}_C = 5 \times 10^{13} \text{ cm}^{-3} \),

\[p_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{6 \times 10^{15}} = 4.5 \times 10^5 \]

\[p_{E}(0) = p_{E0} e^{\left( \frac{V_{CE}}{V_t} \right)} \Rightarrow V_{CE} = V_t \ln \left( \frac{p_{E}(0)}{p_{E0}} \right) \]

\[= 0.0259 \ln \left( \frac{5 \times 10^{13}}{4.5 \times 10^5} \right) = 0.48 \text{ V} \]

10. (C) \( \alpha = \frac{J_{aC}}{J_{aE} + J_{pE}} = \frac{I_{aC}}{I_{aE} + I_{pE}} \)

\[= \frac{118}{12 + 0.2 + 0.1} = 0.787 \]

11. (A) \( \beta = \frac{\alpha}{1 - \alpha} = \frac{0.787}{1 - 0.787} = 3.69 \)

12. (B) \( \gamma = \frac{J_{aE}}{J_{aE} + J_{pE}} = \frac{I_{aE}}{I_{aE} + I_{pE}} = \frac{1.2}{1.2 + 0.1} = 0.923 \)

13. (B) \( n_{E0} = \frac{n_i^2}{N_E} = \frac{(15 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^5 \text{ cm}^{-3} \)

\[n_{p}(0) = n_{p0} e^{\left( \frac{V_{PB}}{V_t} \right)} = 2.25 \times 10^3 e^{\left( \frac{0.5}{0.0256} \right)} = 5.45 \times 10^{11} \text{ cm}^{-3} \]

\[I_C = \frac{eD_nA_{BE}n_{E0}(0)}{x_R} \]

\(= \frac{(1.6 \times 10^{-9})(20)(10^{-3})(5.45 \times 10^{11})}{10^{-4}} = 17.4 \mu\text{A} \)

14. (B) \( \gamma = \frac{1}{1 + \frac{N_E}{D_E} \frac{D_E}{x_E} x_E} \)

\[= \frac{1}{1 + \frac{5 \times 10^{16}}{8} \frac{8}{0.7} \frac{0.7}{15 \times 0.8}} = 0.977 \]

15. (B) \( V_{CE} = V_t \ln \left( \frac{N_B N_{E0}}{n_i^2} \right) \)

\[= 0.0259 \ln \left( \frac{3 \times 10^{16} \times 5 \times 10^{17}}{(15 \times 10^{10})^2} \right) = 0.824 \text{ V} \]

At punch-through
27. (C) \[ I_R = I_S \left( e^\left( \frac{V_{BE}}{V_T} \right) - e^\left( \frac{V_{BC}}{V_T} \right) \right) + \frac{I_S}{\beta_p} \left( e^\left( \frac{V_{BE}}{V_T} \right) - 1 \right) = 0 \]

\[ e^\left( \frac{V_{BE}}{V_T} \right) = \frac{1}{1 + \beta_p} + \frac{\beta_R}{1 + B_F} e^\left( \frac{V_{BC}}{V_T} \right) \]

\[ I_C = I_S \left[ e^\left( \frac{V_{BE}}{V_T} \right) - e^\left( \frac{V_{BC}}{V_T} \right) \right] - \frac{I_S}{\beta_R} \left[ e^\left( \frac{V_{BE}}{V_T} \right) - 1 \right] \]

\[ I_{CEO} = \frac{I_S}{1 + \beta_F} \left( 1 - e^\left( \frac{V_{BE}}{V_T} \right) \right) - \frac{I_S}{\beta_R} \left[ e^\left( \frac{V_{BE}}{V_T} \right) - 1 \right] \]

\[ V_{BE} = -5 \text{ V}, \quad V_t = 0.0259 \text{ V} \]

\[ I_{CEO} = \frac{I_s}{101} (1 - 0) - \frac{I_s}{1} (0 - 1) = 101I_s = 101 \times 10^{-15} \text{ A} \]

28. (D)

<table>
<thead>
<tr>
<th>B-E junction ( V_{BE} )</th>
<th>B-C Junction ( V_{BC} )</th>
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<tbody>
<tr>
<td>Reverse Bias</td>
<td>Forward Bias</td>
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<tr>
<td>Forward bias</td>
<td>Forward-Active</td>
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<tr>
<td>Reverse Bias</td>
<td>Cut-off</td>
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\( V_{BE} = 0, \quad V_{BC} < 0 \), Thus both junction are in reverse bias. Hence cutoff region.

29. (A) \( V_{BE} > 0, \quad V_{BC} = 0 \), Base-Emitter junction forward bias, Base-collector junction reverse bias, Hence forward-active region.

30. (B) \( V_{BE} = 0, \quad V_{BC} > 0 \), Base-Emitter junction reverse bias, Base-collector junction forward bias, Hence reverse-active region.

31. (C) \( V_{BE} = 6 \text{ V}, \quad V_{BC} = 3 \text{ V} \), Both junction are forward bias, Hence saturation region.

32. (C) The current source will forward bias the base-emitter junction and the collector base junction will then be reverse biased. Therefore the transistor is in the forward active region

\[ I_C = I_S e^\left( \frac{V_{BE}}{V_T} \right) \]

\[ I_C = \beta_p I_B = 50 \times 250 \times 10^{-4} = 12.5 \times 10^{-3} \text{ A} \]

\[ V_{BE} = V_t \ln \left( \frac{I_C}{I_s} \right) = 0.0259 \ln \left( \frac{12.5 \times 10^{-3}}{10^{-15}} \right) = 0.84 \text{ V} \]
11. In n-well CMOS fabrication substrate is
   (A) lightly doped \( n \) – type
   (B) lightly doped \( p \) – type
   (C) heavily doped \( n \) – type
   (D) heavily doped \( p \) – type

12. The chemical reaction involved in epitaxial growth in IC chips takes place at a temperature of about
   (A) 500\(^\circ\) C
   (B) 800\(^\circ\) C
   (C) 1200\(^\circ\) C
   (D) 2000\(^\circ\) C

13. A single monolithic IC chip occupies area of about
   (A) 20 mm\(^2\)
   (B) 200 mm\(^2\)
   (C) 2000 mm\(^2\)
   (D) 20,000 mm\(^2\)s

14. Silicon dioxide layer is used in IC chips for
   (A) providing mechanical strength to the chip
   (B) diffusing elements
   (C) providing contacts
   (D) providing mask against diffusion

15. The p-type substrate in a monolithic circuit should be connected to
   (A) any dc ground point
   (B) the most negative voltage available in the circuit
   (C) the most positive voltage available in the circuit
   (D) no where, i.e. be floating

16. The collector-substrate junction in the epitaxial collector structure is, approximately
   (A) a step-graded junction
   (B) a linearly graded junction
   (C) an exponential junction
   (D) None of the above

17. The sheet resistance of a semiconductor is
   (A) an important characteristic of a diffused region especially when used to form diffused resistors
   (B) an undesirable parasitic element
   (C) a characteristic whose value determines the required area for a given value of integrated capacitance
   (D) a parameter whose value is important in a thin-film resistance

18. Monolithic integrated circuit system offer greater reliability than discrete-component systems because
   (A) there are fewer interconnections
   (B) high-temperature metalizing is used
   (C) electric voltage are low
   (D) electric elements are closely matched

19. Silicon dioxide is used in integrated circuits
   (A) because of its high heat conduction
   (B) because it facilitates the penetration of diffusants
   (C) to control the location of diffusion and to protect and insulate the silicon surface.
   (D) to control the concentration of diffusants.

20. Increasing the yield of an IC
   (A) reduces individual circuit cost
   (B) increases the cost of each good circuit
   (C) results in a lower number of good chips per wafer
   (D) means that more transistor can be fabricated on the same size wafer.

21. The main purpose of the metalization process is
   (A) to act as a heat sink
   (B) to interconnect the various circuit elements
   (C) to protect the chip from oxidation
   (D) to supply a bonding surface for mounting the chip

22. In a monolithic-type IC
   (A) each transistor is diffused into a separate isolation region
   (B) all components are fabricated into a single crystal of silicon
   (C) resistors and capacitors of any value may be made
   (D) all isolation problems are eliminated

23. Isolation in ICs is required
   (A) to make it simpler to test circuits
   (B) to protect the transistor from possible “thermal run away”
   (C) to protect the components mechanical damage
   (D) to minimize electrical interaction between circuit components

24. Almost all resistor are made in a monolithic IC
   (A) during the base diffusion
(B) during the collector diffusion
(C) during the emitter diffusion
(D) while growing the epitaxial layer

25. The equation governing the diffusion of neutral atom is
   (A) \( \frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} \)
   (B) \( \frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} \)
   (C) \( \frac{\partial^2 N}{\partial t^2} = D \frac{\partial N}{\partial x} \)
   (D) \( \frac{\partial^2 N}{\partial t^2} = D \frac{\partial N}{\partial x} \)

26. The true statement is
   (A) thick film components are vacuum deposited
   (B) thin film components are made by screen-and-fire process
   (C) thin film resistor have greater precision and are more stable
   (D) thin film resistor are cheaper than the thin film resistor

27. The False statement is
   (A) Capacitor of thin film capacitor made with proper dielectric is not voltage dependent
   (B) Thin film resistors and capacitor need to be biased for isolation purpose
   (C) Thin film resistors and capacitor have smaller stray capacitances and leakage currents.
   (D) None of the above

28. Consider the following two statements
   \( S_1 \) : The dielectric isolation method is superior to junction isolation method.
   \( S_2 \) : The beam lead isolation method is inferior to junction isolation method.

       The true statements is (are)
   (A) \( S_1, S_2 \)  (B) only
   (C) only  (D) Neither nor \( S_2 \)

29. If P is passivation, Q is n-well implant, R is metallization and S is source/drain diffusion, then the order in which they are carried out in a standard n-well CMOS fabrication process is
   (A) S - R - Q - P  (B) R - P - S - Q
   (C) Q - S - R - P  (D) P - Q - R - S

30. For the circuit shown in fig. P2.5.30, the minimum number and the maximum number of isolation regions are respectively
   \( R_2 \)
   \( R_1 \)
   \( Q_1 \)
   \( V_{sg} \)
   \( Q_2 \)

   (A) 2, 6  (B) 3, 6
   (C) 2, 4  (D) 3, 4

31. For the circuit shown in fig. P2.5.31, the minimum number of isolation regions are
   \( Q_1 \)
   \( Q_2 \)
   \( R_1 \)
   \( V_{sg} \)

   (A) 2  (B) 3
   (C) 4  (D) 7

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### SOLUTIONS

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30. (D) The minimum number of isolation region is 3
one containing \( Q_1 \), one containing and one containing both and . The maximum number of isolation region is 4, or one per component.

31. (A) The minimum number of isolation region is two.
One for transistor and one for resistor.

******
CHAPTER 3.1
DIODE CIRCUITS

Statement for Q.1–4:

In the question a circuit and a waveform for the input voltage is given. The diode in circuit has cutin voltage $V_c = 0$. Choose the option for the waveform of output voltage $v_o$.

1. 

![Circuit Diagram](Fig.3.1.1)

2. 

![Circuit Diagram](Fig.3.1.2)

3. 

![Circuit Diagram](Fig.3.1.3)

4. 

![Circuit Diagram](Fig.3.1.4)
For the circuit in fig. P.3.1.5, let cut-in voltage $V_i = 0.7$ V. The plot of $v_o$ versus $v_i$ for $-10 \leq v_i \leq 10$ V is

For the circuit shown in fig. P.3.1.6, each diode has $V_i = 0.7$ V. The plot of $v_o$ versus $v_i$ is

For the circuit shown in fig. P.3.1.7, each diode has $V_i = 0.7$ V. The plot of $v_o$ for $v_i = 10$ V is

A symmetrical 5 kHz square wave whose output varies between +10 V and −10 V is impressed upon the clipping circuit shown in fig. P.3.1.8. If diode has $r_f = 0$ and $r_c = 2 \, \text{M}\Omega$ and $V_i = 0$, the output waveform is
9. In the circuit of fig. P3.1.9, the three signals of fig are impressed on the input terminals. If diode are ideal then the voltage $v_o$ is

10. For the circuit shown in fig. P3.1.10 the input voltage $v_i$ is as shown in fig. Assume the RC time constant large and cutin voltage of diode $V_i=0$. The output voltage $v_o$ is

11. For the circuit shown in fig. P3.1.11, the input voltage $v_i$ is as shown in fig. Assume the RC time constant large and cutin voltage $V_i=0$. The output voltage $v_o$ is

12. In the circuit of fig. P3.1.12, $D_1$ and $D_2$ are ideal diodes. The current $i_1$ and $i_2$ are

13. In the circuit of Fig. P3.1.13 diodes has cutin voltage of 0.6 V. The diode in ON state are

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22. The diodes in the circuit in fig. P3.1.22 has parameters $V_i = 0.6$ V and $r_f = 0$. The current $i_D$ is

- (A) 8.4 mA
- (B) 10 mA
- (C) 7.6 mA
- (D) 0 mA

**Statement for Q.23–25:**

The diodes in the circuit in fig. P3.1.23-25 have linear parameter of $V_i = 0.6$ V and $r_f = 0$.

23. If $v_1 = 10$ V and $v_2 = 0$ V, then $v_o$ is
- (A) 8.93 V
- (B) 7.82 V
- (C) 1.07 V
- (D) 2.18 V

24. If $v_1 = 10$ V and $v_2 = 5$ V, then $v_o$ is
- (A) 9.13 V
- (B) 0.842 V
- (C) 5.82 V
- (D) 1.07 V

25. If $v_1 = v_2 = 0$, then output voltage $v_o$ is
- (A) 0.964 V
- (B) 1.07 V
- (C) 10 V
- (D) 0.842 V

**Statement for Q.26–28:**

The diodes in the circuit in fig. P3.1.26-28 have linear parameters of $V_i = 0.6$ V and $r_f = 0$.

26. If $v_2 = 0$, then output voltage $v_o$ is
- (A) 6.43 V
- (B) 9.43 V
- (C) 7.69 V
- (D) 8.93 V

27. If $v_2 = 5$ V, then $v_o$ is
- (A) 8.93 V
- (B) 12.63 V
- (C) 18.24 V
- (D) 10.56 V

28. If $v_2 = 10$ V, then $v_o$ is
- (A) 10 V
- (B) 9.16 V
- (C) 8.43 V
- (D) 12.13 V

**Statement for Q.29–30:**

The diode in the circuit in fig. P3.1.29–30 has the non-linear terminal characteristic as shown in fig. Let the voltage be $v_s = \cos \omega t$ V.

29. The current $i_D$ is
- (A) $25(1 + \cos \omega t)$ mA
- (B) $5(0.5 + \cos \omega t)$ mA
- (C) $5(1 + \cos \omega t)$ mA
- (D) $5(1 + 0.5 \cos \omega t)$ mA

30. The voltage $v_D$ is
- (A) $0.25(3 + \cos \omega t)$ V
- (B) $0.25(1 + 3 \cos \omega t)$ V
- (C) $0.5(3 + 1 \cos \omega t)$ V
- (D) $0.5(2 + 3 \cos \omega t)$ V

31. The circuit inside the box in fig. P3.1.31. contains only resistor and diodes. The terminal voltage $v_o$ is connected to some point in the circuit inside the box. The largest and smallest possible value of $v_o$ most nearly to is respectively

- (A) 15 V, 6 V
- (B) 24 V, 0 V
- (C) 24 V, 6 V
- (D) 15 V, -9 V
32. In the voltage regulator circuit in fig. P3.1.32 the maximum load current $i_L$ that can be drawn is

(A) 1.4 mA  
(B) 2.3 mA  
(C) 1.8 mA  
(D) 2.5 mA

33. In the voltage regulator shown in fig. P3.1.33 the power dissipation in the Zener diode is

(A) 1 W  
(B) 1.5 W  
(C) 2 W  
(D) 0.5 W

34. The Q-point for the Zener diode in fig. P3.1.34 is

(A) (0.34 mA, 4 V)  
(B) (0.34 mA, 4.93 V)  
(C) (0.94 mA, 4 V)  
(D) (0.94 mA, 4.93 V)

35. In the voltage regulator circuit in fig. P3.1.35 the power rating of Zener diode is 400 mW. The value of $R_L$ that will establish maximum power in Zener diode is

(A) 5 kΩ  
(B) 2 kΩ  
(C) 10 kΩ  
(D) 8 kΩ

Statement for Q.36–38:
In the voltage regulator circuit in fig. P3.1.36–38 the Zener diode current is to be limited to the range $5 \leq i_L \leq 100$ mA.

36. The range of possible load current is

(A) $5 \leq i_L \leq 130$ mA  
(B) $25 \leq i_L \leq 120$ mA  
(C) $10 \leq i_L \leq 110$ mA  
(D) None of the above

37. The range of possible load resistance is

(A) $60 \leq R_L \leq 372$ Ω  
(B) $60 \leq R_L \leq 200$ Ω  
(C) $40 \leq R_L \leq 192$ Ω  
(D) $40 \leq R_L \leq 360$ Ω

38. The power rating required for the load resistor is

(A) 576 mW  
(B) 360 μW  
(C) 480 mW  
(D) 75 μW

39. The secondary transformer voltage of the rectifier circuit shown in fig. P3.1.39 is $v_t = 60 \sin 2\pi 60t$ V. Each diode has a cut in voltage of $V_c = 0.6$ V. The ripple voltage is to be no more than $V_{rip} = 2$ V. The value of filter capacitor will be

(A) 48.8 μF  
(B) 24.4 μF  
(C) 32.2 μF  
(D) 16.1 μF

40. The input to full-wave rectifier in fig. P3.1.40 is

(A) $48.8 \mu F$  
(B) $24.4 \mu F$  
(C) $32.2 \mu F$  
(D) $16.1 \mu F$
(A) 61.2 μF  (B) 41.2 μF
(C) 20.6 μF  (D) 30.6 μF

For the circuit shown in fig. P3.1.41 diode cut-in voltage is \( V_{in} = 0 \). The ripple voltage is to be no more than \( v_{rip} = 4 \) V. The minimum load resistance, that can be connected to the output is

- (A) 6.25 kΩ
- (B) 12.50 kΩ
- (C) 30 kΩ
- (D) None of the above

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**SOLUTIONS**

1. (D) Diode is off for \( v_i < 5 \) V. Hence \( v_o = 5 \) V.
   For \( v_i > 5 \) V, \( v_o = v_i \). Therefore (D) is correct option.

2. (C) Diode will be off if \( v_i + 2 > 0 \). Thus \( v_o = 0 \)
   For \( v_i + 2 < 0 \) V, \( v_i = v_o + 2 = -3 \) V
   Thus (C) is correct option.

3. (D) For \( v_i < 4 \) V the diode is ON and output \( v_o = 4 \) V.
   For \( v_i > 4 \) V diode is off and output \( v_o = v_i \).
   Thus (D) is correct option.

4. (C) During positive cycle when \( v_s < 8 \) V, both diode are OFF \( v_o = v_s \).
   For \( v_s > 8 \) V, \( v_o = 8 \) V, \( D_1 \) is ON.
   During negative cycle when \( |v_s| < 6 \) V, both diode are OFF, \( v_o = v_s \).
   For \( |v_s| > 6 \) V, \( D_2 \) is ON \( v_o = -6 \) V. Therefore (C) is correct.

5. (B) For \( D \) off, \( v_o = \frac{10 - 10}{1 + \frac{1}{20}} = 3.33 \) V.
   For \( v_i \leq 3.33 + 0.7 = 4.03 \) V, \( v_o = 3.33 \) V
   For \( v_i > 4.03 \) V, \( v_o = v_i - 0.7 \)
   For \( v_i = 10 \) V, \( v_o = 9.3 \) V

6. (C) Let \( v_1 \) be the voltage at n-terminal of diode,
   \( v_1 = \frac{15 \times 1}{2 + 1} = 5 \) V
   For \( v_i \leq 5.7 \) V, \( v_o = v_i \)
   \( \frac{v_1 - 15}{2k} + \frac{v_1}{1k} + \frac{v_o - v_i}{1k} = 0 \) \( \Rightarrow \) \( 3v_1 + 2v_o - 2v_i = 15 \)
   \( v_o = v_i + 0.7 \)
   \( 5v_o - 2v_i = 15 + 2.1 = 17.9 \) \( \Rightarrow \) \( v_o = 0.4v_i + 3.42 \)

7. (D) For \( v_o > 0 \), when \( D_1 \) is OFF, Current through \( D_2 \) is
   \( i = \frac{10 - 0.7}{10 + 10} = 0.465 \) mA, \( v_o = 10ki = 4.65 \) V
   \( v_o = v_i \) for \( 0 < v_i < 4.65 \) V.
   For negative values of \( v_o \), the output is negative of positive part. Thus (D) is correct option.

8. (B) The diode conducts (zero resistance) when \( v_i < 25 \) V and \( v_o = v_i \).
   Diode is open (2 MΩ resistance) when \( v_i > 25 \) V and \( v_o = v_i + \frac{v_i - 25}{3} = 5 \) V.
\[ = 50 \times 5 \times (1 + \cos \omega t) \times 10^{-3} + 0.5 \]
\[ = 0.75 + 0.25 \cos \omega t = 0.25(3 + \cos \omega t) \text{ V} \]

31. (D) The output voltage cannot exceed the positive power supply voltage and cannot be lower than the negative power supply voltage.

32. (A) At regulated power supply \( i_x = \frac{30 - 9}{15k} = 1.4 \text{ mA} \) will remain less than 1.4 mA.

33. (D) \( v_{TH} = \frac{75(50)}{75 + 150} = \frac{50}{3} \text{ V} \)

\[ 50 \div 3 > V_Z, \quad R_{TH} = 150 \parallel 75 = 50 \Omega \]

\( i_z = \frac{1}{50} \left( \frac{50}{3} - 15 \right) = 33 \text{ mA}, \quad P = 15i_z = 0.5 \text{ W} \)

34. (A) \( v_{TH} = \frac{3.6(20)}{11 + 3.6} = 4.93 \text{ V} > V_Z \),

\( R_{TH} = 11 \parallel 3.6 = 2.71 \text{ k} \Omega, \quad i_z = \frac{4.93 - 4}{2.71k} = 0.34 \text{ mA} \)

35. (B) \( i_{Z(max)} = \frac{400}{10} = 40 \text{ mA} \)

\[ i_L + i_z = \frac{20 - 10}{222} = 45 \text{ mA} \]

\[ i_L_{(min)} = 45 - 40 = 5 \text{ mA}, \quad R_L = \frac{10}{5m} = 2 \text{ k} \Omega \]

36. (B) Current through 12 \( \Omega \) resistor is

\[ i = \frac{6.3 - 4.8}{12} = 125 \text{ mA} \]

\[ i_L = i_i - i_z = 125 - i_z \quad \Rightarrow \quad 25 \leq i_L \leq 120 \text{ mA} \]

37. (C) \( 25 \leq i_L \leq 120 \text{ mA}, \quad i_L R_L = 4.8 \text{ V} \)

\[ 25 \leq \frac{4.8}{R_L} \leq 120 \text{ mA} \quad \Rightarrow \quad 40 \leq R_L \leq 192 \text{ } \Omega \]

38. (A) \( P_L = i_L V_Z = (120m)(4.8) = 576 \text{ mV} \)

39. (B) \( v_i = 60 \sin 2\pi 60t \text{ V} \)

\[ v_{max} = 60 - 14 = 58.6 \text{ V} \]

\[ C = \frac{v_{max}}{2fV_{rip}} = \frac{58.6}{2(60)10 \times 10^{-3} \times 2} = 24.4 \text{ } \mu\text{F} \]

40. (C) Full wave rectifier

\[ v_i = v_i = 120 \sin 2\pi 60t \text{ V} \]

\[ v_{max} = 120 - 0.7 = 119.3 \text{ V} \]

\[ V_{rip} = 119.3 - 100 = 19.3 \text{ V} \]
Use \( V_{BE(ON)} = 0.7 \) V, \( V_{CE(SAT)} = 0.2 \) V for \textit{npn} transistor if not given in problem.

Statement for Q.1-4:

The common-emitter current gain of the transistor is \( \beta = 75 \). The voltage \( V_{BE} \) in ON state is 0.7 V.

1. \( I_E, R_C = ? \)

   \[ \begin{align*}
   (A) \ & 1.46 \text{ mA}, 6.74 \text{ k}\Omega \\
   (B) \ & 0.987 \text{ mA}, 3.04 \text{ k}\Omega \\
   (C) \ & 1.13 \text{ mA}, 5.98 \text{ k}\Omega \\
   (D) \ & \text{None of the above}
   \end{align*} \]

2. \( V_{EC} = ? \)

   \[ \begin{align*}
   (A) \ & 8.4 \text{ V} \\
   (B) \ & 6.2 \text{ V} \\
   (C) \ & 4.1 \text{ V} \\
   (D) \ & \text{None of the above}
   \end{align*} \]

3. \( I_C, R_C = ? \)

   \[ \begin{align*}
   (A) \ & 0.987 \text{ mA}, 3.04 \text{ k}\Omega \\
   (B) \ & 1.013 \text{ mA}, 2.96 \text{ k}\Omega \\
   (C) \ & 0.946 \text{ mA}, 4.18 \text{ k}\Omega \\
   (D) \ & 1.057 \text{ mA}, 3.96 \text{ k}\Omega \\
   \end{align*} \]

4. \( V_C = ? \)

   \[ \begin{align*}
   (A) \ & 1.49 \text{ V} \\
   (B) \ & 2.9 \text{ V} \\
   (C) \ & 1.78 \text{ V} \\
   (D) \ & 2.3 \text{ V}
   \end{align*} \]
Statement for Q.5-6:

In the circuit of fig.P3.3.5-6  \( V_B = -1 \) V

\[ \beta = ? \]
(A) 103.4  
(B) 135.5  
(C) 134.5  
(D) 102.4

\( V_{CE} = ? \)
(A) 6.4 V  
(B) 4.7 V  
(C) 1.3 V  
(D) 4.2 V

In the circuit shown in fig. P3.3.7 voltage \( V_E = 4 \) V. The value of \( \alpha \) and \( \beta \) are respectively

\[ \text{Fig. P3.3.7} \]

(A) 0.943, 17.54  
(B) 0.914, 17.54  
(C) 0.914, 11.63  
(D) 0.914, 11.63

Statement for Q.8-10:

For the transistor in circuit shown in fig. P3.3.8-10, \( \beta = 200 \). Determine the value of \( I_E \) and \( I_C \) for given value of \( V_B \) in question.

\[ \text{Fig. P3.3.8-10} \]

\( V_B = 0 \) V
(A) 6.43 mA, 2.4 V  
(C) 0 A, 6 V  
(B) 2.18 mA, 3.4 V  
(D) None of the above

\( V_B = 1 \) V
(A) 4 V  
(C) 1 V  
(B) 3 V  
(D) 1.9 V

\( V_B = 2 \) V
(A) 7 V  
(C) 2.6 V  
(B) 1.5 V  
(D) None of the above

Statement for Q.11-12:

The transistor in circuit shown in fig. P3.3.11-12 has \( \beta = 200 \). Determine the value of voltage \( V_o \) for given value of \( V_{BB} \).

\[ \text{Fig. P3.3.11-12} \]

\( V_{BB} = 0 \)
(A) 2.46 V  
(C) 3.33 V  
(B) 1.83 V  
(D) 4.04 V

\( V_{BB} = 1 \)
(A) 4.11 V  
(C) 2.46 V  
(B) 1.83 V  
(D) 3.44 V

\( V_{BB} = 2 \)
(A) 3.18 V  
(C) 0.2 V  
(B) 1.46 V  
(D) None of the above

Statement for Q.14-16:

The transistor shown in the circuit of fig. P3.3.14-16 has \( \beta = 150 \). Determine \( V_o \) for given value of \( I_q \) in question.

\[ \text{Fig. P3.3.14-16} \]

\( I_q = 0.1 \) mA
(A) 1.4 V  
(C) 3.2 V  
(B) 4.5 V  
(D) None of the above
15. $I_q = 0.5$ mA
   (A) 3.16 V  
   (B) 2.52 V  
   (C) 2.14 V  
   (D) 3.94 V

16. $I_q = 2$ mA
   (A) 4.9 V  
   (B) -4.9 V  
   (C) 0.5 V  
   (D) -0.5 V

17. For the circuit in fig. P3.3.17 $V_B = V_C$ and $\beta = 50$. The value of $V_B$ is
   (A) 0.9 V  
   (B) 1.19 V  
   (C) 2.14 V  
   (D) 1.84 V

18. For the circuit shown in fig. P3.3.18, $V_{CB} = 0.5$ V and $\beta = 100$. The value of $I_q$ is
   (A) 1.68 mA  
   (B) 0.909 mA  
   (C) 0.134 mA  
   (D) None of the above

19. For the circuit shown in fig. P3.3.19 the emitter voltage is $V_E = 2$ V. The value of $\alpha$ is
   (A) 0.991  
   (B) 0.939  
   (C) 0.968  
   (D) 0.914

20. For the transistor in fig. P3.3.20, $\beta = 50$. The value of voltage $V_{EC}$ is
   (A) 3.13 V  
   (B) 4.24 V  
   (C) 5.18 V  
   (D) 6.07 V

21. In the circuit shown in fig. P3.3.21 if $\beta = 50$, the power dissipated in the transistor is
   (A) 3.87 mW  
   (B) 10.46 mW  
   (C) 7.49 mW  
   (D) 18.74 mW

22. For the circuit shown in fig. P3.3.22 the Q-point is $V_{CEQ} = 12$ V and $I_{EQ} = 2$ A when $\beta = 60$. The value of resistor $R_C$ and $R_B$ are
   (A) 10 kΩ, 241 kΩ  
   (B) 10 kΩ, 699 kΩ  
   (C) 6 kΩ, 699 kΩ  
   (D) 6 kΩ, 241 kΩ

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29. For the transistor in the circuit of fig. P3.3.29, \( \beta = 100 \). The voltage \( V_B \) is

(A) 3.6 V  
(B) 4.29 V  
(C) 3.9 V  
(D) 4.69 V

30. The current gain of the transistor shown in the circuit of fig. P3.3.30 is \( \beta = 125 \). The Q-point values \( (I_{cQ}, V_{cEQ}) \) are

(A) \( (0.418 \text{ mA}, 20.4 \text{ V}) \)  
(B) \( (0.915 \text{ mA}, 14.8 \text{ V}) \)  
(C) \( (0.915 \text{ mA}, 16.23 \text{ V}) \)  
(D) \( (0.418 \text{ mA}, 18.43 \text{ V}) \)

31. For the circuit shown in fig. P3.3.31, let \( \beta = 75 \). The Q-point \( (I_{cQ}, V_{cEQ}) \) is

(A) \( (4.68 \text{ mA}, 16.46 \text{ V}) \)  
(B) \( (3.12 \text{ mA}, 1.86 \text{ V}) \)  
(C) \( (3.12 \text{ mA}, 8.46 \text{ V}) \)  
(D) \( (4.68 \text{ mA}, 5.22 \text{ V}) \)

32. The current gain of the transistor shown in the circuit of fig. P3.3.32 is \( \beta = 100 \). The values of Q-point \( (I_{cQ}, V_{cEQ}) \) is

(A) \( (1.8 \text{ mA}, 2.1 \text{ V}) \)  
(B) \( (1.4 \text{ mA}, 2.3 \text{ V}) \)  
(C) \( (1.4 \text{ mA}, 1.8 \text{ V}) \)  
(D) \( (1.8 \text{ mA}, 1.4 \text{ V}) \)

33. For the circuit in fig. P3.3.33, let \( \beta = 60 \). The value of \( V_{cEQ} \) is

(A) 2.68 V  
(B) 4.94 V  
(C) 3.73 V  
(D) 5.69 V

34. In the circuit of fig. P3.3.34 Zener voltage is \( V_Z = 5 \text{ V} \) and \( \beta = 100 \). The value of \( I_{cQ} \) and \( V_{cEQ} \) are

(A) \( 12.47 \text{ mA}, 4.3 \text{ V} \)  
(B) \( 12.47 \text{ mA}, 5.7 \text{ V} \)  
(C) \( 10.43 \text{ A, 5.7 V} \)  
(D) \( 10.43 \text{ A, 4.3 V} \)
35. The two transistor in fig. P3.2.35 are identical. If $\beta = 25$, the current $I_{C2}$ is

(A) 28 $\mu$A  (B) 23.2 $\mu$A  (C) 26 $\mu$A  (D) 24 $\mu$A

36. In the shunt regulator of fig. P3.2.26, the $V_Z = 8.2$ V and $V_{BE} = 0.7$ V. The regulated output voltage $V_o$ is

(A) 11.8 V  (B) 7.5 V  (C) 12.5 V  (D) 8.9 V

37. In the series voltage regulator circuit of fig. P3.2.37 $V_{BE} = 0.7$ V, $\beta = 50$, $V_Z = 8.3$ V. The output voltage $V_o$ is

(A) 25 V  (B) 25.7 V  (C) 15 V  (D) 15.7 V

38. In the regulator circuit of fig. P3.2.38 $V_Z = 12$ V, $\beta = 50$, $V_{BE} = 0.7$ V. The Zener current is

(A) 36.63 mA  (B) 36.17 mA  (C) 49.32 mA  (D) 49.78 mA

39. In the bipolar current source of fig. P3.2.39 the diode voltage and transistor BE voltage are equal. If base current is neglected then collector current is

40. In the current mirror circuit of fig. P3.2.40, the transistor parameters are $V_{BE} = 0.7$ V, $\beta = 50$ and the Early voltage is infinite. Assume transistor are matched. The output current $I_o$ is

(A) 1.04 mA  (B) 1.68 mA  (C) 962 $\mu$A  (D) 432 $\mu$A

41. All transistor in the $N$ output mirror in fig. P3.2.41 are matched with a finite gain $\beta$ and early voltage $V_A = \infty$. The expression for each load current is
Consider the basic three transistor current source in fig. P3.2.42. Assume all transistor are matched with finite gain and early voltage $V_A = \infty$. The expression for $I_o$ is

\[
I_o = \frac{I_{ref}}{1 + (1 + N) / (\beta + 1)}
\]

Both of transistor are identical and $\beta > > 1$ and $V_{BE1} = 0.7$ V. The value of resistance $R_1$ and $R_2$ to produce $I_{ref} = 1$ mA and $I_o = 12 \mu$A is ($V_I = 0.026$)

\[
R_1 = \frac{2}{1 + \beta}
\]

\[
R_2 = \frac{2}{1 + \beta}
\]

Consider the wilder current source of fig. P3.2.43.

\[
I_{ref} = \frac{1}{1 + (1 + N) / (\beta + 1)}
\]

\[
I_{ref} = \frac{N}{1 + (1 + N) / (\beta + 1)}
\]

SOLUTIONS

1. (C) $I_E = \frac{12 - 0.7}{10k} \Rightarrow I_E = 1.13 \text{ mA}$

\[
I_c = \frac{75}{75 + 1}(1.13) = 1.12 \text{ mA}
\]

\[
V_{CE} = 12 - 1.13 \times 10 - 1.12 R_c = -12 \Rightarrow 6 \text{ V}
\]

\[
R_c = 5.98 \text{ k}\Omega
\]

2. (C) $8 = 10 \times (75 + 1) I_B + 0.7 + 10 I_B - 2$

\[
I_B = \frac{9.3}{10 + 760} = 12.08 \mu\text{A},
\]

\[
I_c = \beta I_B = 0.906 \text{ mA},
\]

\[
I_E = (\beta + 1) I_B = 0.918 \text{ mA}
\]

\[
8 = 10(0.918) + V_{CE} + 3(0.906) - 8
\]

\[
V_{CE} = 4.1 \text{ V}
\]

3. (A) $I_c = \frac{75}{75 + 1} I_E = \frac{75}{76}(1) = 0.987 \text{ mA}$

\[
R_c = \frac{52}{0.987} = 304 \text{ k}\Omega
\]

4. (A) $5 = (1 + \beta)10kI_B + 20kI_B + 0.7 + \beta 2kI_B$

\[
5 = (760K + 20k + 150k) I_B + 0.7
\]

\[
I_B = 4.62 \mu\text{A}
\]

\[
I_c = \beta I_B = 0.347 \text{ mA}
\]

\[
V_{CE} = 5 - (\beta + 1) I_B R_c = 5 - 760 \times 4.62 \times 10^{-3} = 1.49 \text{ V}
\]

5. (C) $V_B = -I_B R_B$

\[
\Rightarrow I_B = \frac{-V_B}{R_B} = \frac{1}{500k} = 2.0 \mu\text{A}
\]

\[
V_E = 1 - 0.7 = -1.7 \text{ V}
\]

\[
I_E = \frac{V_E - (-3)}{4.8k} = \frac{-1.7 + 3}{4.8k} = 0.271 \text{ mA}
\]

\[
I_E = (\beta + 1) = \frac{0.271}{2 \mu}
\]

\[
\Rightarrow \beta = 134.5
\]

6. (B) $V_{CE} = 3 - V_E = 3 - (-1.7) = 4.7 \text{ V}$

7. (C) $I_E = \frac{5.4}{2k} = 0.5 \text{ mA}$

\[
4 = 0.7 + I_B R_B + I_c R_c - 5\),
\]

\[
8.3 = 100 I_B + 0.5 \times 8
\]

\[
\Rightarrow I_B = 43 \mu\text{A}
\]

\[
I_E = 1 + \beta = \frac{0.5m}{43\mu} = 11.63
\]

***************
\[ \beta = 10.63, \alpha = \frac{\beta}{1 + \beta} \rightarrow \alpha = 0.914 \]

8. (C) \( V_B = 0 \) Transistor is in cut-off region.
\( I_K = 0, V_C = 6 \text{ V} \)

9. (B) \( V_B = 1 \text{ V}, I_E = \frac{1-0.7}{1k} = 0.3 \text{ mA} \)
\( I_C \approx I_E = 0.3 \text{ mA} \)
\( V_C = 6 - I_C R_C = 6 - (0.3)(10) = 3 \text{ V} \)

10. (B) \( V_B = 2 \text{ V}, I_E = \frac{2-0.7}{1} = 1.3 \text{ mA} \)
\( I_C \approx I_E = 1.3 \text{ mA} \)
\( V_C = 6 - (1.3)(10) = -7 \text{ V} \)
Transistor is in saturation. The saturation voltage \( V_{CE} = 0.2 \text{ V} \)
\( V_E = (1.3)(1) = 1.3 \text{ V}, V_C = V_{CE} + V_E = 1.5 \text{ V} \)

11. (C) \( V_{BB} = 0 \), Transistor is in cutoff region
\( V_o = \frac{R_L}{R_C + R_L} V_C = \frac{10(5)}{10} + 5 = 3.33 \text{ V} \)

12. (B) \( I_B = \frac{1-0.7}{50k} = 6 \text{ mA} \)
\( I_C = \beta I_B = 75 \times 6 \mu = 0.45 \text{ mA} \)
\( \frac{5-V_o}{5k} = I_C + \frac{V_o}{10k} \)
\( (1-0.45) = \frac{V_o}{5} + \frac{V_o}{10}, \quad \Rightarrow \quad V_o = 1.83 \text{ V} \)

13. (C) \( I_B = \frac{2-0.7}{50k} = 26 \text{ m\mu}A \)
\( I_C = \beta I_B = 75 \times 26 \text{ m\mu}A = 1.95 \text{ mA} \)
\( V_C = 5 - I_C R_C = 5 - 5 \times 195 = -4.75 \text{ V} \)
Transistor is in saturation, \( V_{CE} = 0.2 \text{ V} = V_C = V_e \)

14. (B) \( I_K = 0.1 \text{ mA} \)
\( I_C = \frac{\beta}{\beta + 1} I_K = \frac{150}{151}(0.1) = 0.0999 \text{ mA} \)
\( V_o = 5 - R_C I_C = 5 - 5(0.0999) = 4.50 \text{ V} \)

15. (B) \( I_K = I_Q = 0.5 \text{ mA} \)
\( I_C = \frac{150}{150+1}(0.5 \text{ m\mu}A) = 0.497 \text{ mA} \)
\( V_o = 5 - R_C I_C = 2.517 \text{ V} \)

16. (D) Transistor is in saturation
\( V_o = V_{CE(sat)} = V_{BB} = 0.2 - 0.7 = -0.5 \text{ V} \)

17. (B) \( I_E = \frac{V_B - 0.7}{1k} \)
\( I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \text{ m\mu}A \)
\( I_C = \frac{6 - V_C}{10}, V_C = V_B \)
\( \frac{50}{51} (V_B - 0.7) = \frac{6 - V_B}{10} \)
\( 10.8 V_B = 12.86, V_B = 1.19 \text{ V} \)

18. (B) \( V_CB = 0.5 \text{ V}, V_C = 0.5 \text{ V} \)
\( I_C = \frac{5 - 0.5}{5k} = 0.9 \text{ mA}, I_Q = \frac{101}{100} = 0.909 \text{ mA} \)

19. (C) \( I_E = \frac{10 - V_E}{10k} = 0.8 \text{ mA} \)
\( V_B = V_K = 0.7 = 1.3 \text{ V} \)
\( I_E = \frac{V_E}{R_E} = \frac{1.3}{50k} = 26 \text{ \mu}A \)
\( \frac{\beta + 1}{\beta} I_E = \frac{0.8 \text{ m\mu}A}{26 \text{ \mu}A} = 30.77 \Rightarrow \beta = 29.77 \)
\( \alpha = \frac{\beta}{\beta + 1} = 0.968 \)

20. (D) \( I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} \text{ m\mu}A = 0.98 \text{ mA} \)
\( V_C = I_C R_C = 0.98(4.7) - 9 = -4.394 \text{ V} \)
\( I_E = \frac{I_E}{\beta + 1} = \frac{1}{51} \text{ m\mu}A = 19.6 \text{ \mu}A \)
\( V_K = I_E R_E + V_{BB} = 50(0.0196) + 0.7 = 1.68 \text{ V} \)
\( V_K = 1.68 - (-4.394) = 6.074 \text{ V} \)

21. (A) \( I_C = \frac{\beta}{\beta + 1} I_E = \frac{50}{51} (0.5) = 0.49 \text{ mA} \)
\( I_E = \frac{0.5}{51} = 9.8 \text{ \mu}A \)
\( V_K = I_E R_E + V_{BB} = (0.0098)(50) + 0.7 = 1.19 \text{ V} \)
\( V_K = I_C R_C = 0.49(4.7) - 9 = -6.7 \text{ V} \)
\( V_K = 1.19 - (-6.7) = 7.89 \text{ V} \)
\( P_Q = V_E V_K + I_K V_{KB} = (0.49)(7.89) + (0.0098)(0.7) \text{ mW} = 3.87 \text{ mW} \)
30. (B) \( R_1 = 58 \, \text{k}\Omega, \, R_2 = 42 \, \text{k}\Omega \)

\[
\begin{align*}
V_{TH} &= \left( \frac{R_2}{R_1 + R_2} \right) V_{CC} = \left( \frac{15}{20 + 15} \right) 10 \, \text{V} = 4.29 \, \text{V} \\
10 &= I_E(1k) + V_{BE} + \frac{I_E}{\beta + 1} (8.57k) + 4.29 \\
10 &= I_E + 0.7 + \frac{I_E}{\beta + 1} (8.57k) + 4.29 \\
\Rightarrow \quad I_E &= 4.62 \, \text{mA} \\
I_B &= \frac{I_E}{\beta + 1} = 0.046 \, \text{mA} \\
V_B &= (8.57)(0.046) + 4.29 = 4.69 \, \text{V}
\end{align*}
\]

31. (D) \( R_1 = 25 \, \text{k}\Omega, \, R_2 = 8 \, \text{k}\Omega \)

\[
\begin{align*}
R_{TH} &= 58 \parallel 42 = 24.36 \, \text{k}\Omega \\
V_{TH} &= \left( \frac{42}{42 + 58} \right) 24 \, \text{V} = 10.1 \, \text{V} \\
10.1 &= I_{BQ}(24.36k) + V_{BE} + (\beta + 1)I_{BQ}(10k) \\
10.1 - 0.7 &= I_{BQ}(24.36k + 1260k) \\
I_{BQ} &= 7.32 \, \text{mA} \\
I_{CQ} &= \beta I_{BQ} = 0.915 \, \text{mA} \\
I_{EQ} &= (\beta + 1)I_{BQ} = 0.922 \, \text{mA} \\
V_{CQ} &= 24 - (0.922)(10) = 14.8 \, \text{V}
\end{align*}
\]

32. (B) \( R_1 = 12 \, \text{k}\Omega, \, R_2 = 2 \, \text{k}\Omega \)

\[
\begin{align*}
R_{TH} &= R_1 || R_2 = 12 || 2 = 1.71 \, \text{k}\Omega \\
V_{TH} &= \left( \frac{2}{12 + 2} \right) 10 - 5 = -3.57 \, \text{V}
\end{align*}
\]

33. (B) \( R_{TH} = 20 \parallel 10 = 6.67 \, \text{k}\Omega \)

\[
\begin{align*}
V_{TH} &= \left( \frac{20}{10 + 20} \right) 10 - 5 = 1.67 \, \text{V}
\end{align*}
\]
41. (A) \( I_{ref} = I_{CR} + I_{BS} = I_{CR} + \frac{I_{ES}}{1 + \beta} \)

\[ I_{ES} = I_{BR} + I_{B1} + I_{B2} + \ldots \ldots + I_{BN} \]

\[ I_{BR} = I_{B1}, \quad I_{CR} = I_{C1} = I_{di} \]

\[ I_{ES} = (1 + N) I_{BR} = \frac{(1 + N) I_{CR}}{\beta} \]

Then \( I_{ref} = I_{CR} + \frac{I_{ES}}{\beta + 1} = I_{CR} + \frac{(1 + N) I_{CR}}{\beta(\beta + 1)} \)

\[ I_{di} = \frac{I_{ref}}{1 + \frac{(1 + N)}{\beta(\beta + 1)}} \]

42. (C) \( I_{ref} = I_{C1} + I_{B1} = I_{B2}, \quad I_{B3} = 2I_{B2} \)

\[ I_{B3} = (1 + \beta)I_{B3} \]

\[ I_{ref} = I_{C1} + \frac{I_{B3}}{1 + \beta} = I_{C1} + \frac{2I_{B2}}{1 + \beta} \]

\[ I_{C1} = I_{C2} = \beta I_{B2} \]

\[ I_{ref} = I_{C2} + \frac{2I_{C2}}{\beta(1 + \beta)} = I_{C2} \left( 1 + \frac{2}{\beta(1 + \beta)} \right) \]

\[ I_{C2} = I_{o} = \frac{I_{ref}}{1 + \frac{2}{\beta(1 + \beta)}} \]

43. (B) If \( \beta >> 1 \) and transistor are identical

\[ I_{ref} \approx I_{C1} = I_{S} e^{\frac{V_{BE1}}{V_{T}}}, \quad I_{o} = I_{C2} = I_{S} e^{\frac{V_{BE2}}{V_{T}}} \]

\[ V_{BE1} = V_{T} \ln \left( \frac{I_{ref}}{I_{S}} \right), \quad V_{BE2} = V_{T} \ln \left( \frac{I_{o}}{I_{S}} \right) \]

\[ V_{BE1} - V_{BE2} = V_{T} \ln \left( \frac{I_{ref}}{I_{o}} \right) \]

From the circuit,

\[ V_{BE1} - V_{BE2} = I_{E2} R_{E} \approx I_{o} R_{E} \]
Statement for Q.1–3:

In the circuit shown in fig. P3.3.1–3 the transistor parameters are as follows:

Threshold voltage $V_{TN} = 2$ V
Conduction parameter $K_n = 0.5$ mA / V$^2$

1. $V_{GS} = (?)$
   (A) 2.05 V  
   (B) 6.43 V  
   (C) 4.86 V  
   (D) 3.91 V

2. $I_D = (?)$
   (A) 1.863 mA  
   (B) 1.485 mA  
   (C) 0.775 mA  
   (D) None of the above

3. $V_{DS} = (?)$
   (A) 4.59 V  
   (B) 3.43 V  
   (C) 5.35 V  
   (D) 6.48 V

Statement for Q.4–6:

In the circuit shown in fig. P3.3.4–6 the transistor parameter are as follows:

$V_{TN} = 2$ V, $k_n' = 60$ $\mu$A / V$^2$, $W/L = 60$

4. $V_{GS} = (?)$
   (A) -3.62 V  
   (B) 3.62 V  
   (C) -0.74 V  
   (D) 0.74 V

5. $I_D = (?)$
   (A) 13.5 mA  
   (B) 10 mA  
   (C) 19.24 mA  
   (D) 4.76 mA

6. $V_{DS} = (?)$
   (A) 2.95 V  
   (B) 11.9 V  
   (C) 3 V  
   (D) 12.7 V
16. The parameter of the transistor in fig. P3.3.16 are \(V_{TN} = 1.2\, \text{V}, \quad K_n = 0.5\, \text{mA/}\text{V}^2 \) and \(\lambda = 0\). The voltage \(V_{DS}\) is

- (A) 1.69 V
- (B) 1.52 V
- (C) 1.84 V
- (D) 0

![Fig. P3.3.16](image)

17. The parameter of the transistor in fig. P3.3.17 are \(V_{TN} = 0.6\, \text{V}\) and \(K_n = 0.2\, \text{mA/}\text{V}^2\). The voltage \(V_S\) is

- (A) 1.72 V
- (B) -1.72 V
- (C) 7.28 V
- (D) -7.28 V

![Fig. P3.3.17](image)

18. In the circuit of fig. P3.3.18 the transistor parameters are \(V_{TN} = 1.7\, \text{V}\) and \(K_n = 0.4\, \text{mA/}\text{V}^2\).

- If \(I_D = 0.8\, \text{mA}\) and \(V_D = 1\, \text{V}\), then value of resistor \(R_S\) and \(R_D\) are respectively

  - (A) 2.36 k\(\Omega\), 5 k\(\Omega\)
  - (B) 5 k\(\Omega\), 2.36 k\(\Omega\)
  - (C) 6.43 k\(\Omega\), 8.4 k\(\Omega\)
  - (D) 8.4 k\(\Omega\), 6.43 k\(\Omega\)

  - (A) 5.84 mW
  - (B) 2.34 mW
  - (C) 0.26 mW
  - (D) 58.4 mW

![Fig. P3.3.18](image)

19. In the circuit of fig. P.3.3.19 the PMOS transistor has parameter \(V_{TP} = -1.5\, \text{V}, \quad k'_p = 25\, \mu\text{A/}\text{V}^2, \quad L = 4\, \mu\text{m}\) and \(\lambda = 0\). If \(I_D = 0.1\, \text{mA}\) and \(V_{SD} = 2.5\, \text{V}\), then value of \(W\) will be

- (A) 15 \(\mu\text{m}\)
- (B) 1.6 \(\mu\text{m}\)
- (C) 32 \(\mu\text{m}\)
- (D) 3.2 \(\mu\text{m}\)

![Fig. P3.3.19](image)

20. The PMOS transistor in fig. P3.3.20 has parameters

\(V_{TP} = -1.2\, \text{V}, \quad \frac{W}{L} = 20, \quad \text{and} \quad k'_p = 30\, \mu\text{A/}\text{V}^2\).

- If \(I_D = 0.5\, \text{mA}\) and \(V_D = -3\, \text{V}\), then value of \(R_S\) and \(R_D\) are

  - (A) 4 k\(\Omega\), 5.8 k\(\Omega\)
  - (B) 4 k\(\Omega\), 5 k\(\Omega\)
  - (C) 5.8 k\(\Omega\), 4 k\(\Omega\)
  - (D) 5 k\(\Omega\), 4 k\(\Omega\)

![Fig. P3.3.20](image)

21. The parameters for the transistor in circuit of fig. P3.3.21 are \(V_{TN} = 2\, \text{V}\) and \(K_n = 0.2\, \text{mA/}\text{V}^2\). The power dissipated in the transistor is

- (A) 5.84 mW
- (B) 2.34 mW
- (C) 0.26 mW
- (D) 58.4 mW

![Fig. P3.3.21](image)
Statement for Q.22–23:

Consider the circuit shown in fig. P3.2.22–33.

The both transistor have parameter as follows

\[ V_{TN} = 0.8 \text{ V}, \quad k_n = 30 \ \mu\text{A} / \text{V}^2 \]

22. If the width-to-length ratios of \( M_1 \) and \( M_2 \) are

\[ \left( \frac{W}{L} \right)_1 = \left( \frac{W}{L} \right)_2 = 40 \]

The output \( V_o \) is

(A) -2.5 V \quad (B) 2.5 V \quad (C) 5 V \quad (D) 0 V

23. If the ratio is \( \left( \frac{W}{L} \right)_1 = 40 \) and \( \left( \frac{W}{L} \right)_2 = 15 \), then \( V_o \) is

(A) 2.91 V \quad (B) 2.09 V \quad (C) 3.41 V \quad (D) 1.59 V

24. In the circuit of fig. P3.324, the transistor parameters are \( V_{TN} = 1 \text{ V} \) and \( k_n = 36 \ \mu\text{A} / \text{V}^2 \). If \( I_D = 0.5 \) mA, \( V_i = 5 \text{ V} \) and \( V_o = 2 \text{ V} \) then the width to-length ratio required in each transistor is

\[ \left( \frac{W}{L} \right)_1 \quad \left( \frac{W}{L} \right)_2 \quad \left( \frac{W}{L} \right)_3 \]

\[ \begin{align*}
(A) & \quad 1.75 & & 6.94 & & 27.8 \\
(B) & \quad 4.93 & & 10.56 & & 50.43 \\
(C) & \quad 35.5 & & 22.4 & & 8.53 \\
(D) & \quad 56.4 & & 38.21 & & 12.56 
\end{align*} \]

25. The transistors in the circuit of fig. P3.3.25 have parameter \( V_{TN} = 0.8 \text{ V}, \ k_n' = 40 \ \mu\text{A} / \text{V}^2 \) and \( \lambda = 0 \). The width-to-length ratio of \( M_2 \) is \( \left( \frac{W}{L} \right)_2 = 1 \). If \( V_o = 0.10 \text{ V} \) when \( V_i = 5 \text{ V} \), then \( \left( \frac{W}{L} \right)_1 \) for \( M_1 \) is

(A) 47.5 \quad (B) 28.4 \quad (C) 40.5 \quad (D) 20.3

Statement for Q.26–27:

All transistors in the circuit in fig. P3.3.26–27 have parameter \( V_{TN} = 1 \text{ V} \) and \( \lambda = 0 \).

\[ \begin{array}{c}
\begin{array}{c}
M_1 \\
\uparrow I_D \\
\downarrow R_D
\end{array} \\
\begin{array}{c}
M_2 \\
\uparrow I_D
\end{array} \\
\begin{array}{c}
M_3 \\
\downarrow R_D
\end{array}
\end{array} \]

The conduction parameter are as follows:

\[ K_{n1} = 400 \ \mu\text{A} / \text{V}^2 \quad K_{n2} = 200 \ \mu\text{A} / \text{V}^2 \quad K_{n3} = 100 \ \mu\text{A} / \text{V}^2 \quad K_{n4} = 80 \ \mu\text{A} / \text{V}^2 \]

26. \( I_{D1} = ? \)

(A) 0.23 mA \quad (B) 0.62 mA \quad (C) 0.46 mA \quad (D) 0.31 mA

27. \( I_{D4} = ? \)

(A) 0.62 mA \quad (B) 0.31 mA \quad (C) 0.46 mA \quad (D) 0.92 mA

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28. For the circuit in fig. P3.3.28 the transistor parameter are $V_{TN} = 0.8 \text{ V}$ and $k_n = 30 \mu\text{A} / \text{V}^2$. If output voltage is $V_o = 0.1 \text{ V}$, when input voltage is $V_i = 42 \text{ V}$, the required transistor width-to-length ratio is

(A) $1.568$ \hspace{1cm} (B) $0.986$ \hspace{1cm} (C) $0.731$ \hspace{1cm} (D) $1.843$

29. For the transistor in fig. P3.3.29 parameters are $V_{TN} = 1 \text{ V}$ and $K_n = 12.5 \mu\text{A} / \text{V}^2$. The Q-point $(I_D, V_{DS})$ is

(A) $(1 \text{ mA, } 8 \text{ V})$ \hspace{1cm} (B) $(0.2 \text{ mA, } 4 \text{ V})$ \hspace{1cm} (C) $(1.17 \text{ mA, } 8 \text{ V})$ \hspace{1cm} (D) $(0.23 \text{ mA, } 3.1\text{V})$

30. For an $n$-channel JFET, the parameters are $I_{DSS} = 6 \text{ mA}$ and $V_p = -3 \text{ V}$. If $V_{DS} > V_{DSS} \text{ (sat)}$ and $V_{GS} = -2 \text{ V}$, then $I_D$ is

(A) $16.67 \text{ mA}$ \hspace{1cm} (B) $0.67 \text{ mA}$ \hspace{1cm} (C) $5.55 \text{ mA}$ \hspace{1cm} (D) $1.67 \text{ mA}$

31. For the circuit in fig. P3.3.32 the transistor parameters are $V_p = -3.5 \text{ V}$, $I_{DSS} = 18 \text{ mA}$, and $\lambda = 0$. The value of $V_{DS}$ is

(A) $2.7 \text{ V}$ \hspace{1cm} (B) $2.85 \text{ V}$ \hspace{1cm} (C) $-1.30 \text{ V}$ \hspace{1cm} (D) $1.30 \text{ V}$

32. A $p$-channel JFET biased in the saturation region with $V_{SD} = 5 \text{ V}$ has a drain current of $I_D = 2.8 \text{ mA}$, and $I_D = 0.3 \text{ mA}$ at $V_{GS} = 3 \text{ V}$. The value of $I_{DSS}$ is

(A) $10 \text{ mA}$ \hspace{1cm} (B) $5 \text{ mA}$ \hspace{1cm} (C) $7 \text{ mA}$ \hspace{1cm} (D) $2 \text{ mA}$

Statement for Q.33–34:

For the p-channel transistor in the circuit of fig. P3.3.33–34 the parameters are $I_{DSS} = 6 \text{ mA, } V_p = 4 \text{ V}$ and $\lambda = 0$.

33. The value of $I_{DQ}$ is

(A) $8.86 \text{ mA}$ \hspace{1cm} (B) $6.39 \text{ mA}$ \hspace{1cm} (C) $4.32 \text{ mA}$ \hspace{1cm} (D) $1.81 \text{ mA}$

34. The value of $V_{SD}$ is

(A) $-4.28 \text{ V}$ \hspace{1cm} (B) $2.47 \text{ V}$ \hspace{1cm} (C) $4.28 \text{ V}$ \hspace{1cm} (D) $2.19 \text{ V}$

35. The transistor in the circuit of fig. P3.3.35 has parameters $I_{DSS} = 8 \text{ mA}$ and $V_p = -4 \text{ V}$. The value of $V_{DSQ}$ is

(A) $2.7 \text{ V}$ \hspace{1cm} (B) $2.85 \text{ V}$ \hspace{1cm} (C) $-1.30 \text{ V}$ \hspace{1cm} (D) $1.30 \text{ V}$
1. (A) \( R_1 = 32 \, k\Omega \), \( R_2 = 18 \, k\Omega \), \( V_{DD} = 10 \, V \)

\[
V_G = \left( \frac{R_2}{R_1 + R_2} \right) V_{DD} = \left( \frac{18}{18 + 32} \right) 10 = 3.6 \, V
\]

Assume transistor in saturation region

\[
I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2
\]

\[
R_S = 2 \, k\Omega, \quad K_n = 0.5 \, mA / V^2
\]

\[
3.6 - V_{GS} = (2)(0.5)(V_{GS} - 0.8)^2 \quad \Rightarrow \quad V_{GS} = 2.05 \, V
\]

2. (C) \( I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.05}{2k} = 0.775 \, mA \)

3. (C) \( V_{DS} = V_{DD} - I_D(R_D + R_S) \)

\[
= 10 - 0.775(4 + 2) = 5.35 \, V
\]

\( V_{DS} > V_{DS} \) as assumed.

4. (B) \( R_1 = 14 \, k\Omega \), \( R_2 = 6 \, k\Omega \), \( R_S = 0.5 \, k\Omega \), \( R_D = 1.2 \, k\Omega \)

\[
V_G = \left( \frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left( \frac{6}{14 + 6} \right) 20 - 10 = -4 \, V
\]

Assume transistor in saturation

\[
I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS} + 10}{R_S} = K_n(V_{GS} - V_{TN})^2
\]

\[
K_n = \frac{k_n W}{2L} = 0.60(60 \times 10^{-6}) = 1.8 \, mA / V^2
\]

\[
\Rightarrow \quad -4 - V_{GS} + 10 = (0.5)(1.8)(V_{GS} - 2)^2
\]

\[
\Rightarrow \quad V_{GS} = 3.62, \quad -0.74, \quad V_{GS} \quad \text{will be positive.}
\]

5. (D) \( I_D = \frac{V_G - V_{GS} + 10}{R_S} = \frac{-4 - 3.62 + 10}{0.5k} = 4.76 \, mA \)

6. (B) \( 10 = I_D(R_S + R_D) + V_{DS} - 10 \)

\( V_{DS} = 20 - 4.76(12 + 0.5) = 119 \, V \)

\( V_{DS} > V_{DS} \), Assumption is correct.

7. (B) \( R_1 = 8 \, k\Omega \), \( R_L = 22 \, k\Omega \), \( R_S = 0.5 \, k\Omega \), \( R_D = 2 \, k\Omega \)

\[
V_G = \left( \frac{R_2}{R_1 + R_2} \right) (20) - 10 = \left( \frac{22}{8 + 22} \right) 20 - 10 = 4.67 \, V
\]

Assume transistor in saturation

\[
I_D = \frac{10 - V_S}{R_S} = K_n(V_{SG} + V_{TP})^2
\]

\[
V_S = V_G + V_{SG}
\]

\[
10 - (4.67 + V_{SG}) = (0.5)(1)(V_{GS})
\]

\[
\Rightarrow \quad V_{SG} = 3.77 \, V, \quad -1.77 \, V, \quad V_{SG} \quad \text{is positive voltage.}
\]

8. (A) \( I_D = \frac{10 - V_S}{R_S} = \frac{10 - (V_G + V_{GS})}{R_S} \)

\[
= \frac{10 - (4.67 + 3.77)}{0.5} = 312 \, mA
\]

9. (C) \( 10 = I_D(R_S + R_D) + V_{SD} - 10 \)

\( V_{SD} = 20 - I_D(R_S + R_D) = 20 - 2.12(2 + 0.5) = 12.2 \, V \)

10. (C) Assume transistor in saturation.

\[
I_D = 0.4 \, mA, \quad 0.4 = K_n(V_{GS} + V_{TP})^2
\]

\[
0.4 = (0.2)(V_{SG} - 0.8)^2 \quad \Rightarrow \quad V_{SG} = \sqrt{2} + 0.8 = 2.21 \, V
\]

\( V_G = 0, \quad V_{SG} = V_S - V_G = S
\]

11. (A) \( V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3 \, V \)

\( V_{SD} = V_S - V_D = 2.21 - (-3) = 5.21 \, V \)

12. (C) \( R_1 = 145 \, k\Omega, \quad R_2 = 55 \, k\Omega, \quad R_S = 0.6 \, k\Omega, \quad R_D = 0.8 \, k\Omega \)

\[
V_G = \left( \frac{R_1}{R_1 + R_2} \right) 10 - 5 = \left( \frac{55}{145 + 55} \right) 10 - 5 = -2.25 \, V
\]

Assume transistor in saturation.

\[
I_D = \frac{V_S - (-5)}{R_S} = K_n(V_{GS} - V_{TN})^2
\]

\[
V_S = V_G - V_{GS}
\]

\[
-2.25 - V_{GS} = 5 = (0.6)(0.5)(V_{GS} - (-1))^2
\]

\[
\Rightarrow \quad V_{GS} = 1.24, -6.58 \, V
\]

\( V_{GS} \) is positive. Thus (D) is correct option.

13. (D) \( I_D = \frac{V_S + 5}{R_S} = \frac{V_G - V_{GS} + 5}{R_S} = \frac{-225 - 124 + 5}{0.6k} \)

\[
= 252 \, mA, \quad \text{Therefore (D) is correct option.}
\]

14. (B) \( 5 = I_D(R_S + R_D) + V_{DS} - 5 \)

\( V_{DS} = 10 - I_D(R_S + R_D) = 10 - 2.52(0.8 + 0.6) = 6.47 \, V \)

\( V_{DS} > V_{DS} \), Assumption is correct.

15. (B) \( I_S = 50 \, \mu A = I_D = K_n(V_{GS} - V_{TN})^2 \)

\[
50 \times 10^{-6} = 0.5 \times 10^{-6}(V_{GS} - 12)^2 \quad \Rightarrow \quad V_{GS} = 1.516 \, V
\]

\( V_G = 0, \quad V_S = V_G - V_{GS} = -1.516 \, V
\]

\( V_{DS} = V_D - V_S = 5 - (-1.516) = 6.516 \, V
\]

16. (B) \( I_D = 50 \mu A = K_n(V_{GS} - V_{TN})^2 \)

\[
\Rightarrow \quad 50 \times 10^{-6} = 0.5 \times 10^{-6}(V_{GS} - 12)^2
\]
$V_{gs} = 1.52 \text{ V}, \quad V_{gs} = V_{ds}$

**17. (B)**

$I_d = K_p (V_{gs} - V_{tn})^2$

$\Rightarrow \quad 0.25 = 0.2 (V_{gs} - 0.6)^2 \quad \Rightarrow \quad V_{gs} = 1.72 \text{ V}, \quad V_{ds} = 1.72 \text{ V}$

$V_{gs} = V_g - V_s, \quad V_g = 0, \quad V_s = -1.72 \text{ V}$

**18. (A)**

$I_d = \frac{5 - V_d}{R_d} = 0.8 \text{ mA}, \quad R_d = 6 - 1 = 5 \text{ k}\Omega$

$I_d = K_p (V_{gs} - V_{tn})^2$

$\Rightarrow \quad 0.8 = (0.4) (V_{gs} - 1.7)^2 \quad \Rightarrow \quad V_{gs} = 3.11 \text{ V}$

$V_{gs} = V_g - V_s, \quad V_g = 0, \quad V_s = -3.11 \text{ V}$

$I_d = 0.8 \text{ mA} = \frac{-3.11 - (-5)}{R_s} \quad \Rightarrow \quad R_s = 2.36 \text{ k}\Omega$

**19. (C)**

$V_{sd} = V_{sg}, \quad I_d = \frac{K_p}{2} \frac{W}{L} (V_{gs} + V_{tp})^2$

$10^{-4} = \left( \frac{25}{2} \right) \left( \frac{W}{4} \right) (2.5 - 15)^2 \quad \Rightarrow \quad W = 32 \mu\text{m}$

**20. (D)**

$K_p = \left( \frac{30 \times 10^{-6}}{2} \right) (20) = 0.3 \\text{ mA/ V}^2$

$I_d = K_p (V_{sg} + V_{tp})^2 \quad \Rightarrow \quad 0.5 = 0.3 (V_{sg} - 12)^2$

$\Rightarrow \quad V_{sg} = 2.49 \text{ V}, \quad V_d = 0$

$V_s = V_{sg} = 2.49 \text{ V}$

$I_d = \frac{5 - V_d}{R_s} \quad \Rightarrow \quad R_s = \frac{5 - 2.49}{0.5} = 5.02 \text{ k}\Omega$

$I_d = \frac{5 - V_d}{R_d} \quad \Rightarrow \quad R_d = \frac{-3 + 5}{0.5} = 4 \text{ k}\Omega$

**21. (B)**

Assume transistor in saturation

$I_d = \frac{10 - V_{gs}}{10k} = K_p (V_{gs} - V_{tn})^2$

$10 - V_{gs} = (10)(0.2)(V_{gs} - 2)^2$

$\Rightarrow \quad V_{gs} = 3.77 \text{ V}, \quad -0.27 \text{ V}, \quad V_{gs} \quad \text{will be} \quad 3.77 \text{ V}$

$V_{gs} = V_{ds} = 3.77 \text{ V}$

$I_d = \frac{10 - 3.77}{10k} = 0.623 \text{ mA}$

Power = $I_d V_{ds} = 2.35 \text{ mW}$

$V_{ds} > V_{gs} - V_{tn}$ assumption is correct.

**22. (B)**

For both transistor $V_{ds} = V_{gs}$,

$V_{ds} > V_{gs} - V_{tn}$ Therefore both transistor are in saturation.

$I_{in} = I_{d2} \quad \Rightarrow \quad K_{n1} (V_{gs1} - V_{tn1})^2 = K_{n2} (V_{gs2} - V_{tn2})^2$

$K_{n1} = K_{n2}, \quad V_{tn1} = V_{tn2}$

$V_{gs1} = V_{gs2} = \frac{5}{2} \text{ V}$

$V_s = V_{gs2} = 2.5 \text{ V}$

**23. (A)**

$\left( \frac{W}{L} \right) > \left( \frac{W}{L} \right)_2 \quad \text{thus} \quad V_{gs1} < V_{gs2}$

$40 (V_{gs1} - 0.8)^2 = 15 (V_{gs2} - 0.8)^2$

$V_{gs2} = 5 - V_{gs1}$

$1.63 (V_{gs1} - 0.8) = (5 - V_{gs1} - 0.8)$

$V_{gs1} = 2.09, \quad V_{gs2} = 2.91 \text{ V}, \quad V_s = V_{gs2} = 2.91 \text{ V}$

**24. (A)** Each transistor is biased in saturation because

$V_{ds} = V_{gs}$ and $V_{ds} > V_{gs} - V_{tn}$

For $M_2, \quad V_2 = 2 \text{ V} = V_{gs3}$

$I_d = 0.5 = \left( \frac{36 \times 10^{-3}}{2} \right) \left( \frac{W}{L} \right)_3 (2 - 1)^2 \quad \Rightarrow \quad \left( \frac{W}{L} \right)_3 = 27.8$

For $M_2, \quad V_{gs2} = V_1 - V_2 = 5 - 2 = 3 \text{ V}$

$I_d = 0.5 = \left( \frac{36 \times 10^{-3}}{2} \right) \left( \frac{W}{L} \right)_2 (3 - 1)^2 \quad \Rightarrow \quad \left( \frac{W}{L} \right)_2 = 6.94$

For $M_1, \quad V_{gs1} = 10 - V_1 = 10 - 5 = 5 \text{ V}$

$I_d = 0.5 = \left( \frac{36 \times 10^{-3}}{2} \right) \left( \frac{W}{L} \right)_1 (5 - 1)^2 \quad \Rightarrow \quad \left( \frac{W}{L} \right)_1 = 1.74$

**25. (D)**

$M_3$ is in saturation because

$V_{gs2} = V_{ds2} > V_{gs2} - V_{tn}$

$M_1$ is in non saturation because

$V_{gs1} = V_1 = 5 \text{ V}, \quad V_{gs1} = V_0 = 0 \text{ V}$

$V_{gs2} = V_{gs1} - V_{tn}, \quad I_{in} = I_d2$

$\left( \frac{W}{L} \right)_4 \left( 2V_{gs1} - V_{tn} \right) V_{ds1} - V_{gs2}^2 \Rightarrow \left( \frac{W}{L} \right)_2 \left( 2V_{gs2} - V_{tn2} \right)^2$

$\Rightarrow \quad \left( \frac{W}{L} \right) \left[ 2(5 - 0.8)(0.1) - (0.1)^2 \right] = (1)(5 - 0.1 - 0.8)^2$

$\left( \frac{W}{L} \right) (0.83) = 16.81 \quad \Rightarrow \quad \left( \frac{W}{L} \right)_4 = 20.3$

**26. (B)**

$I_{in} = K_{n1} (V_{gs1} - V_{tn})^2 = K_{n2} (V_{gs2} - V_{tn})^2$

$V_{gs1} = 5 - V_{gs2} \quad \Rightarrow \quad (5 - V_{gs2} - 1)^2 = 200 (V_{gs2} - 1)^2$

$\Rightarrow \quad V_{gs2} = 2.76 \text{ V}, \quad V_{gs2} = 2.24 \text{ V}$

$I_{in} = 400 \times 10^{-6} (2.24 - 1)^2 = 0.62 \text{ mA}$

**27. (B)**

$V_{gs2} = V_{gs3} = 276 \text{ V}$

$I_{d4} = K_{n4} (V_{gs4} - V_{tn})^2 = K_{n4} (V_{gs3} - V_{tn})^2$

$= 100 \times 10^{-6} (2.76 - 1)^2 = 0.31 \text{ mA}$

**28. (C)**

$V_{gs} = 4.2 \text{ V}, \quad V_{ds} = 0.1 \text{ V}$

$V_{ds} < V_{gs} - V_{tn}$, Thus transistor is in non saturation.

$I_d = \frac{5 - 0.1}{10k} = 0.49 \text{ mA}$

$I_d = \frac{K_p}{2} \frac{W}{L} \left( 2 (V_{gs} - V_{tn}) V_{ds} - V_{ds}^2 \right)$
1. If the transistor parameter are \( \beta = 180 \) and Early voltage \( V_A = 140 \) V and it is biased at \( I_{CQ} = 2 \) mA, the values of hybrid-\( \pi \) parameter \( g_m, r_e \) and \( r_o \) are respectively
   (A) 14 A/V, 2.33 k\( \Omega \), 90 k\( \Omega \)
   (B) 14 A/V, 90 k\( \Omega \) , 2.33 k\( \Omega \)
   (C) 77 mA/V, 2.33 k\( \Omega \) , 70 k\( \Omega \)
   (D) 77.2 A/V, 70 k\( \Omega \), 2.33 k\( \Omega \)

   **Statement for Q.2–3.**

   Consider the circuit of fig. P3.4.2–3. The transistor parameters are \( \beta = 120 \) and \( V_A = \infty \).

2. The hybrid-\( \pi \) parameter values of \( g_m \), \( r_e \) and \( r_o \) are
   (A) 24 mA/V, \( \infty \), 5 k\( \Omega \)
   (B) 24 mA/V, 5 k\( \Omega \) , \( \infty \)
   (C) 48 mA/V, 10 k\( \Omega \) , 18.4 k\( \Omega \)
   (D) 48 mA/V, 18.4 k\( \Omega \), 10 k\( \Omega \)

3. The small signal voltage gain \( A_v = v_o/v_s \) is
   (A) \(-4.38\)  
   (B) \(4.38\)  
   (C) \(-1.88\)  
   (D) \(1.88\)

4. The nominal quiescent collector current of a transistor is 1.2 mA. If the range of \( \beta \) for this transistor is \( 80 \leq \beta \leq 120 \) and if the quiescent collector current changes by \( \pm 10 \) percent, the range in value for \( r_e \) is
   (A) 1.73 k\( \Omega \) < \( r_e \) < 2.59 k\( \Omega \)
   (B) 1.93 k\( \Omega \) < \( r_e \) < 2.59 k\( \Omega \)
   (C) 1.73 k\( \Omega \) < \( r_e \) < 2.59 k\( \Omega \)
   (D) 1.56 k\( \Omega \) < \( r_e \) < 2.88 k\( \Omega \)

   **Statement for Q.5–6:**

   Consider the circuit in fig. P3.4.5.6. The transistor parameter are \( \beta = 100 \) and \( V_A = \infty \).

5. If Q-point is in the center of the load line and \( I_{CQ} = 0.5 \) mA, the values of \( V_{BB} \) and \( R_C \) are
   (A) 10 k\( \Omega \) , 0.95 V  
   (B) 10 k\( \Omega \) , 1.45 V  
   (C) 48 k\( \Omega \) , 0.95 V  
   (D) 48 k\( \Omega \) , 1.45 V
Statement for Q.14–15:

Consider the common Base amplifier shown in fig. P3.4.14–15. The parameters are \( g_m = 2 \text{ mS} \) and \( r_o = 250 \text{ k}\Omega \). Find the Thevenin equivalent faced by load resistance \( R_L \).

14. The Thevenin voltage \( v_{TH} \) is
   (A) 263\( v_i \)          (B) 132\( v_i \)
   (C) 346\( v_i \)          (D) 498\( v_i \)

15. The Thevenin equivalent resistance \( R_{TH} \) is
   (A) 384 \text{ k}\Omega          (B) 697 \text{ k}\Omega
   (C) 408 \text{ k}\Omega          (D) 915 \text{ k}\Omega

Statement for Q.16–17:

The common-base amplifier is drawn as a two-port in fig. P3.4.16–17. The parameters are \( \beta = 100 \), \( g_m = 3 \text{ mS} \), and \( r_o = 800 \text{ k}\Omega \).

16. The h-parameter \( h_{21} \) is
   (A) 2.46          (B) 0.9
   (C) 0.5          (D) 0.67

17. The h-parameter \( h_{12} \) is
   (A) \( 3.8 \times 10^{-4} \)          (B) \( 4.83 \times 10^{-3} \)
   (C) \( 3.8 \times 10^{4} \)          (D) \( 4.83 \times 10^{-3} \)

18. For an \( n \)-channel MOSFET biased in the saturation region, the parameters are \( K_s = 0.5 \text{ mA}/\text{V}^2 \), \( V_{TN} = 0.8 \text{ V} \) and \( \lambda = 0.01 \text{ V}^{-1} \), and \( I_{DQ} = 0.75 \text{ mA} \). The value of \( g_m \) and \( r_o \) are
   (A) 0.68 mS, 603 k\Omega          (B) 1.22 mS, 603 k\Omega
   (C) 1.22 mS, 133 k\Omega          (D) 0.68 mS, 133 k\Omega

Statement for Q.21–23:

For the circuit shown in fig. P3.4.21–23 transistor parameters are \( V_{TN} = 2 \text{ V} \), \( K_s = 0.5 \text{ mA}/\text{V}^2 \) and \( \lambda = 0.01 \text{ V}^{-1} \). The transistor is in saturation.

19. For an \( n \)-channel MOSFET biased in the saturation region, the parameters are \( V_{TN} = 1 \text{ V} \), \( \frac{1}{2} \mu_s C_m = 18 \text{ mA}/\text{V}^2 \) and \( \lambda = 0.015 \text{ V}^{-1} \) and \( I_{DQ} = 2 \text{ mA} \). If transconductance is \( g_m = 3.4 \text{ mA}/\text{V} \), the width-to-length ratio is
   (A) 80.6          (B) 43.2
   (C) 190          (D) 110

20. In the circuit of fig. P3.4.20, the parameters are \( g_m = 1 \text{ mA}/\text{V} \), \( r_o = 50 \text{ k}\Omega \). The gain \( A_v = v_o/v_i \) is
   (A) \( -8.01 \)          (B) 8.01
   (C) 14.16          (D) \(-14.16 \)

21. If \( I_{DQ} \) is to be 0.4 mA, the value of \( V_{GSQ} \) is
   (A) 5.14 V          (B) 4.36 V
   (C) 2.89 V          (D) 1.83 V

22. The values of \( g_m \) and \( r_o \) are
   (A) 0.89 mS, \( \infty \)          (B) 0.89 mS, 0
   (C) 1.48 mS, 0          (D) 1.48 mS, \( \infty \)

23. The small signal voltage gain \( A_v \) is
   (A) 14.3          (B) \(-14.3 \)
   (C) \(-8.9 \)          (D) 8.9
Statement for Q.33–34:
Consider the source-follower circuit in fig. P3.4.33-34. The values of parameter are \( g_m = 2 \, \text{mS} \) and \( r_o = 100 \, \text{k}\Omega \).

33. The voltage gain \( A_v \) is
(A) 0.89
(B) −0.89
(C) 2.79
(D) −2.79

34. The output resistance \( R_o \) is
(A) 100 k\Omega
(B) 0.498 k\Omega
(C) 1.33 k\Omega
(D) None of the above

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SOLUTIONS

1. (C) \( g_m = \frac{I_{CQ}}{V_i} = \frac{2}{0.0259} = 77.2 \, \text{mA/V} \)
\[ r_x = \frac{\beta V_i}{I_{CQ}} = \frac{180}{772} = 2.33 \, \text{k}\Omega \]
\[ r_o = \frac{V_A}{I_{CQ}} = \frac{140}{2} = 70 \, \text{k}\Omega \]

2. (B) \( I_{BQ} = \frac{2 - 0.7}{250} = 52 \, \mu\text{A} \)
\[ I_{CQ} = \beta I_B = (120)(5.2 \mu\text{A}) = 0.642 \, \text{mA} \]
\[ g_m = \frac{I_{CQ}}{V_i} = \frac{0.624}{0.0259} = 24 \, \text{mA/V} \]
\[ r_x = \frac{\beta V_i}{I_{CQ}} = \frac{120}{24} = 5 \, \text{k}\Omega, \quad r_o = \infty \]

3. (C) \( A_v = -g_m R_C \left( \frac{r_x}{r_x + R_B} \right) = \frac{\beta R_C}{r_x + R_B} \]
\[ = -(24m)(4k) \left( \frac{5k}{5k + 250k} \right) = -1.88 \]

4. (D) \( r_n = \frac{\beta V_i}{I_{CQ}} \)
\[ r_{n,max} = \frac{(120)(0.0259)}{108m} = 2.88 \, \text{k}\Omega, \]
\[ r_{n(min)} = \frac{(80)(0.0259)}{132m} = 156 \, \text{k}\Omega \]

5. (A) \( V_{ECQ} = \frac{1}{2} V_{CC} = 5 \, \text{V} \)
\[ V_{ECQ} = 10 - I_{CQ} R_C = 5 \]
\[ \Rightarrow 10 - (0.5m)R_C = 5 \]
\[ R_C = 10 \, \text{k}\Omega, \]
\[ I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 5 \, \mu\text{A} \]
\[ V_{EBQ} + I_{BQ}R_B = V_{BE} \]
\[ \Rightarrow 0.7 + (5\mu)(50k) = 0.95 \, \text{V} \]

6. (D) \( g_m = \frac{I_{CQ}}{V_i} = \frac{0.5}{0.0259} = 19.3 \, \text{mA/V} \)
\[ r_x = \frac{\beta V_i}{I_{CQ}} = \frac{(100)(0.0259)}{0.5m} = 5.18 \, \text{k}\Omega, \quad r_o = \infty \]

7. (B) \( I_{CQ} = \left( \frac{100}{1001} \right)0.35 = 0.347 \, \text{mA} \)

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The small-signal equivalent circuit is as shown in Fig. S3.4.7

\[ V_s = \frac{r_n \| 10k}{500 + r_n \| 10k} (v_i) \]
\[ v_o = -g_m V_s (r_n \| 7k) \]
\[ g_m = \frac{I_{cq}}{V_i} = \frac{0.347mA}{0.0259} = 13.13 \text{ mA/V} \]
\[ r_n = \frac{\beta V_i}{I_{cq}} = \frac{100}{13.13} = 7.6 \text{ kΩ} \]
\[ r_n = \frac{100}{I_{cq}} = 288 \text{ kΩ} \]
\[ r_n \| 7k = \frac{288 \times 7}{288 + 7} = 6.83 \text{ kΩ}, r_n \| 10k = \frac{7.6 \times 10}{7.6 + 10} = 4.32 \text{ kΩ} \]
\[ A_v = -13.13 \left( \frac{4.32k}{500 + 4.32k} \right) (6.83k) = -80 \]

8. (C) DC Analysis: \[ I_{cq} = I_{eq} \]
\[ V_{eq} = 5 = 10 - I_{cq} (R_c + R_k) \]
\[ 5 = 10 - I_{cq} (1.2k + 0.2k) \rightarrow I_{cq} = 3.57 \text{ mA} \]
\[ I_{eq} = \frac{357}{150} = 23.8 \mu A \]

AC Analysis:

9. (A) DC Analysis: \[ V_{TH} = \frac{50}{10 + 50} (12) = 10 \text{ V} \]
\[ R_{TH} = 11 \| 50 = 8.33 \text{ kΩ} \]
\[ I_{BQ} = \frac{-12 - 0.7 - 10}{8.33k + (101)\text{k}} = 119 \mu A \]
\[ I_{cq} = \beta I_{BQ} = 119 \text{ mA}, I_{eq} = 12 \text{ mA} \]
\[ V_{eq} = 12 - (1.20)(1 - (1.19)) = 8.42 \text{ V} \]

AC Analysis:

For \( 1 \leq v_{ec} \leq 11 \text{ V} \), \[ \Delta v_{eq} = 11 - 8.42 = 258 \text{ V} \]
\[ \Rightarrow \text{ Output voltage swing} = 5.16 \text{ V peak to peak} \]

10. (B) \[ V_{acq} = 8.42 \text{ V} \]

11. (B) Since the B–C junction is not reverse biased, the transistor continues to operate in the forward-active mode
12. (C) \[ r_o = \frac{V_o}{I_{cQ}} \Rightarrow I_{cQ} = \frac{V_o}{r_o} = \frac{75}{200k} = 0.375 \text{ mA} \]

13. (B) \[ r_e = \frac{\beta V_i}{I_{cQ}} = \frac{75(0.0259)}{1m} = 194 \text{ k}\Omega \]

14. (D) The equivalent circuit is shown below.

Removing the \( R_L \), \(-V_o = \frac{v_v r_v}{270 + r_v} \)

\[ v_{th} = -r_e g_m V_i - V_o = \frac{v_v r_v (1 + g_m r_o)}{270 + r_v} \]

\[ r_e = \frac{\beta}{g_m} = \frac{100}{2m} = 50 \text{ k}\Omega \]

\[ v_{th} = \frac{v_v 50k (1 + (2m)/250k)}{270 + 50k} = 498 v_i \]

15. (A) The equivalent small-signal circuit is shown in fig. S3.4.15.

\[ I_{sc} = g_m V_s + \frac{V_s}{r_o} = 2mV_s + \frac{V_s}{250k} = 2004 mV_s \]

\[ v_1 \left( \frac{1}{39k} + \frac{1}{r_o} + \frac{1}{r_o} \right) - \frac{v_2}{r_o} = -g_m v_1 \]

\[ v_1 = \frac{1}{\frac{800k}{39k} + \frac{1}{r_o} + \frac{1}{r_o} + g_m} = \frac{1}{800k} \]

\[ v_2 = 3.8 \times 10^{-4} \]
$R_s = 4 \text{k}\Omega, \ v_{gs} = 0.84v_i$,  
$v_n = -g_{m}v_{gs}(R_o) = -(1.41\text{m})(0.84)(100\text{k}||5\text{k})$  
$\Rightarrow \ \frac{v_n}{v_i} = A_v = -5.6$

28. (A) $R_o = R_D||r_o||100\text{k} = 4.76 \text{k}\Omega$

29. (A) As shown in fig. S3.4.27, $R_i = R_i||R_s = 20.6 \text{k}\Omega$

30. (C) From the DC analysis:  
$V_{GSQ} = 1.5 \text{V}, \ I_{DSQ} = 0.5 \text{mA}$  
$g_m = 2K_n(V_{GS} - V_{TH}) = 2(1\text{m})(15 - 0.8) = 1.4 \text{mA/V}$  
$r_o = \left|\frac{\partial I_{DSQ}}{\partial V_{DSQ}}\right| = \infty$

The resulting small-signal equivalent circuit is shown in fig. S5.4.30

![Small-signal equivalent circuit](image)

$v_o = -g_mv_{gs}R_D, \ v_i = v_{gs} + g_mv_{gs}R_s$  
$\Rightarrow \ \frac{v_o}{v_i} = -\frac{g_mR_D}{1 + g_mR_s} = -\frac{(1.4\text{m})(7\text{k})}{1 + (1.4\text{m})(0.5\text{k})} = -5.76$

31. (B) Since the DC gate current is zero, $V_g = -V_{GSQ}$  
$I_{DSQ} = I_g = K_n(V_{GSQ} - V_{TH})^2$  
$\Rightarrow \ 0.5 = (V_{GSQ} - 0.8)^2$  
$V_{GSQ} = 1.51 \text{V} = V_S$  
$V_{DSQ} = 5 - (0.5\text{m})(7\text{k}) - (1.51) = 301 \text{V}$

The transistor is therefore biased in the saturation region. The small-signal equivalent circuit is shown in fig.S3.4.31.

![Small-signal equivalent circuit](image)

$v_o = -g_mv_{gs}(7\text{k})$

32. (A) The small-signal equivalent circuit is shown in fig. S3.4.34

![Small-signal equivalent circuit](image)

$v_o = -g_mv_{gs}(R_D||R_L), \ v_i = -v_{gs}$  
$A_v = \frac{v_o}{v_i} = g_m(R_D||R_L) = (2\text{m})(5\text{k}||4\text{k}) = 4.44$

33. (A) The small-signal equivalent circuit is shown in fig. S3.4.33

![Small-signal equivalent circuit](image)

$v_o = g_mv_{gs}(R_L||r_o)$  
$v_i = v_{gs} + v_o = v_{gs} + g_mv_{gs}(R_L||r_o)$  
$v_o = \frac{1}{1 + g_m(R_L||r_o)}$  
$v_o = A_v = \frac{g_m(R_L||r_o)}{1 + g_m(R_L||r_o)}$  
$R_L||r_o = 4\text{k}||100\text{k} = 100\text{k} = 3.86 \text{k}\Omega$  
$A_v = \frac{(2\text{m})(3.85\text{k})}{1 + (2\text{m})(3.85\text{k})} = 0.89$

34. (B) $R_o = \frac{1}{g_m}||r_o$  
$= \left(\frac{1}{2}\right)||(100) = 0.498 \text{k}\Omega$

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CHAPTER 3.5

OPERATIONAL AMPLIFIERS

1. \( A_v = \frac{v_o}{v_i} = ? \)

(A) -10  
(B) 10  
(C) -11  
(D) 11

2. \( A_v = \frac{v_o}{v_i} = ? \)

(A) -10  
(B) 10  
(C) 13.46  
(D) -13.46

3. The input to the circuit in fig. P3.5.3 is \( v_i = 2 \sin \omega t \) mV. The current \( i_o \) is

(A) 600 k\( \Omega \) \( \quad \) (B) 450 k\( \Omega \)  
(C) 4.5 M\( \Omega \)  
(D) 6 M\( \Omega \)

4. In circuit shown in fig. P3.5.4, the input voltage \( v_i \) is 0.2 V. The output voltage \( v_o \) is

(A) 6 V  
(B) -6 V  
(C) 8 V  
(D) -8 V

5. For the circuit shown in fig. P3.5.5 gain is \( A_v = \frac{v_o}{v_i} = -10 \). The value of \( R \) is

(A) 600 k\( \Omega \)  
(B) 450 k\( \Omega \)  
(C) 4.5 M\( \Omega \)  
(D) 6 M\( \Omega \)
6. For the op-amp circuit shown in fig. P3.5.6 the voltage gain \( A_v = \frac{v_o}{v_i} \) is

(A) 8 
(B) 8 
(C) -10 
(D) 10

[Fig. P3.5.6]

7. For the op-amp shown in fig. P3.5.7 open loop differential gain is \( A_{od} = 10^3 \). The output voltage \( v_o \) for \( v_i = 2 \) V is

(A) -1.996 
(B) -1.998 
(C) -2.004 
(D) -2.006

[Fig. P3.5.7]

8. The op-amp of fig. P3.5.8 has a very poor open-loop voltage gain of 45 but is otherwise ideal. The closed-loop gain of amplifier is

(A) 20 
(B) 4.5 
(C) 4 
(D) 5

[Fig. P3.5.8]

9. For the circuit shown in fig. P3.5.9 the input voltage \( v_i \) is 1.5 V. The current \( i_o \) is

(A) -1.5 mA 
(B) 1.5 mA 
(C) -0.75 mA 
(D) 0.75 mA

[Fig. P3.5.9]

10. In the circuit of fig. P3.5.10 the output voltage \( v_o \) is

(A) 2.67 V 
(B) -2.67 V 
(C) -6.67 V 
(D) 6.67 V

[Fig. P3.5.10]

11. In the circuit of fig. P3.5.11 the voltage \( v_{11} \) is \((1 + 2 \sin \omega t) \) mV and \( v_{12} = -10 \) mV. The output voltage \( v_o \) is

(A) \(-0.4(1 + \sin \omega t) \) mV 
(B) \(0.4(1 + \sin \omega t) \) mV 
(C) \(0.4(1 + 2 \sin \omega t) \) mV 
(D) \(-0.4(1 + 2 \sin \omega t) \) mV

[Fig. P3.5.11]

12. For the circuit in fig. P3.5.12 the output voltage is \( v_o = 2.5 \) V in response to input voltage \( v_i = 5 \) V. The finite open-loop differential gain of the op-amp is

(A) \(5 \times 10^4\) 
(B) 250.5 
(C) \(2 \times 10^4\) 
(D) 501

[Fig. P3.5.12]

13. \( v_o = ? \)

[Fig. P3.5.13]
28. For the circuit shown in fig. P3.5.28 the input resistance is

\[ R = \frac{38 \text{ k} \Omega}{240 \text{ k} \Omega} \]

\begin{itemize}
  \item (A) 38 k\Omega
  \item (B) 17 k\Omega
  \item (C) 25 k\Omega
  \item (D) 47 k\Omega
\end{itemize}

29. In the circuit of fig. P3.5.29 the op-amp slew rate is \( SR = 0.5 \text{ V/\mu s} \). If the amplitude of input signal is 0.02 V, then the maximum frequency that may be used is

\[ f_{\text{max}} = \frac{0.02 \text{ V}}{0.5 \text{ V/\mu s}} \]

\begin{itemize}
  \item (A) \( 0.55 \times 10^6 \text{ rad/s} \)
  \item (B) 0.55 rad/s
  \item (C) \( 1.1 \times 10^6 \text{ rad/s} \)
  \item (D) 1.1 rad/s
\end{itemize}

30. In the circuit of fig. P3.5.30 the input offset voltage and input offset current are \( V_o = 4 \text{ mV} \) and \( I_o = 150 \text{ nA} \). The total output offset voltage is

\[ V_{o_{\text{total}}} = V_o + I_o R_{\text{in}} \]

\begin{itemize}
  \item (A) 479 mV
  \item (B) 234 mV
  \item (C) 168 mV
  \item (D) 116 mV
\end{itemize}

31. \( i_o = ? \)

\[ i_o = \frac{1}{5} \text{ mA} \]

\begin{itemize}
  \item (A) \( -8 \text{ mA} \)
  \item (B) 18 A
  \item (C) \( -36 \text{ A} \)
  \item (D) 36 A
\end{itemize}

Statement for Q.32–33:

Consider the circuit shown below

32. If \( v_i = 2 \text{ V} \), then output \( v_o \) is

\[ v_o = 4 \text{ V} \]

\begin{itemize}
  \item (A) 4 V
  \item (B) \( -4 \text{ V} \)
  \item (C) 3 V
  \item (D) \( -3 \text{ V} \)
\end{itemize}

33. If \( v_i = -2 \text{ V} \), then output \( v_o \) is

\[ v_o = -6 \text{ V} \]

\begin{itemize}
  \item (A) \( -6 \text{ V} \)
  \item (B) 6 V
  \item (C) \( -3 \text{ V} \)
  \item (D) 3 V
\end{itemize}

34. \( v_o(t) = ? \)

\[ v_o(t) = \frac{1}{5} e^{t \text{ ms}} \text{ V} \]

\begin{itemize}
  \item (A) \( e^{\frac{t}{15}} u(t) \text{ V} \)
  \item (B) \( -e^{\frac{t}{15}} u(t) \text{ V} \)
  \item (C) \( e^{\frac{1}{15} t} u(t) \text{ V} \)
  \item (D) \( -e^{\frac{1}{15} t} u(t) \text{ V} \)
\end{itemize}

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35. The circuit shown in fig. P3.5.35 is at steady state before the switch opens at \( t = 0 \). The \( v_c(t) \) for \( t > 0 \) is

\[
(A) \ 10 - 5e^{-12.5t} \ V \quad (B) \ 5 + 5e^{-12.5t} \ V \\
(C) \ 5 + 5e^{\frac{t}{12.5}} \ V \quad (D) \ 10 - 5e^{\frac{t}{12.5}} \ V
\]

36. The LED in the circuit of fig. P3.5.36 will be on if \( v_i \) is

\[
(A) > 10 \ V \quad (B) < 10 \ V \\
(C) > 5 \ V \quad (D) < 5 \ V
\]

37. In the circuit of fig. P3.5.37 the CMRR of the op-amp is 60 dB. The magnitude of the \( v_o \) is

\[
(A) \ 1 \ mV \quad (B) \ 100 \ mV \\
(C) \ 200 \ mV \quad (D) \ 2 \ mV
\]

38. The analog multiplier \( X \) of fig. P3.5.38 has the characteristics \( v_o = v_i v_2 \). The output of this circuit is

\[
(A) \ v_i v_2 \quad (B) -v_i v_2 \\
(C) \ -\frac{v_i}{v_2} \quad (D) \ \frac{v_i}{v_2}
\]

39. If the input to the ideal comparator shown in fig. P3.5.39 is a sinusoidal signal of 8 V (peak to peak) without any DC component, then the output of the comparator has a duty cycle of

\[
(A) \ \frac{1}{2} \quad (B) \ \frac{1}{3} \\
(C) \ \frac{1}{6} \quad (D) \ \frac{1}{12}
\]

40. In the op-amp circuit given in fig. P3.5.40 the load current \( i_L \) is

\[
(A) \ -\frac{v_i}{R_s} \quad (B) \ \frac{v_i}{R_s} \\
(C) \ -\frac{v_i}{R_L} \quad (D) \ \frac{v_i}{R_L}
\]

41. In the circuit of fig. P3.5.41 output voltage is \( v_o = ti \sin V \) for a certain set of \( \omega, R, \) an C. The \( |v_o| \) will be 2 V if

\[
(A) \ \omega \ is \ doubled \quad (B) \ \omega \ is \ halved \\
(C) \ R \ is \ doubled \quad (D) \ None \ of \ the \ above
\]
42. In the circuit of fig. P3.5.42, the 3 dB cutoff frequency is

(A) 10 kHz  
(B) 1.59 kHz  
(C) 354 Hz  
(D) 689 Hz

43. The phase shift oscillator of fig. P3.5.43 operates at $f = 80$ kHz. The value of resistance $R_F$ is

(A) 148 kΩ  
(B) 236 kΩ  
(C) 438 kΩ  
(D) 814 kΩ

44. The value of $C$ required for sinusoidal oscillation of frequency 1 kHz in the circuit of fig. P3.5.44 is

(A) $\frac{1}{2\pi} \mu F$  
(B) $2\pi \mu F$  
(C) $\frac{1}{2\pi \sqrt{6}} \mu F$  
(D) $2\pi \sqrt{6} \mu F$

45. In the circuit shown in fig. P3.5.45 the op-amp is ideal. If $\beta_F = 60$, then the total current supplied by the 15 V source is

(A) 123.1 mA  
(B) 98.3 mA  
(C) 49.4 mA  
(D) 168 mA

46. In the circuit in fig. P3.5.46 both transistor $Q_1$ and $Q_2$ are identical. The output voltage at $T = 300$ K is

(A) $2 \log_{10} \left( \frac{V_2}{V_1} \cdot \frac{R_2}{R_1} \right)$  
(B) $\log_{10} \left( \frac{V_2}{V_1} \cdot \frac{R_1}{R_2} \right)$  
(C) $2.303 \log_{10} \left( \frac{V_2}{V_1} \cdot \frac{R_1}{R_2} \right)$  
(D) $4.605 \log_{10} \left( \frac{V_2}{V_1} \cdot \frac{R_1}{R_2} \right)$

47. In the op-amp series regulator circuit of fig. P8.3.47 $V_z = 62$ V, $V_{BE} = 0.7$ V and $\beta = 60$. The output voltage $v_o$ is

(A) 35.8 V  
(B) 24.8 V  
(C) 29.8 V  
(D) None of the above
26. (C) \( v_{o+} = v_{o-} = 0 \) V, current through 6 V source
\[ i = \frac{v}{3R} = 2 \text{ mA}, \quad v_o = -2m(3k + 2k) = -10 \text{ V} \]

27. (D) \( v_+ = v_{(1)} = \frac{v_+}{1 + 3} = \frac{v_+}{2} + \frac{v_{(2)}}{2 + 1} \)
\[ v_+ = v_- = \frac{v_+ + 2v}{3} = \frac{v_+}{2} = -8 \]

28. (B) Since op-amp is ideal

29. (C) Closed loop gain \( A = \left| \frac{R_F}{R_1} \right| = \frac{240k}{10k} = 24 \)

The maximum output voltage \( v_{om} = 24 \times 0.02 = 0.48 \) V
\[ \omega \leq \frac{SR}{v_{om}} = \frac{0.5}{0.48} = 1.1 \times 10^6 \text{ rad/s} \]

30. (A) The offset due to \( V_{io} \) is \( v_o = \left( 1 + \frac{R_1}{R_3} \right) V_{io} \)
\[ v_o = \left( 1 + \frac{500}{5} \right) 4m = 404 \text{ mV} \]
Due to \( I_{io} \), \( v_o = R_F I_{io} = (500k)(150n) = 75 \text{ mV} \)
Total offset voltage \( v_o = 404 + 75 = 479 \text{ mV} \)

31. (A) \( 6 = -\frac{v_o}{6k}, \quad i_o = -6 + \frac{v_o}{3k} \)
\[ i_o = -6 + \frac{-6(6k)}{3k} = -18 \text{ A} \]

32. (B) If \( v_i > 0 \), then \( v_o < 0 \), \( D_1 \) blocks and \( D_2 \) conducts
\[ A_v = -\frac{6k}{3k} = -2 \quad \Rightarrow \quad v_o = -2(2) = -4 \text{ V} \]

33. (D) If \( v_i < 0 \), then \( v_o > 0 \), \( D_2 \) blocks and \( D_1 \) conduct
\[ A_v = -\frac{3k}{2k} = -1.5, \quad v_o = (2)(-1.5) = 3 \text{ V} \]

34. (A) Voltage follower \( v_o = v_+ = v_\)
\[ v_o (0^-) = 5m(250 \parallel 1000) = 1 \text{ V}, \quad v_o (\infty) = 0 \]
\[ \tau = 8m(1000 + 250) = 10 \text{ s} \]

35. (A) \( v_o (0^-) = 5 \text{ V} = v_o (0^-) = 5 \text{ V} \)
For \( t > 0 \) the equivalent circuit is shown in fig. S3.5.35

\[ \tau = 20k \times 4\mu = 0.08 \text{ s} \]
\[ v_o = 10 + (5 - 10)e^{-\frac{t}{60\times 10^3}} = 10 - 5e^{-12.5x} \text{ V for } t > 0 \]

36. (C) \( v_o = \frac{(10)(10k)}{10k + 10k} = 5 \text{ V} \)
When \( v_o > 5 \text{ V} \), output will be positive and LED will be on. Hence (C) is correct.

37. (B) \( v_o = \frac{(2)}{2R} = 1 \text{ V}, \quad v_o = \frac{(2)}{2R} = 1 \text{ V}, \quad v_o = 0 \)
\[ V_{CM} = \frac{v_+ + v_-}{2} = 1, \quad v_o = \frac{R_F}{1} V_{CM} \]
\[ CMRR = 60 \text{ dB} = 10^3, \quad v_o = \frac{100}{1} \cdot \frac{1}{10^3} = 100 \text{ mV} \]

38. (C) \( v_o = 0 = v_o \),
Let output of analog multiplier be \( v_o \).
\[ \frac{v_o}{R} = \frac{-v_p}{R} \quad \Rightarrow \quad v_o = -v_p, \quad v_p = v_o v_o \]
\[ v_o = -v_o v_o, \quad v_o = \frac{-v_o}{v_o} \]

39. (B) When \( v_i > 2 \text{ V} \), output is positive. When \( v_i < 2 \text{ V} \), output is negative.
40. (A) \( \frac{v_i - v_o}{R_1} = \frac{v_i - v_o}{R_1} \Rightarrow 2v_i = v_o + v_o \)

Thus when \( R_1 \) and \( R \) changed, the transfer function is unchanged.

41. (D) This is a all pass circuit

42. (B) Let \( R_1 = 3 \) k\( \Omega \), \( R_2 = 6 \) k\( \Omega \), \( C = 50 \) nF

43. (B) The oscillation frequency is

44. (A) This is Wien-bridge oscillator. The ratio \( \frac{R_2}{R_1} = 21 \) k\( \Omega \) is greater than 2. So there will be oscillation

**Fig. S3.5.44**

Frequency = \( \frac{1}{2\pi RC} \) \( \Rightarrow \) \( 1 \times 10^3 = \frac{1}{2\pi(1k)C} \)

\( C = \frac{1}{2\pi} \mu\text{F} \)

45. (C) \( v_c = 5 \) V = \( v_c = v_E \),

The input current to the op-amp is zero.

\( i_{in} = i_2 = i_1 = i_z + \alpha i_E \)

\( = \frac{15 - 5}{47k} \times \frac{60}{61/100} = 49.4 \text{ mA} \)

46. (B) \( v_o = \frac{333}{20} (v_{in} - v_{s2}) \)

47. (B) \( v_i = v_o = \frac{10v_o}{10 + 30} = \frac{v_o}{4} \)

48. \( v_o = 4v_z = 62 \times 4 = 24.8 \text{ V} \)
1. The 1001102 is numerically equivalent to
   1. 26_{16}  
   2. 36_{10}  
   3. 46_{8}  
   4. 21_{24}

   The correct answer are
   (A) 1, 2, and 3  
   (B) 2, 3, and 4  
   (C) 1, 2, and 4  
   (D) 1, 3, and 4

2. If \((211)_x = (152)_x\), then the value of base \(x\) is
   (A) 6  
   (B) 5  
   (C) 7  
   (D) 9

3. 11001, 1001 and 111001 correspond to the 2’s complement representation of the following set of numbers
   (A) 25, 9 and 57 respectively  
   (B) -6, -6 and -6 respectively  
   (C) -7, -7 and -7 respectively  
   (D) -25, -9 and -57 respectively

4. A signed integer has been stored in a byte using 2’s complement format. We wish to store the same integer in 16-bit word. We should copy the original byte to the less significant byte of the word and fill the more significant byte with
   (A) 0  
   (B) 1  
   (C) equal to the MSB of the original byte  
   (D) complement of the MSB of the original byte.

5. A computer has the following negative numbers stored in binary form as shown. The wrongly stored number is
   (A) -37 as 1101 1011  
   (B) -89 as 1010 0111  
   (C) -48 as 1110 1000  
   (D) -32 as 1110 0000

6. Consider the signed binary number \(A = 01010110\) and \(B = 11101100\) where \(B\) is the 1’s complement and MSB is the sign bit. In list-I operation is given, and in list-II resultant binary number is given.

<table>
<thead>
<tr>
<th>List–I</th>
<th>List–II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. (A + B)</td>
<td>1.01 00 0011</td>
</tr>
<tr>
<td>Q. (B - A)</td>
<td>2.01 10 1001</td>
</tr>
<tr>
<td>R. (A - B)</td>
<td>3.01 00 0010</td>
</tr>
<tr>
<td>S. (-A - B)</td>
<td>4.1001 0101</td>
</tr>
<tr>
<td>5.1011 1100</td>
<td></td>
</tr>
<tr>
<td>6.1001 0110</td>
<td></td>
</tr>
<tr>
<td>7.1011 1101</td>
<td></td>
</tr>
<tr>
<td>8.0110 1010</td>
<td></td>
</tr>
</tbody>
</table>

   The correct match is
   (A) 3 4 2 5
   (B) 3 6 8 7
   (C) 1 4 8 7
   (D) 1 6 2 5
28. The simplified form of a logic function \( Y = A(B + C(AB + A\overline{C})) \) is
(A) \( \overline{A} \overline{B} \)  \hspace{1cm} (B) \( AB \)
(C) \( \overline{AB} \)  \hspace{1cm} (D) \( A\overline{B} \)

29. The reduced form of the Boolean expression of \( Y = (AB) \cdot (\overline{AB}) \) is
(A) \( A + B \)  \hspace{1cm} (B) \( \overline{A} + \overline{B} \)
(C) \( AB + \overline{AB} \)  \hspace{1cm} (D) \( A\overline{B} + AB \)

30. If \( X\overline{Y} + \overline{X}Y = Z \) then \( X\overline{Z} + \overline{X}Z \) is equal to
(A) \( \overline{Y} \)  \hspace{1cm} (B) \( Y \)
(C) \( 0 \)  \hspace{1cm} (D) \( 1 \)

31. If \( XY = 0 \) then \( X \oplus Y \) is equal to
(A) \( X + Y \)  \hspace{1cm} (B) \( \overline{X} + \overline{Y} \)
(C) \( XY \)  \hspace{1cm} (D) \( \overline{X}\overline{Y} \)

32. From a four-input OR gate the number of input condition, that will produce HIGH output are
(A) 1  \hspace{1cm} (B) 3
(C) 15  \hspace{1cm} (D) 0

33. A logic circuit control the passage of a signal according to the following requirements:

1. Output \( X \) will equal \( A \) when control input \( B \) and \( C \) are the same.

2. \( X \) will remain HIGH when \( B \) and \( C \) are different.

The logic circuit would be

![Logic Circuit Diagram](A) \hspace{2cm} (B) \hspace{2cm} (C) \hspace{2cm} (D)

34. The output of logic circuit is HIGH whenever \( A \) and \( B \) are both HIGH as long as \( C \) and \( D \) are either both LOW or both HIGH. The logic circuit is
(A) \hspace{2cm} (B) \hspace{2cm} (C) \hspace{2cm} (D)

35. In fig. P4.1.35 the input condition, needed to produce \( X = 1 \), is

![Logic Circuit Diagram](A) \hspace{2cm} (B) \hspace{2cm} (C) \hspace{2cm} (D)

(A) \( A = 1, \ B = 1, \ C = 0 \)  \hspace{1cm} (B) \( A = 1, \ B = 1, \ C = 1 \)
(C) \( A = 0, \ B = 1, \ C = 1 \)  \hspace{1cm} (D) \( A = 1, \ B = 0, \ C = 0 \)

36. Consider the statements below:
1. If the output waveform from an OR gate is the same as the waveform at one of its inputs, the other input is being held permanently LOW.
2. If the output waveform from an OR gate is always HIGH, one of its inputs is being held permanently HIGH.

The statement, which is always true, is
(A) Both 1 and 2  \hspace{1cm} (B) Only 1
(C) Only 2  \hspace{1cm} (D) None of the above

37. To implement \( y = ABCD \) using only two-input NAND gates, minimum number of requirement of gate is
(A) 3  \hspace{1cm} (B) 4
(C) 5  \hspace{1cm} (D) 6

38. If the \( X \) and \( Y \) logic inputs are available and their complements \( \overline{X} \) and \( \overline{Y} \) are not available, the minimum number of two-input NAND required to implement \( X \oplus Y \) is
(A) 4  \hspace{1cm} (B) 5
(C) 6  \hspace{1cm} (D) 7
Statement for Q.39–40:

A Boolean function \( Z = ABC \) is to be implement using NAND and NOR gate. Each gate has unit cost. Only \( A, B \) and \( C \) are available.

39. If both gate are available then minimum cost is
(A) 2 units  
(B) 3 units  
(C) 4 units  
(D) 6 units

40. If NAND gate are available then minimum cost is
(A) 2 units  
(B) 3 units  
(C) 4 units  
(D) 6 units

41. In fig. P.4.1.41 the LED emits light when

(A) both switch are closed  
(B) both switch are open  
(C) only one switch is closed  
(D) LED does not emit light irrespective of the switch positions

42. If the input to the digital circuit shown in fig. P.4.1.42 consisting of a cascade of 20 XOR gates is \( X \), then the output \( Y \) is equal to

(A) \( X \)  
(B) \( \overline{X} \)  
(C) 0  
(D) 1

43. A Boolean function of two variables \( x \) and \( y \) is defined as follows :
\[
f(0, 0) = f(0, 1) = f(1, 1) = 1; f (1, 0) = 0
\]
SOLUTIONS

1. (D) $100110_2 = 2^5 + 2^2 + 2^1 = 38_{10}$

$26_{10} = 2 \times 16 + 6 = 38_{10}$

$46_{10} = 4 \times 8 + 6 = 38_{10}$

$212_{10} = 2 \times 4^2 + 4^1 = 38_{10}$

So $36_{10}$ is not equivalent.

2. (C) $2x^2 + x + 1 = 64 + 5 \times 8 + 2 \Rightarrow x = 7$

3. (C) All are 2's complement of 7

$11001 \Rightarrow 00110$

$+ 1$

$00111 = 7_{10}$

$1001 \Rightarrow 0110$

$+ 1$

$0111 = 7_{10}$

$111001 \Rightarrow 000110$

$+ 1$

$000111 = 7_{10}$

4. (C) See a example

42 in a byte $00101010$

42 in a word $00000000 01010100$

$-42$ in a byte $11010110$

$-42$ in a word $11111111 11010110$

Therefore (C) is correct.

5. (C) $48_{10} = 00110000_2$

$-48_{10} = 11001111$

$+ 1$

$11010000$

6. (D) Here $\overline{A}, \overline{B}$ are 1's complement

$A + B,
\begin{align*}
\overline{A} &= 01010110 \\
\overline{B} &= 11101100
\end{align*}$

$A + 11101100$

$01100010$

$B - A = B + \overline{A},
\begin{align*}
B &= 11101100 \\
\overline{A} &= 01010110
\end{align*}$

$B + 01010110$

$110010101$

$+ 1$

$10010110$

$A - B = A + \overline{B},
\begin{align*}
A &= 01010110 \\
\overline{B} &= 00010011
\end{align*}$

$+ 00010011$

$01101000$

$-A - B = \overline{A} + \overline{B},
\begin{align*}
\overline{A} &= 10101000 \\
\overline{B} &= 00010011
\end{align*}$

$+ 00010011$

$10111000$

7. (B) Here $\overline{A}, \overline{B}$ are 2's complement

$A + B,
\begin{align*}
A &= 01000110 \\
B &= 11101000
\end{align*}$

$+ 10001100$

$10101100$

Discard the carry 1

$A - B = A + \overline{B},
\begin{align*}
A &= 01000110 \\
\overline{B} &= 11010101
\end{align*}$

$+ 01110001$

$B - A, B
\begin{align*}
B &= 11010001 \\
A &= 10111010
\end{align*}$

$+ 10111010$

$10110100$

Discard the carry 1

$-A - B = \overline{A} + \overline{B},
\begin{align*}
\overline{A} &= 10111010 \\
\overline{B} &= 11010011
\end{align*}$

$+ 11010011$

$11101110$

8. (B) $11_{10} = 1011_2$

$0.3 \quad 2F_{-1} \quad B_i \quad F_i$

$\begin{array}{cccc}
0.6 & 0 & 0.6 \\
1.2 & 1 & 0.2 \\
0.4 & 0 & 0.4 \\
0.8 & 0 & 0.8 \\
1.6 & 1 & 0.6 \\
\end{array}$

Repeat from the second line $0.3_{10} = 0.01001_2$

9. (C)

$\begin{align*}
\begin{array}{ccccccc}
b_4 & b_3 & b_2 & p_3 & b_1 & p_2 & p_3 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\end{align*}$

$C_i^* = b_4 \oplus b_3 \oplus b_1 \oplus p_3 = 0$

$C_2^* = b_4 \oplus b_3 \oplus b_1 \oplus p_2 = 1$

$C_3^* = b_4 \oplus b_3 \oplus b_2 \oplus p_3 = 1$
\[ C_3^2 C_2^2 C_1^2 = 110 \] which indicate position 6 in error.

Transmitted code 1001100.

10. (D) \[ X = MNQ + M \overline{N}Q + \overline{M}NQ \]
\[ = MQ + \overline{M}NQ = Q(M + \overline{M}N) = Q(M + N) \]

11. (A) The logic circuit can be modified as shown in fig. S. 4.1.11

![Fig. S4.1.11](image)

Now \( Z = AB + (C + D)E \)

12. (D) You can see that input to last XNOR gate is same. So output will be HIGH.

13. (D) \[ Z = \overline{A} + (\overline{AB} + \overline{BC}) + \overline{C} \]
\[ = \overline{A} + (\overline{A} + \overline{B} + \overline{B} + \overline{C}) + \overline{C} = \overline{A} + \overline{B} + \overline{C} \]
\[ ABC = \overline{A} + \overline{B} + \overline{C} \]
\[ AB + \overline{BC} + \overline{AC} = \overline{A} + \overline{B} + \overline{C} + \overline{A} + \overline{B} + \overline{C} = \overline{A} + \overline{B} + \overline{C} \]

14. (C) \( (X + \overline{Y})(X + Y) = XY + \overline{X} \overline{Y} \)
\( (X + Y)(X + \overline{Y})(X + Y) = (X + Y)(XY + \overline{X} \overline{Y}) \)
\[ = XY + XY = XY \]

15. (B) Using duality
\[ (A + B)(\overline{A} + C)(B + C) = (A + B)(\overline{A} + C) \]
Thus (B) is correct option.

16. (B) \[ Z = (\overline{AB})(CD)(EF) = AB + CD + EF \]

17. (A) \( X = (\overline{A} \overline{B} + \overline{AB})(A + \overline{B}) = (\overline{AB} + \overline{A} \overline{B})(\overline{AB}) = \overline{AB} \)

18. (B) \( Y = (A \oplus B) \cdot \overline{C} = (\overline{AB} + AC) \cdot \overline{C} \)
\[ = (\overline{AB} + \overline{AB}) + \overline{C} = \overline{A} \overline{B} + AB + \overline{C} \]

19. (C) \( Z = A(A + \overline{A})BC = ABC \)

20. (A) \( Z = AB(\overline{B} + C) = ABC \)

21. (A) \( Z = (\overline{A} + \overline{B}) \cdot BC = (AB) \cdot BC = ABC \)

22. (A) \( A(A + \overline{B})(A + \overline{B} + C) \)

Therefore No gate is required to implement this function.

23. (A)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>(A + BC)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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</tbody>
</table>

Fig. S 4.1.23

24. (B) \( X = \overline{A}BC + AB\overline{C} + A\overline{B} \overline{C} = \overline{B}C + AB\overline{C} \)

25. (B) \( (A + B)(B + \overline{C}) = (AB)(\overline{BC}) = \overline{ABC} \)
\( (A + B)(B + \overline{C}) = (A + B) + (B + \overline{C}) = A + B + \overline{C} \)
\( (\overline{A} + \overline{B})(\overline{B} + \overline{C}) = (\overline{A} + \overline{B}) + (B + C) = \overline{A} + B + C \)
\( = A\overline{B} + B + C = A + B + C \)
Thus (B) is correct.

26. (D) \( AC + B\overline{C} = AC(B + \overline{B}) + (A + \overline{A})B\overline{C} \)
\[ = ABC + \overline{ABC} + ACB + \overline{ACB} \]

27. (D) \( F = A + \overline{AB} + \overline{A} \overline{B} \overline{C} \overline{D} \overline{E} = A + \overline{AB} + \overline{A} \overline{B} \overline{C} \overline{D} \overline{E} \)
\[ = A + \overline{AB} + \overline{A} \overline{B} \overline{C} \overline{D} \overline{E} \]
\[ = A + A(B + \overline{B})(C + \overline{D} + \overline{E}) = A + B + C + D + E \]

28. (B) \( A(B + C(\overline{AB} + \overline{AC})) = AB + AC(\overline{AB} \cdot \overline{AC}) \)
\[ = AB + AC(\overline{A} + \overline{B})(\overline{A} + \overline{C}) \]
\[ = AB + AC(\overline{A} + \overline{C} + \overline{A} \overline{C} + B \overline{C}) = AB \]

29. (C) \( (\overline{AB}) \cdot (\overline{AC}) = \overline{AB} + \overline{AC} = \overline{A}B + \overline{A}C \)

30. (B) \( X\overline{Z} + \overline{XZ} = X(\overline{XY} + \overline{X} \overline{Y}) + \overline{X}(X \overline{Y} + \overline{X} \overline{Y}) \)
\[ = X(\overline{XY} + \overline{X} \overline{Y}) + \overline{X} \overline{Y} = XY + \overline{X} \overline{Y} \]

31. (A) \( X \oplus Y = XY + \overline{X} \overline{Y} = (XY + \overline{XY}) = (\overline{XY}) = X + Y \)
A function with don’t care condition is as follows:
\[ f(a, b, c, d) = \Sigma m(0, 2, 3, 5, 7, 8, 9, 10, 11) + \Sigma d(4, 15) \]

The minimized expression for this function is:
(A) \( \overline{a}b + \overline{b}d + cd + \overline{a}b\overline{d} \)
(B) \( \overline{a}b + b\overline{d} + cd + a\overline{b}d \)
(C) \( a\overline{b} + \overline{b}d + \overline{b}c + \overline{a}bd \)
(D) Above all

A function with don’t cares is as follows:
\[ g(X, Y, Z) = \Sigma m(5, 6) + \Sigma d(1, 2, 4) \]

For above function consider following expression:
1. \( XYZ + X\overline{Y}Z \)
2. \( X\overline{Y} + X\overline{Z} \)
3. \( X\overline{Z} + \overline{X}Z + Y\overline{Z} \)
4. \( Y\overline{Z} + \overline{Y}Z \)

The solution for \( g \) are
(A) 1, 2, and 3
(B) 1, 2, and 4
(C) 1, and 4
(D) 1, and 3

A logical function of four variable is given as
\[ f(A, B, C, D) = (\overline{A} + B)(C(B + CD)) \]

The function as a sum of product is
(A) \( \overline{A} + B + AC\overline{D} + BCD \)
(B) \( \overline{A} + BC + \overline{A}CD + BCD \)
(C) \( \overline{A}B + BC + \overline{A}CD + BCD \)
(D) \( AB + \overline{A}B + \overline{A}CD + BCD \)

A combinational circuit has input A, B, and C and its K-map is as shown in fig. P4.2.10. The output of the circuit is given by

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Fig. P4.2.1

(A) \( (\overline{A}B + AB)\overline{C} \)
(B) \( (AB + \overline{A}B)\overline{C} \)
(C) \( \overline{A}BC \)
(D) \( A \oplus B \oplus C \)

The Boolean Expression \( Y = (A + B)(\overline{A} + C) \) is equal to

(A) \( AC + \overline{A}B \)
(B) \( AC + \overline{A}B + BC \)
(C) \( \overline{A}B + BC + \overline{A}BC \)
(D) Above all

In the logic circuit of fig. P4.2.12 the redundant gate is

(A) 1
(B) 2
(C) 3
(D) 4

If function \( W, X, Y, \) and \( Z \) are as follow

\[ W = R + \overline{P}Q + \overline{R}S \]
\[ X = PQ\overline{R} + \overline{P}\overline{Q}R + P\overline{Q} + RS \]
\[ Y = RS + PR + \overline{P}Q + \overline{P}\overline{Q} \]
\[ Z = R + S + PQ + \overline{P}\overline{Q}R + P\overline{Q}S \]

Then
(A) \( W = Z, \ X = \overline{Z} \)
(B) \( W = Z, \ X = Y \)
(C) \( W = Y \)
(D) \( W = Y = \overline{Z} \)

In a certain application four inputs \( A, B, C, D \) are fed to logic circuit, producing an output which operates a relay. The relay turns on when \( f(A, B, C, D) = 1 \) for the following states of the inputs \( (ABCD) : 0000, 0010, 0101, 0110, 1101 and 1110. \) States 1000 and 1001 do not occur, and for the remaining states the relay is off. The minimized Boolean expression \( f \) is
(A) \( \overline{A}C\overline{D} + BC\overline{D} + B\overline{C}D \)
(B) \( B\overline{C}D + BC\overline{D} + \overline{A}CD \)
(C) \( ABD + B\overline{C}D + \overline{B}CD \)
(D) \( \overline{A}BD + B\overline{C}D + BC\overline{D} \)

There are four Boolean variables \( x_1, x_2, x_3, \) and \( x_4. \) The following function are defined on sets of them
\[ f(x_1, x_2, x_3) = \Sigma m(3, 4, 5) \]
\[ g(x_4, x_3, x_2) = \Sigma m(1, 6, 7) \]
\[ h(x_4, x_3, x_2, x_1) = f \oplus g \]

Then \( h(x_4, x_3, x_2, x_1) \) is
(A) \( \Sigma m(3, 12, 13) \)
(B) \( \Sigma m(3, 6) \)
(C) \( \Sigma m(3, 12) \)
(D) \( 0 \)
Statement for Q.16–17:

A switching function of four variable, \( f(w, x, y, z) \) is to equal the product of two other function \( f_1 \) and \( f_2 \) of the same variable \( f = f_1 f_2 \). The function \( f \) and \( f_1 \) are as follows:

\[
\begin{align*}
f &= \Sigma m(4, 7, 15) \\
f_1 &= \Sigma m(0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 15)
\end{align*}
\]

16. The number of fully specified function, that will satisfy the given condition, is
   (A) 32 (B) 16 (C) 4 (D) 1

17. The simplest function for \( f_2 \) is
   (A) \( x \) (B) \( \bar{x} \) (C) \( y \) (D) \( \bar{y} \)

18. A four-variable switching function has minterms \( m_6 \) and \( m_9 \). If the literals in these minterms are complemented, the corresponding minterm numbers are
   (A) \( m_3 \) and \( m_6 \) (B) \( m_9 \) and \( m_6 \) (C) \( m_2 \) and \( m_6 \) (D) \( m_6 \) and \( m_9 \)

19. The minimum function that can detect a “divisible by 3” 8421 BCD code digit (representation \( D_0 D_1 D_2 D_3 \)) is given by
   (A) \( D_0 D_1 + D_1 D_2 + D_2 D_3 \)
   (B) \( D_0 D_1 + D_1 D_2 + D_2 D_1 + D_3 D_2 \)
   (C) \( D_0 D_1 + D_1 D_2 + D_2 D_1 \)
   (D) \( D_0 D_1 + D_1 D_2 + D_2 D_1 + D_3 D_2 D_1 \)

20. \( f(x_2, x_1, x_0) = ? \)
   (A) \( \Pi(1, 2, 4, 5, 7) \) (B) \( \Sigma(1, 2, 4, 5, 7) \)
   (C) \( \Sigma(0, 3, 6) \) (D) None of Above

21. For a binary half subtractor having two input \( A \) and \( B \), the correct set of logical expressions for the outputs \( D = (A - B) \) and \( X \) (borrow) are
   (A) \( D = AB + \overline{AB}, \ X = \overline{AB} \)
   (B) \( D = \overline{AB} + \overline{AB}, \ X = AB \)
   (C) \( D = \overline{AB} + \overline{AB}, \ X = AB \)
   (D) \( D = AB + \overline{AB}, \ X = \overline{AB} \)

22. \( f_1 f_2 = ? \)
   (A) \( x_0 x_1 x_2 \) (B) \( x_0 \oplus x_1 \oplus x_2 \)
   (C) 1 (D) 0

23. The logic circuit shown in fig. P4.2.23 implements
   (A) \( D(A \cup C + \overline{AC}) \) (B) \( D(B \oplus C + \overline{AC}) \)
   (C) \( D(B \oplus C + \overline{AB}) \) (D) \( D(B \cup C + \overline{AB}) \)

Statement for Q.24–25:

The building block shown in fig. P4.2.24–25 is an active high output decoder.
24. The output $X$ is
   (A) $AB + BC + CA$  (B) $A + B + C$
   (C) $ABC$  (D) None of the above

25. The output $Y$ is
   (A) $A + B$  (B) $B + C$
   (C) $C + A$  (D) None of the above

26. A logic circuit consist of two $2 \times 4$ decoder as shown in fig. P4.2.26.

The output of decoder are as follow

- $D_0 = 1$ when $A_0 = 0$, $A_1 = 0$
- $D_1 = 1$ when $A_0 = 1$, $A_1 = 0$
- $D_2 = 1$ when $A_0 = 0$, $A_1 = 1$
- $D_3 = 1$ when $A_0 = 1$, $A_1 = 1$

The value of $f(x, y, z)$ is
   (A) $0$  (B) $z$
   (C) $\bar{z}$  (D) $1$

**Statement for Q.27-29:**

A MUX network is shown in fig. P4.2.27-29.

27. $Z_1 = ?$
   (A) $a + b + c$  (B) $ab + ac + bc$
   (C) $a \cup b \cup c$  (D) $a \oplus b \oplus c$

28. $Z_2 = ?$
   (A) $ab + bc + ca$  (B) $a + b + c$
   (C) $abc$  (D) $a \cup b \cup c$

29. This circuit act as a
   (A) Full adder  (B) Half adder
   (C) Full subtractor  (D) Half subtractor

30. The network shown in fig. P4.2.30 implements

   (A) NOR gate  (B) NAND gate
   (C) XOR gate  (D) XNOR gate

31. The MUX shown in fig. P4.2.31 is $4 \times 1$ multiplexer. The output $Z$ is

   (A) $ABC$  (B) $A \oplus B \oplus C$
   (C) $A \cup B \cup C$  (D) $A + B + C$

32. The output of the $4 \times 1$ multiplexer shown in fig. P4.2.32 is

   (A) $X + Y$  (B) $\overline{X} + \overline{Y}$
   (C) $XY + X$  (D) $XY$
33. The MUX shown in fig. P4.2.33 is a 4 × 1 multiplexer. The output $Z$ is

\[ (A) \ A \oplus \ C \quad (B) \ A \cup C \quad (C) \ B \oplus C \quad (D) \ B \cup C \]

34. $f =$?

\[ (A) \ \overline{w}x\overline{y}z + \overline{w}x\overline{y} + xy + yz \]
\[ (B) \ wxyz + \overline{w}xyz + \overline{x}y + yz \]
\[ (C) \ w\overline{x}\overline{y}z + w\overline{x}\overline{y} + xy + zx \]
\[ (D) \ \overline{w}xyz + uwxyz + gy + z\overline{x} \]

35. For the logic circuit shown in fig. P4.2.35 the output $Y$ is

\[ (A) \ A \oplus B \quad (B) \ A \oplus \overline{B} \quad (C) \ A \oplus B \oplus C \quad (D) \ A \oplus B \oplus C \]

36. The 4-to-1 multiplexer shown in fig. P4.2.36 implements the Boolean expression

\[ f(w, x, y, z) = \Sigma(4, 5, 7, 8, 10, 12, 15) \]

The input to $I_1$ and $I_3$ will be

\[ (A) \ y\overline{z}, \ y + \overline{z} \quad (B) \ y + z, \ y \cup z \]
\[ (C) \ y + z, \ y \oplus z \quad (D) \ x + \overline{y}, \ y \oplus z \]

37. The 8-to-1 multiplexer shown in fig. P4.2.37 realizes the following Boolean expression

\[ (A) \ w\overline{x}z + \overline{w}\overline{x}\overline{z} + wyz + x\overline{y}\overline{z} \]
\[ (B) \ wxz + \overline{w}xyz + wy\overline{z} + \overline{w}x\overline{y} \]
\[ (C) \ w\overline{x}\overline{z} + w\overline{y}\overline{z} + w\overline{y}z + wz \]
\[ (D) \ \text{MUX is not enable} \]

Statement for Q.38-40:

A PLA realization is shown in fig. P4.2.38–40
38. \( f_1(x_2, x_1, x_0) = ? \)
(A) \( x_2 \overline{x}_0 + x_1 x_0 \)
(C) \( x_2 + x_0 \)
(B) \( x_2 x_0 + x_1 \overline{x}_2 \)
(D) \( x_2 \lor x_0 \)

39. \( f_2(x_2, x_1, x_0) = ? \)
(A) \( \sum(m(1, 2, 5, 6)) \)
(B) \( \sum(m(1, 2, 6, 7)) \)
(C) \( \sum(2, 3, 4) \)
(D) None of the above

40. \( f_3(x_2, x_1, x_0) = ? \)
(A) \( \Pi M(0, 4, 6, 7) \)
(B) \( \Pi M(2, 4, 5, 7) \)
(C) \( \Pi M(1, 2, 3, 5) \)
(D) \( \Pi M(2, 3, 4, 7) \)

41. If the input \( X_1X_2X_3X_0 \) to the ROM in fig. P4.2.41 are \( \text{8–4–2–1 BCD numbers} \), then output \( Y_3Y_2Y_1Y_0 \) are

(A) 2–4–2–1 BCD number
(B) gray code number
(C) excess 3 code converter
(D) none of the above

42. It is desired to generate the following three Boolean function
\[
\begin{align*}
f_1 &= a\overline{b}c + \overline{a}b\overline{c} + bc \\
f_2 &= a\overline{b}c + ab + \overline{a}b\overline{c} \\
f_3 &= \overline{a}\overline{b}\overline{c} + abc + \overline{a}c
\end{align*}
\]
by using an OR gate array as shown in fig. P4.2.42 where \( P_1 \) and \( P_5 \) are the product terms in one or more of the variable \( a, \overline{a}, b, \overline{b}, c \) and \( \overline{c} \).

43. The terms \( P_1, P_2, P_3, P_4 \) and \( P_5 \) are
(A) \( \overline{a}b, ac, b\overline{c}, bc, \overline{a}b \)
(B) \( a\overline{b}, b\overline{c}, ac, \overline{a}b, bc \)
(C) \( ac, \overline{a}b, b\overline{c}, \overline{a}b, bc \)
(D) Above all

44. The circuit shown in fig. P4.2.43 has 4 boxes each described by input \( P, Q, R \) and output \( Y, Z \) with \( Y = P \oplus Q \oplus R \) and \( Z = RQ + \overline{P}Q + Q\overline{P} \).

(A) adder giving \( P + Q \)
(B) subtractor giving \( P - Q \)
(C) subtractor giving \( Q - P \)
(D) adder giving \( P + Q + R \)

45. The circuit shown in fig. P4.2.44 converts

(A) BCD to binary code
(B) Binary to excess
(C) Excess–3 to Gray Code
(D) Gray to Binary code

**********
16. (A) \( f = \Sigma m(4, 7, 15) \),
\( f'_z = \Sigma m(0, 1, 2, 3, 4, 7, 8, 9, 10, 11, 15) \)
\( f'_2 = \Sigma m(4, 7, 15) + \Sigma dc(5, 6, 12, 13, 14) \)
There are 5 don't care condition. So \( 2^5 = 32 \) different functions \( f'_2 \).
17. (A) \( f_2 = \Sigma m(4, 7, 15) + \Sigma dc(5, 6, 12, 13, 14) \), \( f_2 = x \)

\[
Z = R + S + PQ + \overline{P}Q \overline{R} + P\overline{Q}S = R + S + P\overline{Q}P \overline{R} P\overline{Q} S \\
= R + S + (\overline{P} + Q)(P + Q + R)(\overline{P} + Q + S) \\
= R + S + (PQ + \overline{P}R + \overline{Q}P + QR)(\overline{P} + Q + S) \\
= R + S + PQ + PQR + P\overline{R} + Q\overline{R} + \overline{Q}RS
\]

We can see that \( W = Z \), \( X = \overline{Z} \)

14. (D) \( \overline{ABD} + \overline{BCD} + \overline{BCD} \)

18. (B) \( m_0 = \overline{ABC}D \), \( m_0 = A\overline{BC}D \)
After complementing literal \( m'_0 = \overline{ABC}D = m_0 \), \( m'_0 = A\overline{BC}D = m_0 \)

19. (B) 0, 3, 6 and 9 are divisible by 3

\[
D_2D_1D_0D_4 \\
00: 1 1 1 \\
01: \times \times \times \\
11: \times \times \times \\
10: 1 \times \times
\]

20. (B) \( f = \Sigma m(0, 3, 6) = \Sigma m(1, 2, 4, 5, 7) \)

21. (C) \( D = A\overline{B} + \overline{A}B \), \( X = \overline{A}B \)
22. (D) \( f_1 = \Sigma m(0, 2, 4, 6), \)
\( f_2 = \Sigma m(1, 3, 5, 7), \) \( f_1f_2 = 0 \)

23. (D) 
\[ Z = D(\overline{AB}C + \overline{ABC} + \overline{A}BC + ABC) \]
\[ = D(\overline{AB}C + C) + BC(A + A) + \overline{AB}C \]
\[ = D(\overline{AB} + BC + A\overline{AB}) = D(\overline{B}(A + \overline{A}) + BC) \]
\[ = D(\overline{A}B + \overline{B}C + BC) = D(B \cup C + AB) \]

24. (A) \( X = \Sigma m(3, 5, 6, 7), \) \( X = AB + BC + CA \)

25. (D) \( Y = \Sigma m(1, 3, 5, 7), \) \( Y = C \)

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<td>01</td>
<td>1</td>
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Fig. S4.2.24

26. (D) \( D_0 = \overline{A}_1\overline{A}_0, \) \( D_1 = \overline{A}_1A_0, \) \( D_2 = A_1\overline{A}_0, \)

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Fig. S4.2.25

\[ D_3 = A_1A_0 \]
For first decoder \( A_0 = x, \) \( A_1 = y, \) \( D_2 = y\overline{x}, \) \( D_3 = xy \)
For second decoder \( D_1 = D_2D_3 = y\overline{xy} = 0, \) \( A_0 = z \)
\( f = D_0 + D_1 = \overline{A}_1\overline{A}_0 + \overline{A}_1A_0 = \overline{A}_1 = 1 \)

27. (D) The output of first MUX is
\( Z_o = ab + \overline{a}\overline{b} = (a \oplus b) \)
This is input to select \( S_0 \) of both second-level MUX
\( Z_i = CS_0 + \overline{C}S_0 = C \oplus \overline{S}_0 = a \oplus b \oplus c \)

28. (A) \( Z_2 = bS_0 + c\overline{S}_0 \)
\( = b(ab + c\overline{b}) + c(ab + a\overline{b}) = ab + a\overline{bc} + \overline{abc} \)
\( = a(b + \overline{b}c) + \overline{abc} = ab + ac + \overline{abc} \)

29. (A) The equation of \( Z_i \) is the equation of sum of \( A \) and \( B \) with carry and equation of 2 is the resultant carry. Thus, it is a full adder.

30. (B) \( f_1 = \overline{C}D + CB = CB, \) \( S = F_1 \)
\( f = \overline{f}_1 + f_1\overline{A} = \overline{CB} + CB\overline{A} = \overline{CB} + \overline{A} \)
\( = \overline{C} + B + \overline{A} = A \overline{B} \)

31. (D) \( Z = \overline{ABC} + \overline{AB} + A\overline{B} + AB \)
\( = \overline{A}(B + C) + A(B + B) = A(B + C) + A = A + B + C \)

32. (A) \( Z = \overline{X}Y + X\overline{Y} + XY, \) \( Z = X + Y \)

33. (A) \( Z = \overline{ABC} + \overline{AB} + A\overline{B} + AB \)
\( = \overline{AC} + A\overline{C} = A \oplus C \)

34. (A) The output from the upper first level multiplexer is \( f_a \) and from the lower first level multiplexer is \( f_b \)
\( f_a = \overline{w}x + w\overline{x}, \) \( f_b = \overline{w}x + wx = x \)
\( f = f_0\overline{y} + f_1y\overline{z} + yz = (\overline{w}x + w\overline{x})\overline{yz} + xy\overline{z} + yz \)
\( = \overline{w}x\overline{yz} + w\overline{x}\overline{yz} + xy + yz \)

35. (D) Output is 1 when even parity

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Fig. S4.2.35
Therefore \( Y = A \oplus B \oplus C \)

36. (B) \( I_1 = \overline{y} + z, \) \( I_2 = y + z \)

<table>
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<th>00</th>
<th>01</th>
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<th>10</th>
</tr>
</thead>
</table>
| \( S_0 \) 00 | 0 | 0 | 0 | 0 | \( I_0 = 0 \)
| 01 | 1 | 1 | 1 | 0 | \( I_1 = \overline{y} + z \)
| \( w \) 11 | 1 | 0 | 1 | 0 | \( I_2 = yz + \overline{y}z = y \cup z \)
| 10 | 1 | 0 | 0 | 1 | \( I_3 = \overline{z} \)

Fig. S 4.2.36
37. (C) Let \( z = 0 \), Then
\[
f = \overline{w} \overline{x} y + \overline{w} x y + w \overline{x} \overline{y} + w x \overline{y} = \overline{w} \overline{x} + w \overline{y}
\]
If we put \( z = 0 \) in given option then
(A) \( \overline{w} \overline{x} + x \overline{y} \)  
(B) \( w y + \overline{w} \overline{x} \overline{y} \)  
(C) \( \overline{w} \overline{x} + w \overline{y} \)
Since MUX is enable so option (C) is correct.

38. (C) \( f = x_o x_2 + x_0 \overline{x}_1 x_2 + \overline{x}_0 x_2 = x_o x_2 (1 + \overline{x}_1) + \overline{x}_0 x_2 \)
\( f = x_o x_2 + \overline{x}_0 x_2 \)
39. (B) \( f_2 = \overline{x}_0 x_1 + x_1 x_2 + x_0 \overline{x}_1 \overline{x}_2 \)
\( = \overline{x}_0 x_1 + \overline{x}_0 x_2 + x_1 x_2 + x_0 x_1 x_2 + x_0 \overline{x}_1 \overline{x}_2 \)
\( = x_2 x_1 \overline{x}_0 + \overline{x}_2 x_1 x_0 + \overline{x}_2 x_1 x_0 + x_2 x_1 x_0 \)
\( f_2(x_2, x_1, x_0) = \Sigma m(1, 2, 6, 7) \)
40. (C) \( f_3 = \overline{x}_0 x_1 + x_1 x_2 \)
\( = x_2 x_1 \overline{x}_0 + \overline{x}_2 x_1 x_0 + x_2 x_1 x_0 + x_2 x_1 \overline{x}_0 \)
\( f_3(x_2, x_1, x_0) = \Sigma m(0, 4, 6, 7) \)
\( f_3(x_2, x_1, x_0) = \Sigma m(0, 1, 2, 3) \)
41. (A)
Let \( X_3 X_2 X_1 X_0 \) be 1001 then \( Y_3 Y_2 Y_1 Y_0 \) will be 1111.
Let \( X_3 X_2 X_1 X_0 \) be 1000 then \( Y_3 Y_2 Y_1 Y_0 \) will be 1110
Let \( X_3 X_2 X_1 X_0 \) be 0110 then \( Y_3 Y_2 Y_1 Y_0 \) will be 1100

<table>
<thead>
<tr>
<th>( P_n )</th>
<th>( Q_n )</th>
<th>( R_n )</th>
<th>( Z_n )</th>
<th>( Y_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 1 )</td>
<td>( n = 2 )</td>
<td>( n = 3 )</td>
<td>( n = 4 )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

42. (A) \( f_1 = a \overline{b} c + \overline{a} b \overline{c} + b c = a c + a b \)
\( f_2 = a \overline{b} c + a b + a b \overline{c} = a b + b \overline{c} \)
\( f_3 = a b \overline{c} + a b c + a c = a b \overline{c} + b c \)
Thus \( P_1 = a \overline{b} \), \( P_2 = a c \), \( P_3 = b \overline{c} \), \( P_4 = b c \), \( P_5 = a b \)
43. (B) Let \( P = 1001 \) and \( Q = 1010 \) then
\( Y_n = P_n \oplus Q_n \oplus R_n \), \( Z_n = R_n Q_n + \overline{P_n} R_n + Q_n \overline{R_n} \)
output is 1111 which is 2’s complement of –1. So it gives \( P - Q \). Let another example \( P = 1101 \) and \( Q = 0110 \) then output is 00111. It gives \( P - Q \).
So (B) is correct.

44. (D) Let input be 1010
output will be 1101
Let input be 0110
output will be 0100
This convert gray to Binary code.

So this converts 2–4–2–1 BCD numbers.
11. The circuit shown in fig. P4.3.11 is

(A) a MOD–2 counter
(B) a MOD–3 counter
(C) generate sequence 00, 10, 10, 00.....
(D) generate sequence 00, 10, 00, 10, 00.....

12. The counter shown in fig. P4.3.12 is a

(A) MOD–8 up counter
(B) MOD–8 down counter
(C) MOD–6 up counter
(D) MOD–6 down counter

13. The counter shown in fig. P4.3.13 counts from

(A) 000 to 111
(B) 111 to 000
(C) 100 to 000
(D) 000 to 100

14. The mod-number of the asynchronous counter shown in fig. P4.2.13 is

(A) 24 (B) 48
(C) 25 (D) 36

15. The frequency of the pulse at z in the network shown in fig. P4.3.15 is

(A) 10 Hz (B) 160 Hz
(C) 40 Hz (D) 5 Hz

16. The three-stage Johnson counter as shown in fig. P4.2.16 is clocked at a constant frequency of \( f_c \) from the starting state of \( Q_2Q_1Q_0 = 101 \). The frequency of output \( Q_2Q_1Q_0 \) will be

(A) \( J_2 \)
(B) \( K_2 \)
(C) \( J_1 \)
(D) \( K_1 \)
17. The counter shown in the fig. P4.3.17 has initially $Q_0Q_1Q_2 = 000$. The status of $Q_2Q_1Q_0$ after the first pulse is

(A) 0 0 1  
(B) 0 1 0  
(C) 1 0 0  
(D) 1 0 1

18. A 4 bit ripple counter and a 4 bit synchronous counter are made by flip flops having a propagation delay of 10 ns each. If the worst case delay in the ripple counter and the synchronous counter be $R$ and $S$ respectively, then

(A) $R = 10 \text{ ns}$, $S = 40 \text{ ns}$  
(B) $R = 40 \text{ ns}$, $S = 10 \text{ ns}$  
(C) $R = 10 \text{ ns}$, $S = 30 \text{ ns}$  
(D) $R = 30 \text{ ns}$, $S = 10 \text{ ns}$

19. A 4 bit modulo-6 ripple counter uses $JK$ flip-flop. If the propagation delay of each FF is 50 ns, the maximum clock frequency that can be used is equal to

(A) 5 MHz  
(B) 10 MHz  
(C) 4 MHz  
(D) 20 MHz

20. The initial contents of the 4-bit serial-in-parallel-out right-shift, register shown in fig. P4.3.20 is 0 1 1 0. After three clock pulses are applied, the contents of the shift register will be

(A) 0 0 0 0  
(B) 0 1 0 1  
(C) 1 1 1 1  
(D) 1 0 1 0

21. In the circuit shown in fig. P4.3.21 is PIPO 4-bit register, which loads at the rising edge of the clock. The input lines are connected to a 4 bit bus. Its output acts as the input to a $16 \times 4$ ROM whose output is floating when the enable input $E$ is 0. A partial table of the contents of the ROM is as follows

<table>
<thead>
<tr>
<th>Address</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0011</td>
<td>1111</td>
<td>0100</td>
<td>1010</td>
<td>1011</td>
<td>1000</td>
<td>0010</td>
</tr>
</tbody>
</table>

The clock to the register is shown below, and the data on the bus at time $t_1$ is 0110.

22. A 4-bit right shift register is initialized to value 1000 for $(Q_3, Q_2, Q_1, Q_0)$. The $D$ input is derived from $Q_3$, $Q_2$ and $Q_1$ through two XOR gates as shown in fig. P4.2.22. The pattern 1000 will appear at

(A) 1 1 1 1  
(B) 1 0 1 1  
(C) 1 0 0 0  
(D) 0 0 1 0
Statement for Q.23–24:
The 8-bit left shift register and D-flip-flop shown in fig. P4.3.22–23 is synchronized with the same clock. The
D-flip-flop is initially cleared.

23. The circuit acts as
(A) Binary to 2's complement converter
(B) Binary to Gray code converter
(C) Binary to 1's complement converter
(D) Binary to Excess–3 code converter

24. If initially register contains byte B7, then after 4 clock pulse contents of register will be
(A) 73
(B) 72
(C) 7E
(D) 74

Statement for Q.25–26:
A Mealy system produces a 1 output if the input has been 0 for at least two consecutive clocks followed immediately by two or more consecutive 1's.

25. The minimum state for this system is
(A) 4
(B) 5
(C) 8
(D) 9

26. The flip-flop required to implement this system are
(A) 2
(B) 3
(C) 4
(D) 5

27. The output of a Mealy system is 1 if there has been a pattern of 11000, otherwise 0. The minimum state for this system is
(A) 4
(B) 5
(C) 6
(D) 7

28. To count from 0 to 1024 the number of required flip-flop is
(A) 10
(B) 11
(C) 12
(D) 13

29. Four memory chips of 16 × 4 size have their address buses connected together. This system will be of size
(A) 64 × 4
(B) 32 × 8
(C) 16 × 16
(D) 256 × 1

30. The address bus width of a memory of size 1024 × 8 bits is
(A) 10 bits
(B) 13 bits
(C) 8 bits
(D) 18 bits

31. For the circuit of Fig. P4.3.31 consider the statement:

Assertion (A) : The circuit is sequential
Reason (R) : There is a loop in circuit
Choose correct option
(A) Both A and R true and R is the correct explanation of A
(B) Both A and R true but R is not a correct explanation of A
(C) A is true but R is false
(D) A is false

***************
SOLUTIONS

1. (C) Given FF is a negative edge triggered T flip-flop. So at the negative edge of clock \( V_i \) FF will invert the output if there is 1 at input.

2. (A) At first rising edge of clock, \( D \) is HIGH. So \( Q \) will be high till 2nd rising edge of clock. At 2nd rising edge, \( D \) is low so \( Q \) will be LOW till 3rd rising edge of clock. At 3rd rising edge, \( D \) is HIGH, so \( Q \) will be HIGH till 4th rising edge. At 4th rising edge \( D \) is HIGH so \( Q \) will be HIGH till 5th rising edge. At 5th rising edge, \( D \) is LOW, so \( Q \) will be LOW till 6th rising edge.

3. (C)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Q )</th>
<th>( S )</th>
<th>( R )</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. S4.3.3

4. (D) \( Q^* = x \oplus Q \)
\[ Q_1^* = x_1 \oplus Q_0 = x_1 \bar{Q} + \bar{x}_1 0 = x_1 \]
\[ Q_2^* = x_2 \oplus x_1 \]
\[ Q_3^* = x_3 \oplus x_2 \oplus x_1 \]
\[ Q_4^* = x_4 \oplus x_3 \oplus x_2 \oplus x_1 \]
So this generate the even parity and check odd parity.

5. (C)

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( S )</th>
<th>( R )</th>
<th>( Q )</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \times )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \times )</td>
</tr>
</tbody>
</table>

Fig. S4.3.5

\[ Q^* = \bar{A}B + AQ = \bar{A}B + \bar{B}Q \]

6. (D) \[ Q^* = LM + LM\bar{Q} \]
\[ = L(M + M\bar{Q}) \]
\[ = LM + L\bar{Q} \]

<table>
<thead>
<tr>
<th>( L )</th>
<th>( M )</th>
<th>( Q^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( \bar{Q} )</td>
</tr>
</tbody>
</table>

Fig. S4.3.6

7. (D)

<table>
<thead>
<tr>
<th>Initially</th>
<th>( J )</th>
<th>( K )</th>
<th>( Q )</th>
<th>( \bar{Q} )</th>
<th>( Q_{n+1} )</th>
<th>( \bar{Q}_{n+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clock 1st</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2nd</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3rd</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4th</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5th</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. S4.3.7

Therefore sequence is 010101.

8. (A) \[ A \quad B \quad X \quad Y \]
\[ 1 \quad 1 \quad 0 \quad 1 \]
\[ 1 \quad 0 \quad 0 \quad 1 \]

\( X \) and \( Y \) are fixed at 0 and 1.

9. (D) \[ Z = \bar{X}Q + Y\bar{Q} \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( \bar{Q} )</td>
</tr>
</tbody>
</table>

Fig. S4.3.9

Comparing from the truth table of \( J-K \) FF
\( Y = J, \; X = K \)
10. (C)

![Diagram](image)

11. (B)

<table>
<thead>
<tr>
<th>Present State</th>
<th>FF Input</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_A ; Q_B$</td>
<td>$T_A ; T_B$</td>
<td>$Q_A^+ ; Q_B^+$</td>
</tr>
<tr>
<td>0 0</td>
<td>0 1</td>
<td>0 1</td>
</tr>
<tr>
<td>0 1</td>
<td>1 1</td>
<td>1 0</td>
</tr>
<tr>
<td>1 0</td>
<td>1 0</td>
<td>0 0</td>
</tr>
<tr>
<td>1 1</td>
<td>1 1</td>
<td>0 0</td>
</tr>
</tbody>
</table>

![Table](image)

From table it is clear that it is a MOD–3 counter.

12. (B) It is a down counter because 0 state of previous FFs change the state of next FF. You may trace the following sequence, let initial state be 0 0 0

<table>
<thead>
<tr>
<th>FF C</th>
<th>FF B</th>
<th>FF A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J ; K ; \bar{C}$</td>
<td>$J ; K ; \bar{B}$</td>
<td>$J ; K ; \bar{A}$</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1 1 1</td>
<td>1 1 1</td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 0 0</td>
<td>1 1 0</td>
</tr>
<tr>
<td>0 0 0</td>
<td>1 1 0</td>
<td>1 1 1</td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 0 1</td>
<td>1 1 0</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1 1 1</td>
<td>1 1 1</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0 0 0</td>
<td>1 1 0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>1 1 0</td>
<td>1 1 1</td>
</tr>
<tr>
<td>0 0 0</td>
<td>0 0 1</td>
<td>1 1 0</td>
</tr>
</tbody>
</table>

![Table](image)

We see that 1 0 1 repeat after every two cycles, hence frequency will be $f_c / 2$.

13. (C) It is a down counter because the inverted FF output drive the clock inputs. The NAND gate will clear FFs A and B when the count tries to recycle to 111. This will produce as result of 100. Thus the counting sequence will be 100, 011, 010, 001, 000, 100 etc.

14. (A) It is a 5 bit ripple counter. At 11000 the output of NAND gate is LOW. This will clear all FF. So it is a Mod–24 counter. Note that when 11000 occur, the CLR input is activated and all FF are immediately cleared. So it is a MOD 24 counter not MOD 25.

15. (D) 10-bit ring counter is a MOD–10, so it divides the 160 kHz input by 10. Therefore, $w = 16$ kHz. The four-bit parallel counter is a MOD–16. Thus, the frequency at $x = 1$ kHz. The MOD–25 ripple counter produces a frequency at $y = 40$ Hz. $(1\text{ kHz}/25 = 40 \text{ Hz})$. The four-bit Johnson Counter is a MOD-8. This, the frequency at $z = 5$ Hz.

16. (D)

<table>
<thead>
<tr>
<th>$\bar{Q}_0 ; Q_0$</th>
<th>$Q_2 ; \bar{Q}_2$</th>
<th>$Q_1 ; \bar{Q}_1$</th>
<th>$Q_0^+ ; Q_0^+ ; Q_0^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_2 ; K_2$</td>
<td>$J_1 ; K_1$</td>
<td>$J_0 ; K_0$</td>
<td>$Q_2^+ ; Q_2^+ ; Q_2^+$</td>
</tr>
<tr>
<td>0 1</td>
<td>1 0</td>
<td>0 1</td>
<td>0 1 0</td>
</tr>
<tr>
<td>1 0</td>
<td>0 1</td>
<td>1 0</td>
<td>1 0 1</td>
</tr>
<tr>
<td>0 1</td>
<td>1 0</td>
<td>0 1</td>
<td>0 1 0</td>
</tr>
<tr>
<td>1 0</td>
<td>0 1</td>
<td>1 0</td>
<td>1 0 1</td>
</tr>
</tbody>
</table>

![Table](image)

We see that 1 0 1 repeat after every two cycles, hence frequency will be $f_c / 2$.

17. (C) At first cycle

$J_2K_2 = 10 \Rightarrow Q_2 = 1,$

$J_1K_1 = 00 \Rightarrow Q_1 = 1,$

$J_0K_0 = 00 \Rightarrow Q_0 = 0$

18. (B) In ripple counter delay $4T_p = 40$ ns.

The synchronous counter are clocked simultaneously, then its worst delay will be equal to 10 ns.

19. (A) 4 bit uses 4 FF

Total delay $Nt_o = 4 \times 50 \text{ ns} = 200 \times 10^{-9} \text{ s}$

$f = \frac{1}{200 \times 10^{-9}} = 5 \text{ Mhz}$

20. (D) At pulse 1 input, $1 \oplus 0 = 1$

So contents are 1 0 1 1,

At pulse 2 input $1 \oplus 1 = 0'$

So contents are 0 1 0 1,

At pulse 3 input 0 $\oplus 1 = 1$, contents are 1 0 1 0
Statement for Q.1–2:

Consider the DL circuit of fig. P4.4.1–2.

1. For positive logic the circuit is a
   (A) AND  (B) OR  (C) NAND  (D) NOR

2. For negative logic the circuit is a
   (A) AND  (B) OR  (C) NAND  (D) NOR

3. The diode logic circuit of fig. P4.4.3 is a
   (A) AND  (B) OR  (C) NAND  (D) NOR

In the circuit shown in fig. P.4.4.4, the output Z is

(A) \( AB + \overline{C} \)  
(B) \( \overline{ABC} \)  
(C) \( ABC \)  
(D) \( ABC \)

Statement for Q.5–7:

Consider the AND circuit shown in fig. P4.4.5–7.

The binary input levels are \( V(0) = 0 \text{ V} \) and \( V(1) = 25 \text{ V} \). Assume ideal diodes. If \( V_1 = V(0) \) and \( V_2 = V(1) \), then \( V_o \) is to be at 5 V. However, if \( V_1 = V_2 = V(1) \), then \( V_o \) is to rise above 5 V.

If \( V_o = 20 \text{ V} \) and \( V_1 = V_2 = V(1) \), the diode current \( I_{D1}, I_{D2}, \) and \( I_{D0} \) are

(A) 1 mA, 1 mA, 4 mA  
(B) 1 mA, 1 mA, 5 mA  
(C) 5 mA, 5mA, 1 mA  
(D) 0, 0, 0

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6. If $V_{ss} = 40 \text{ V}$ and both input are at HIGH level then, diode current $I_{D1}, I_{D2}$ and $I_{po}$ are respectively

(A) 0.4 mA, 0.4 mA, 0
(B) 0, 0, 1 mA
(C) 0.4 mA, 0.4 mA, 1 mA
(D) 0, 0, 0

7. The maximum value of $V_{ss}$ which may be used is

(A) 30 V
(B) 25 V
(C) 125 V
(D) 20 V

8. The ideal inverter in fig. P4.4.8 has a reference voltage of 2.5 V. The forward voltage of the diode is 0.75 V. The maximum number of diode logic circuit, that may be cascaded ahead of the inverter without producing logic error, is

(A) 3
(B) 4
(C) 5
(D) 9

9. Consider the TTL circuit in fig. P4.4.9. The value of $V_{H}$ and $V_{L}$ are respectively

(A) 5 V, 0 V
(B) 4.8 V, 0 V
(C) 4.8 V, 0.2 V
(D) 5 V, 0.2 V

Statement Q.10–11:

Consider the resistor transistor logic gate of fig. P4.4.10-11.

10. For positive logic the gate is

(A) AND
(B) OR
(C) NAND
(D) NOR

11. For negative logic the gate is

(A) AND
(B) OR
(C) NAND
(D) NOR

Statement for Q.12–13:

Consider the RTL circuit of fig. P4.4.12–13.

12. If $V_{o1}$ is taken as the output, then circuit is a

(A) AND
(B) OR
(C) NAND
(D) NOR

13. If $V_{o2}$ is taken as output, then circuit is a

(A) AND
(B) OR
(C) NAND
(D) NOR

Statement for Q.14–15:

Consider the TTL circuit of fig. P4.4.14. If either or both $V_{i1}$ and $V_{i2}$ are logic LOW, $Q_1$ is driven to saturation.
22. The circuit shown in fig. P4.4.22 is

\[ +V_{DD} \]

\[ A \]

\[ B \]

\[ M_2 \]

\[ Y \]

\[ M_1 \]

Fig. P4.4.22

(A) NAND  
(B) NOR  
(C) AND  
(D) OR

23. The circuit shown in fig. P4.4.23 acts as a

\[ +V_{DD} \]

\[ M_3 \]

\[ A \]

\[ Y \]

\[ B \]

\[ M_1 \]

\[ M_2 \]

Fig. P4.4.23

(A) NAND  
(B) NOR  
(C) AND  
(D) OR

24. The circuit shown in fig. P4.4.24 implements the function

\[ +V_{DD} \]

\[ A \]

\[ B \]

\[ C \]

\[ \overline{C} \]

\[ \overline{A} \]

\[ \overline{B} \]

\[ \overline{C} \]

\[ Y \]

Fig. P4.4.24

(A) \( ABC + \overline{ABC} \)  
(B) \( ABC + (A + B + C) \)  
(C) \( ABC + (A + B + C) \)  
(D) None of the above

25. The circuit shown in fig. P4.4.25 implements the function

\[ +V_{DD} \]

\[ C \]

\[ A \]

\[ B \]

\[ D \]

\[ Y \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

Fig. P4.4.25

(A) \((A + B)C + D\)  
(B) \((AB + C)\overline{D}\)  
(C) \((A + B)C + \overline{D}\)  
(D) \((AB + C)D\)

26. Consider the CMOS circuit shown in fig. P4.4.26. The output \(Y\) is

\[ +V_{DD} \]

\[ C \]

\[ A \]

\[ B \]

\[ Y \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

Fig. P4.4.26

(A) \((A + C)\overline{B}\)  
(B) \((A + B)\overline{C}\)  
(C) \(AB + C\)  
(D) \(AB + \overline{C}\)

27. The CMOS circuit shown in fig. P4.4.27 implements

\[ +5\,V \]

\[ \text{Logic Input} \]

\[ A \text{ to } E \]

\[ \text{PMOS Network} \]

\[ Y \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

\[ E \]

Fig. P4.4.27

(A) \(AB + CD + E\)  
(B) \((A + B)(C + D)\overline{E}\)  
(C) \(AB + CD + E\)  
(D) \((A + B)(C + D)E\)
13. (B) The $Q_4$ stage is simply an inverter (a NOT gate). Thus output $V_{o2}$ is the logic complement of $V_{o1}$. Therefore this is a OR gate.

14. (A) When $Q_1$ is saturated, $V_{o1}$ is logic LOW otherwise $V_{o1}$ is logic HIGH. The following truth table shows AND logic.

<table>
<thead>
<tr>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_{o1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

15. (C) The $Q_2$ stage is simply an inverter. Thus output $V_{o2}$ is the logic complement of $V_{o1}$.

16. (C) If $V_1 = V_2 \leq V_L$, $V_{o1} = V_{CC}$. If $V_1(V_2) > V_H$, while $V_2(V_1) \leq V_L$, $Q_1(Q_2)$ is ON and $Q_2(Q_1)$ is OFF and $V_{o1} \approx V_{CC}$. If $V_1 = V_2 > V_H$, both $Q_1$ and $Q_2$ are ON and $V_{o1} \approx 2V_{CE(out)}$. The truth table shows NAND logic.

<table>
<thead>
<tr>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_{o1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_L$</td>
<td>0</td>
<td>$V_{CC}$</td>
</tr>
<tr>
<td>1</td>
<td>$V_L$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$V_{CC}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$V_{CC}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$2V_{CE(out)}$</td>
</tr>
</tbody>
</table>

17. (A) The $Q_3$ stage is simply an inverter. Hence AND logic.

18. (C) For each successive gate, that has a transistor in saturation, the current required is

$$I_{R(sat)} = \frac{I_{C(sat)}}{\beta} = \frac{V_{CC} - V_{CE(sat)}}{\beta R_C} = \frac{5 - 0.2}{50(640)} = 0.15 \text{ mA}$$

19. (A) Let $V_1 = V_2 = 0 \text{ V}$, then $M_3$, is ON and $M_4$ will be OFF and $M_4$ ON, hence $V_o = -V_{DD}$. Let $V_1 = 0 \text{ V}$ and $V_2 = -V_{DD}$ then $M_3$ will be ON, $M_1$ OFF $M_2$ OFF, $M_2$ ON, hence $V_o = -V_{DD}$. Let $V_1 = -V_{DD}$ and $V_2 = 0 \text{ V}$, then $M_3$ OFF, $M_4$ ON, $M_2$ OFF hence $V_o = -V_{DD}$. Finally if $V_1 = V_2 = -V_{DD}$, $M_3$ and $M_4$ will be OFF and $M_1$, $M_2$ will be ON, hence $V_o = 0 \text{ V}$. Thus the given CMOS gate satisfies the function of a negative NAND gate.

20. (C) If $V_A = V_{DD}$ then $M_1$ is ON and $V_{o1} = 0 \text{ V}$. If $V_A = V_{DD}$ and $V_L = 0 \text{ V}$ then $M_3$ and $M_4$ are ON but $M_1$ is OFF hence $V_{o1} = 0 \text{ V}$. If $V_A = 0 \text{ V}$ and either or both $V_B$, $V_C$, are $0 \text{ V}$ then $M_1$ is OFF and either or both $M_2$ and $M_3$, will be OFF, which implies no current flowing through $M_1$, hence $V_{o1} = -V_{DD}$. Thus given circuit satisfies the logic equation $A + B C$.

21. (A) Let $V_1 = V_2 = 0 \text{ V} = V(0)$ then $M_4$, and $M_3$ will be ON and $M_1$, $M_2$ OFF hence $V_o = V_{DD} = V(1)$. Let $V_1 = 0 \text{ V}$, $V_2 = V_{DD}$ then $M_4$, and $M_2$ will be ON but $M_3$ and $M_1$ will be OFF hence $V_o = 0 \text{ V}$. Let $V_1 = V_{DD}$, $V_2 = 0 \text{ V}$, then $M_4$, and $M_3$ will be OFF and $M_1$ ON hence $V_o = 0 \text{ V} = V(0)$. Finally if $V_1 = V_2 = V_{DD}$, $M_1$ and $M_2$ will be ON but $M_4$ will be OFF hence $V_o = 0 \text{ V} = V(0)$. Thus the given CMOS satisfy the function of a positive NOR gate.

22. (A) If either one or both the inputs are $V(0) = 0 \text{ V}$ the corresponding FET will be OFF, the voltage across the load FET will be $0 \text{ V}$, hence the output is $V_{DD}$. If both inputs are $V(1) = V_{DD}$, both $M_1$ and $M_2$ are ON and the output is $V(0) = 0 \text{ V}$. It satisfy NAND gate.

23. (B) If both the inputs are at $V(0) = 0 \text{ V}$, the transistor $M_1$ and $M_2$ are OFF, hence the output is $V(1) = V_{DD}$. If either one or both of the inputs are at $V(1) = V_{DD}$, the corresponding FET will be ON and the output will be $V(0) = 0 \text{ V}$. Hence it is a NOR gate.
24. (B) If all inputs $A$, $B$ and $C$ are HIGH, then input to inverter is LOW and output $Y$ is HIGH. If all inputs are LOW, then input to inverter is also LOW and output $Y$ is HIGH. In all other case the input to inverter is HIGH and output $Y$ is LOW.

Hence $Y = ABC + \overline{ABC} = ABC + (A + B + C)$

25. (C) The operation of circuit is given below:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$P_A$</th>
<th>$P_B$</th>
<th>$P_C$</th>
<th>$P_D$</th>
<th>$N_A$</th>
<th>$N_B$</th>
<th>$N_C$</th>
<th>$N_D$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>×</td>
<td>×</td>
<td>1</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>OFF</td>
<td>×</td>
<td>×</td>
<td>ON</td>
<td>LOW</td>
<td></td>
</tr>
<tr>
<td>×</td>
<td>×</td>
<td>0</td>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>HIGH</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>HIGH</td>
<td></td>
</tr>
<tr>
<td>×</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>OFF</td>
<td>OFF</td>
<td>×</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>LOW</td>
<td></td>
</tr>
</tbody>
</table>

$Y = (A + B'C + D$

26. (B) The operation of this circuit is given below:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$P_A$</th>
<th>$P_B$</th>
<th>$P_C$</th>
<th>$P_D$</th>
<th>$N_A$</th>
<th>$N_B$</th>
<th>$N_C$</th>
<th>$N_D$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
<td>×</td>
<td>0</td>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>HIGH</td>
<td></td>
</tr>
<tr>
<td>×</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>OFF</td>
<td>OFF</td>
<td>×</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>LOW</td>
<td></td>
</tr>
</tbody>
</table>

$Y = (A + B'C$

27. (B) If input $E$ is LOW, output will not be LOW. It must be HIGH. Option (B) satisfy this condition.

28. (A) In this circuit parallel combination are OR gate and series combination are AND gate.

Hence $Y = (A + B)(C + D)(E + F)$

29. (A) When an output is HIGH, it may be as low as $V_{OH(min)} = 2.4$ V. The minimum voltage that an input will respond to as a HIGH is $V_{HI(min)} = 2.0$ V. A negative noise spike that can drive the actual voltage below 2.0 V if its amplitude is greater than $V_{NH} = V_{OH(min)} - V_{HI(min)} = 2.4 - 2.0 = 0.4$ V

30. (A) When an output is LOW, it may be as high as $V_{OL(max)} = 0.4$ V. The maximum voltage that an input will respond to as a LOW is $V_{IL(max)} = 0.8$ V. A positive noise spike can drive the actual voltage above the 0.8 V level if its amplitude is greater than $V_{NL} = V_{IL(max)} - V_{OL(max)} = 0.8 - 0.4 = 0.4$ V

31. (B) A positive noise spike can drive the voltage above 1.0 V level if the amplitude is greater than $V_{NH} = V_{OH(min)} - V_{HI(min)} = 0.9$ V,

A negative noise spike can drive the voltage below 3.5 V if the amplitude is greater than $V_{NL} = V_{IL(max)} - V_{OL(max)} = 0.5 + 1.5 = 2.0$ V.

32. (B) $V_{IH(min)} = V_{OH(min)} - V_{NH} = 0.8 - 0.5 = 0.3$ V,

$V_{IL(max)} = V_{OL(max)} + V_{NL} = 0.5 + 0.3 = 0.8$ V.

33. (C) $V_{NH} = V_{OH(min)} - V_{HI(min)}, V_{NL} = V_{IL(max)} - V_{OL(max)}$

$V_{NH} = 27$ (for LS) -20 (for ALS) = 7 V,

$V_{NL} = 0.8$ (for ALS) -0.5 (for LS) = 0.3 V

34. (B) $V_{NH} = 2.5$ (for ALS) -2.0 (for LS) = 0.5 V,

$V_{NL} = 0.8$ (for LS) - 0.4 (for ALS) = 0.4 V

35. (D) $V_{NH(min)} = 0.5$ V, $V_{NL(min)} = 0.3$ V

36. (B) fanout (LOW) $= \frac{I_{OL(max)}}{I_{IH(max)}} = \frac{8m}{0.1m} = 80$

fanout (HIGH) $= \frac{I_{OH(max)}}{I_{HI(max)}} = \frac{400\mu}{20\mu} = 20$

The fanout is chosen the smaller of the two.

37. (B) In HIGH state the loading on the output of gate 1 is equivalent to six 74LS input load.

Hence load = $6 \times I_{HI} = 6 \times 20\mu = 120\mu A$

38. (C) The NAND gate represent only a single input load in the LOW state. Hence only five loads in the LOW state.

load = $5I_{IL} = 5 \times 0.4 = 2$ mA

******
1. After an arithmetic operation, the flag register of 8085 µP has the following contents

<table>
<thead>
<tr>
<th>$D_7$</th>
<th>$D_6$</th>
<th>$D_5$</th>
<th>$D_4$</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>×</td>
<td>1</td>
<td>×</td>
<td>0</td>
<td>×</td>
<td>1</td>
</tr>
</tbody>
</table>

The contents of accumulator after operation may be
(A) 75 (B) 6C (C) DB (D) B6

2. In an 8085 microprocessor, the instruction CMP B has been executed while the contents of accumulator is less than that of register B. As a result carry flag and zero flag will be respectively
(A) set, reset (B) reset, set (C) reset, reset (D) set, set

3. Consider the following 8085 instruction

MVI A, A9H  
MVI B, 57H  
ADD B  
ORA A

The flag status (S, Z, CY) after the instruction ORA A is executed, is
(A) (0, 1, 1) (B) (0, 1, 0) (C) (1, 0, 0) (D) (1, 0, 1)

4. Consider the following set of 8085 instructions

MVI A, 8EH  
ADI 73H  
JC DSPLY  
OUT PORT1  
HLT  

DSPLY : XRA A  
OUT PORT1  
HLT

The output at PORT1 is
(A) 00 (B) FEH (C) 01H (D) 11H

5. Consider the following 8085 assembly program

MVI A, DATA1  
MOV B, A  
SUI 51H  
JC DLT  
MOV A, B  
SUI 82H  
JC DSPLY  

DLT : XRA A  
OUT PORT1  
HLT  

DSPLY : MOV A, B  
OUT PORT2  
HLT

This program will display
(A) the bytes from 51H to 82H at PORT2  
(B) 00H AT PORT1  
(C) all byte at PORT1  
(D) the bytes from 52H to 81H at PORT 2

6. It is desired to mask is the high order bits ($D_7 - D_4$) of the data bytes in register C. Consider the following set of instruction

(a) MOV A, C  
ANI F0H  
MOV C, A  
HLT
(b) MOV A, C
MVI B, F0H
ANA B
MOV C, A
HLT

(c) MOV A, C
MVI B, 0FH
ANA B
MOV C, A
HLT

(d) MOV A, C
ANI 0FH
MOV C, A
HLT

The instruction set, which execute the desired operation are
(A) a and b (B) c and d
(C) only a (D) only d

Consider the following 8085 instruction
XRA A
MVI B, 4AH
SUI 4FH
ANA B
HLT

The contents of register A and B are respectively
(A) 05, 4A (B) 4F, 00
(C) B1, 4A (D) None of the above

Consider the following 8085 assembly program :
MVI B, 89H
MOV A, B
MOV C, A
MVI D, 37H
OUT PORT1
HLT

The output at PORT1 is
(A) 89 (B) 37
(C) 00 (D) None of the above

Consider the sequence of 8085 instruction given below
LXI H, 9258H
MOV A, M
CMA
MOV M, A

By this sequence of instruction the contents of memory location
(A) 9258H are moved to the accumulator
(B) 9258H are compared with the contents of the accumulator
(C) 8529H are complemented and stored at location 529H
(D) 5829H are complemented and stored at location 85892H

10. Consider the sequence of 8085 instruction
MVI A, 5EH
ADI A2H
MOV C, A
HLT

The initial contents of resistor and flag are as follows
A   C   S   Z   CY
**××** **××** 0 0 0

After execution of the instructions the contents of register and flags are

A   C   S   Z   CY
(A) 10H 10H 0 0 1
(B) 10H 10H 1 0 0
(C) 00H 00H 1 1 0
(D) 00H 00H 0 1 1

11. It is desired to multiply the number 0AH by 0BH and store the result in the accumulator. The numbers are available in register B and C respectively. A part of the 8085 program for this purpose is given below :
MVI A, 00H
LOOP : ---------------
-------------
-------------
-------------
-------------
HLT
END

The sequence of instruction to complete the program would be
(A) JNZ LOOP
ADD B
DCR C
(B) ADD B
JNZ LOOP
DCR C
(C) DCR C
JNZ LOOP
ADD B
(D) ADD B
DCR C
JNZ LOOP
12. Consider the following assembly language program:

```
MVI B, 87H
MOV A, B
START : JMP NEXT
MVI B, 00H
XRA B
OUT PORT1
HLT
NEXT : XRA B
JP START1
OUT PORT2
HLT
```

The execution of the above program in an 8085 will result in

(A) an output of 87H at PORT1
(B) an output of 87H at PORT2
(C) infinite looping of the program execution with accumulator data remaining at 00H
(D) infinite looping of the program execution with accumulator data alternating between 00H and 87H.

13. Consider the following 8085 program

```
MVI A, DATA1
ORA A,
JM DSPLY
OUT PORT1
CMA
DSPLY : ADI 01H
OUT PORT1
HLT
```

If DATA1 = A7H, the output at PORT1 is

(A) 47H  (B) 58H
(C) 00  (D) None of the above

**Statement for Q.14–15:**

Consider the following program of 8085 assembly language:

```
LXI H 4A02H
LDA 4A00H
MOV B, A
LDA 4A01H
CMP B
JZ FNSH
JC GRT
MOV M, A
JMP FNSH
MOV M, B
FNSH : HLT
```

14. If the contents of memory location 4A00H, 4A01H and 4A02H, are respectively A7H, 98H and 47H, then after the execution of program contents of memory location 4A02H will be respectively

(A) A7H  (B) 98H
(C) 47H  (D) None of the above

15. The memory requirement for this program is

(A) 20 Byte  (B) 21 Byte
(C) 23 Byte  (D) 18 Byte

16. The instruction, that does not clear the accumulator of 8085, is

(A) XRA A  (B) ANI 00H
(C) MVI A, 00H  (D) None of the above

17. The contents of some memory location of an 8085 µP based system are shown:

<table>
<thead>
<tr>
<th>Address Hex.</th>
<th>Contents (Hex.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>02</td>
</tr>
<tr>
<td>3001</td>
<td>30</td>
</tr>
<tr>
<td>3002</td>
<td>00</td>
</tr>
<tr>
<td>3003</td>
<td>30</td>
</tr>
</tbody>
</table>

Fig. P4.6.17

The program is as follows

```
LHLD 3000H
MOV E, M
INX H
MOV D, M
LDAX D
MOV L, A
INX D
LDAX D
MOV H, A
```

The contents if HL pair after the execution of the program will be

(A) 0030 H  (B) 3000 H
(C) 3002 H  (D) 0230H

18. Consider the following loop

```
XRA A
LXI B, 0007H
LOOP : DCX B
JNZ LOOP
```

This loop will be executed

(A) 1 times  (B) 8 times
(C) 7 times  (D) infinite times
19. Consider the following loop

```
LXI H, 000AH
LOOP : DCX B
MOV A, B
ORA C
JNZ LOOP
```

This loop will be executed
(A) 1 time   (B) 10 times
(C) 11 times (D) infinite times

20. The contents of accumulator after the execution of following instruction will be

```
MVI A, A7H
ORA A
RLC
```

(A) CFH   (B) 4FH
(C) 4EH   (D) CEH

21. The contents of accumulator after the execution of following instructions will be

```
MVI A, B7H
ORA A
RAL
```

(A) 6EH   (B) 6FH
(C) EEH   (D) EFH

22. The contents of the accumulator after the execution of the following program will be

```
MVI A, C5H
ORA A
RAL
```

(A) 45H   (B) C5H
(C) C4H   (D) None of the above

23. Consider the following set of instruction

```
MVI A, BYTE1
RLC
MOV B, A
RLC
ADD B
```

If BYTE1 = 07H, then content of A, after the execution of program will be
(A) 46H   (B) 70H
(C) 38H   (D) 68H

24. Consider the following program

```
MVI A, BYTE1
RRC
```

If BYTE1 = 32H, the contents of A after the execution of program will be
(A) 08H   (B) 8CH
(C) 12H   (D) None of the above

25. Consider the following program

```
MVI A, DATA
MVI B, 64H
MVI C, C8H
CMP B
JC RJCT
CMP C
JNC RJCT
OUT PORT1
HLT
RJCT: SUB A
OUT PORT1
HLT
```

If the following sequence of byte is loaded in accumulator,

```
DATA (H) 58 64 73 B4 C8 FA
```

then sequence of output will be
(A) 00, 00, 73, B4, 00, FA
(B) 58, 64, 00, 00, C8, FA
(C) 58, 00, 00, 00, C8, FA
(D) 00, 64, 73, B4, 00, FA

26. Consider the following instruction to be executed by a 8085. The input port has an address of 01H and has a data 05H to input:

```
IN 01H
ANI 80H
```

After execution of the two instruction the contents of flag register are

(A) 1 0 x 1 x 1 x 0
(B) 0 1 x 0 x 1 x 0
(C) 0 1 x 1 x 1 x 0
(D) 0 1 x 1 x 0 x 0
### Chapter 4.6: Microprocessor

18. (A) The instruction XRA will set the Z flag. LXI and DCX does not alter the flag. Hence this loop will be executed 1 times.

19. (B) LXI B, 000AH ;00 → C, 0AH → B
LOOP: DCX B ;CB – 1 → B,
      MOV A, B ;B → A
      ORA C ;A OR C → A, set flag
      JNZ LOOP

Hence this loop will be executed 0AH or ten times.

20. (B) MVI A, B7H ;B7H
      ORA A ;Set Flags, CY = 1
      RLC ;Rotate accumulator left

The contents of bit $D_7$ are placed in bit $D_6$.

Before RLC 10100111
After RLC 01011111

21. (A) RAL instruction rotate the accumulator left through carry.

$D_7 \rightarrow CY$ , $CY \rightarrow D_6$ , ORA reset the carry.

Before RAL 10110111 0
After RAL 01101110 1

22. (A) RRC instruction rotate the accumulator right and $D_0$ is placed in $D_7$.

MVI A, C5H ;C5H
ORA A ;Reset Carry flag
RAL ;Rotate A left through carry
RRC ;Rotate A right, A = 45H

23. (A) This program multiply BYTE1 by 10. Hence content of A will be 46H.

07A7 07C8 10H

24. (B) Contents of Accumulator A = 0011 0010
After First RRC = 0001 1001
After second RRC = 1000 1100

25. (D) This program will display the number between 64H to C8H including 64H. C8H will not be displayed. Thus (D) is correct option.

26. (C) 05H AND 80H = 00
After the ANI instruction S, Z and P are modified to reflect the result of operation. CY is reset and AC is set. Thus,
\[ S = 0, \quad Z = 1, \quad AC = 1, \quad P = 1, \quad CY = 0 \]

27. (B) \[ ACl \quad 56H \quad \rightarrow A + 56H + CY \rightarrow A \]
\[ 37H + 56H + 1 = 8EH \]

28. (C) Instruction load the register pairs HL with 01FFH. SHLD instruction store the contents of L in the memory location 2050H and content of H in the memory location 2051H. Contents of HL are not altered.

29. (B) At a time 8085 can drive only a digit. In a second each digit is refreshed 500 times. Thus time given to each digit is:
\[ \frac{1}{(5 \times 500)} = 0.4 \text{ ms} \]

30. (C) The stack pointer register SP point to the upper memory location of stack. When data is pushed on stack, it stores above this memory location.

31. (B) Line 5 push the content of HL register pair on stack. The contents of L will go to 03FFH and contents of H will go to 03FEH. Hence memory location 03FEH contain 22H.

32. (C) Contents of register pair B lie on the top of stack when POP H is executed, HL pair will be loaded with the contents of register pair B.

33. (C) The instruction PUSH B store the contents of BC at stack. The POP PSW instruction copy the contents of BC in to PSW. The contents of register C will be copied into flag register.
\[ D_0 = 1 = \text{carry flag}, \quad D_6 = 0 = \text{zero flag}. \]
Hence zero flag will be reset and carry will be set.

34. (A) \[ \text{MVI A DATA1} \quad ; \text{DATA1} \rightarrow A \]
\[ \text{ORA A} \quad ; \text{Set flag} \]
\[ \text{JP DSPLY} \quad ; \text{If A is positive, then} \]
\[ \text{jump to DSPLY} \]
\[ \text{XRA A} \quad ; \text{Clear A} \]
\[ \text{DSPLY OUT PORT1} \quad ; A \rightarrow \text{PORT2} \]
\[ \text{HLT} \]
If DATA1 is positive, it will be displayed at port1 otherwise 00.
30. \( y(t) = u(t) * h(t) \), where \( h(t) = \begin{cases} e^{2t}, & t < 0 \\ e^{-2t}, & t > 0 \end{cases} \)

(A) \( \frac{1}{2} e^{2t} u(-t - 1) + \frac{5}{6} - \frac{1}{3} e^{-2t} u(-t) \)

(B) \( \frac{1}{2} e^{2t} u(-t - 1) + \frac{5}{6} - \frac{1}{3} e^{-2t} u(-t) \)

(C) \( \frac{1}{2} e^{2t} + \frac{1}{6} [5 - 3e^{2t} - 2e^{-2t}] u(t) \)

(D) \( \frac{1}{2} e^{2t} + \frac{1}{6} [5 - 3e^{2t} - 2e^{-2t}] u(-t) \)

Statement for Q.31-34:

The impulse response of LTI system is given. Determine the step response.

31. \( h(t) = e^{-t} \)

(A) \( 2 + e^{-t} - e^{-t} \)

(B) \( e^t u(-t + 1) + 2 - e^{-t} \)

(C) \( e^t u(-t + 1) + [2 - e^{-t}] u(t) \)

(D) \( e^t + [2 - e^{-t} - e^t] u(t) \)

32. \( h(t) = \delta^2(t) \)

(A) \( 1 \)

(B) \( u(t) \)

(C) \( \delta^3(t) \)

(D) \( \delta(t) \)

33. \( h(t) = u(t) - u(t - 4) \)

(A) \( tu(t) + (1 - t) u(t - 4) \)

(B) \( tu(t) + (1 - t) u(t - 4) \)

(C) \( 1 + t \)

(D) \( (1 + t) u(t) \)

34. \( h(t) = y(t) \)

(A) \( u(t) \)

(B) \( t \)

(C) \( 1 \)

(D) \( tu(t) \)

Statement for Q.35-38:

The system described by the differential equations has been specified with initial condition. Determine the output of the system and choose correct option.

35. \( \frac{dy(t)}{dx} + 10y(t) = 2x(t), \ y(0^-) = 1, \ x(t) = u(t) \)

(A) \( \frac{1}{2} (1 + 4e^{10t}) u(t) \)

(B) \( \frac{1}{2} (1 + 4e^{10t}) \)

(C) \( -\frac{1}{2} (1 + 4e^{10t}) u(t) \)

(D) \( -\frac{1}{2} (1 + 4e^{10t}) \)

36. \( \frac{d^2y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 4y(t) = \frac{dx(t)}{dt}, \ y(0^-) = 0, \ \frac{dy(t)}{dt} \bigg|_{t=0} = 1, \ x(t) = \sin t \ u(t) \)

(A) \( \frac{5}{4} \sin t + \frac{3}{4} \cos t + \frac{1}{6} e^{-t} - \frac{13}{61} e^{-4t}, \ t \geq 0 \)

(B) \( \frac{5}{2} \sin t + \frac{3}{4} \cos t - \frac{13}{51} e^{-4t} + \frac{1}{6} e^{-t}, \ t \geq 0 \)

(C) \( \frac{5}{4} \sin t + \frac{3}{4} \cos t - \frac{13}{51} e^{-4t} + \frac{1}{6} e^{-t}, \ t \geq 0 \)

(D) \( \frac{3}{4} \sin t + \frac{5}{34} e^{-4t} - \frac{13}{51} e^{-4t}, \ t \geq 0 \)

37. \( \frac{d^2y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) = 2x(t), \ y(0^-) = -1, \ \frac{dy(t)}{dt} \bigg|_{t=0} = 1, \ x(t) = e^{-t} u(t) \)

(A) \( \frac{2}{3} e^{-t} - \frac{5}{6} e^{-2t} + \frac{5}{6} e^{-4t}, \ t \geq 0 \)

(B) \( \frac{3}{2} + \frac{5}{6} e^{-2t} + \frac{5}{6} e^{-4t}, \ t \geq 0 \)

(C) \( 2 + 5(3e^{-2t} + e^{-4t}), \ t \geq 0 \)

(D) \( 2 - 5(3e^{-2t} + e^{-4t}), \ t \geq 0 \)

38. \( \frac{d^2y(t)}{dt^2} + 3y(t) = 3dx(t) \), \( y(0^-) = -1, \ \frac{dy(t)}{dt} \bigg|_{t=0} = 1, \ x(t) = 2te^{-t} u(t) \)

(A) \( \sin t + 4 \cos t - 3e^{-3t} + t, \ t \geq 0 \)

(B) \( 4 \sin t - \cos t - 3e^{-t}, \ t \geq 0 \)

(C) \( \sin t - 4 \cos t + 3e^{-3t} + t, \ t \geq 0 \)

(D) \( 4 \sin t + \cos t - 3te^{-t}, \ t \geq 0 \)

39. The raised cosine pulse \( x(t) \) is defined as

\[
 x(t) = \begin{cases} 
 \frac{1}{2} \cos \omega t + 1, & -\frac{\pi}{\omega} \leq t \leq \frac{\pi}{\omega} \\
 0, & \text{otherwise}
\end{cases}
\]

The total energy of \( x(t) \) is

(A) \( \frac{3\pi}{4\omega} \)

(B) \( \frac{3\pi}{8\omega} \)

(C) \( \frac{3\pi}{2\omega} \)

(D) \( \frac{3\pi}{\omega} \)

40. The sinusoidal signal \( x(t) = 4 \cos(200t + \pi/6) \) is passed through a square law device defined by the input output relation \( y(t) = x^2(t) \). The DC component in the signal is

(A) \( 3.46 \)

(B) \( 4 \)

(C) \( 2.83 \)

(D) \( 8 \)
41. The impulse response of a system is \( h(t) = \alpha (t - 0.5) \). If two such systems are cascaded, the impulse response of the overall system will be
(A) \( 0.5 \alpha (t - 0.25) \)  
(B) \( \alpha (t - 0.25) \)  
(C) \( \alpha (t - 1) \)  
(D) \( 0.5 \alpha (t - 1) \)

42. The impulse response of a system is shown in the figure below.

[Image of impulse response]

The output of the system is zero everywhere except for the
(A) \( 0 < t < 5 \)  
(B) \( 0 < t < 8 \)  
(C) \( 1 < t < 5 \)  
(D) \( 1 < t < 8 \)

43. Consider the impulse response of two LTI systems:

\( S_1 : h_1(t) = e^{-(t-2)/\alpha} u(t) \)
\( S_2 : h_2(t) = e^{-\alpha} \cos 2t u(t) \)

The stable system is
(A) \( S_1 \)  
(B) \( S_2 \)  
(C) Both \( S_1 \) and \( S_2 \)  
(D) None

44. The non-invertible system is
(A) \( y(t) = x(t - 4) \)  
(B) \( y(t) = \int_{-\infty}^{t} x(\tau) d\tau \)  
(C) \( y(t) = \frac{dx(t)}{dt} \)  
(D) None of the above

45. A continuous-time linear system with input \( x(t) \) and output \( y(t) \) yields the following input-output pairs:

\( x(t) = e^{2t} \Rightarrow y(t) = e^{j2t} \)
\( x(t) = e^{-2t} \Rightarrow y(t) = e^{-j2t} \)

If \( x_1(t) = \cos (2t - 1) \), the corresponding \( y_1(t) \) is
(A) \( \cos (5t - 1) \)  
(B) \( e^{-j} \cos (5t - 1) \)  
(C) \( \cos (5t - 1) \)  
(D) \( e^{j} \cos (5t - 1) \)

46. The \( y(t) = x(t) \ast h(t) \) is

[Images of input and output signals]

47. If \( dy(t)/dt \) contains only three discontinuities, the value of \( a \) is
(A) 1  
(B) 2  
(C) 3  
(D) 0

48. Consider the signal \( x(t) = \alpha (t + 2) - \alpha (t - 2) \). The value of \( E_x \) for the signal \( y(t) = \int_{-\infty}^{t} x(\tau) d\tau \) is
(A) 4  
(B) 2  
(C) 1  
(D) \( \infty \)

49. The response of a system \( S \) to a complex input \( x(t) = e^{j2t} \) is specified as \( y(t) = e^{j2t} \). The system
(A) is definitely LTI  
(B) is definitely not LTI  
(C) may be LTI  
(D) information is insufficient

50. The response of a system \( S \) to a complex input \( x(t) = e^{j2t} \) is specified as \( y(t) = \cos 8t \). The system
(A) is definitely LTI  
(B) is definitely not LTI  
(C) may be LTI  
(D) information is insufficient

51. The auto-correlation of the signal \( x(t) = e^{-t} u(t) \) is

(A) \( \frac{1}{2} e^{t} u(-t) + \frac{1}{2} e^{-t} u(t) \)  
(B) \( \frac{e^{t}}{2} + \frac{1}{2} (e^{-t} - e^{t}) u(t) \)  
(C) \( \frac{1}{2} e^{-t} u(-t) + \frac{1}{2} e^{-t} u(t) \)  
(D) \( \frac{1}{2} e^{t} u(-t) - \frac{1}{2} e^{-t} u(t) \)

www.gatehelp.com
1. \( A \) \( \frac{2\pi}{T} = 60\pi \Rightarrow T = \frac{\pi}{30} \)

2. \( C \) \( T_1 = \frac{2\pi}{5} \), \( T_2 = \frac{2\pi}{7} \), \( \text{LCM} \left( \frac{2\pi}{5}, \frac{2\pi}{7} \right) = 2\pi \)

3. \( D \) Not periodic because of \( t \).

4. \( D \) Not periodic because least common multiple is infinite.

5. \( C \) \( y(t) \) is not periodic although \( \sin t \) and \( 6\cos 2\pi t \) are independently periodic. The fundamental frequency can’t be determined.

6. \( C \) This is energy signal because \( E_e = \int_0^\infty |x(t)|^2 dt = \int_0^\infty e^{-4t}dt = \frac{1}{4} \)

7. \( A \) \( \|x(t)\| = 1 \), \( E_e = \int_0^\infty |x(t)|^2 dt = \infty \)

So this is a power signal not a energy.

8. \( D \) \( v(t) \) is sum of 3 unit step signal starting from, 1, 2, and 3, all signal ends at 4.

9. \( A \) The function 1 does not describe the given pulse. It can be shown as follows:

10. \( B \)

11. \( C \)

12. \( D \) Multiplication by 5 will bring contraction on time scale. It may be checked by \( x(5 \times 0.8) = x(4) \).

13. \( A \) Division by 5 will bring expansion on time scale. It may be checked by \( y(t) = x\left(\frac{20}{5}\right) = x(4) \).

14. \( C \) \( y(t) = \begin{cases} 1, \quad &\text{for } -5 < t < -4 \\ -1, \quad &\text{for } 4 < t < 5 \\ 0, \quad &\text{otherwise} \end{cases} \)

\[ E = \int_{-5}^{-4} (1)^2 dt + \int_4^{5} (-1)^2 dt = 2 \]

15. \( D \) \( E = 2\int_0^5 x^2(t)dt = 2\int_0^1 (1)^2 dt + 2\int_4^5 (5-t)^2 dt \]

\[ = 8 + \frac{2}{3} \cdot 26 = \frac{792}{3} \]

16. \( B \) Let \( x_1(t) = v(t) \) then \( y_1(t) = u\{v(t)\} \)

Let \( x_2(t) = kv(t) \) then \( y_2(t) = u\{kv(t)\} \neq ky_1(t) \)

(Not homogeneous not linear)

\[ y_1(t) = u\{v(t)\}, \quad y_2(t) = u\{v(t-t_o)\} = y_1(t-t_o) \quad \text{(Time invariant)} \]

The response at any time depends only on the excitation at time \( t = t_o \) and not on any future value.

(Causal)

17. \( C \) \( y_1(t) = v(t-5) - v(3-t) \)

\[ y_2(t) = kv(t-5) - kv(3-t) = ky_1(t) \quad \text{(Homogeneous)} \]

Let \( x_1(t) = v(t) \) then \( y_1(t) = v(t-5) - v(3-t) \)

Let \( x_2(t) = 2v(t) \) then \( y_2(t) = v(t-5) - 2v(3-t) \)

Let \( x_3(t) = x(t) + w(t) \)

Then \( y_3(t) = v(t-5) + v(t-5) - v(3-t) - w(3-t) \)

\[ = y_1(t) + y_2(t) \quad \text{(Additive)} \]

Since it is both homogeneous and additive, it is also linear.

\[ y_1(t) = v(t-5) - v(3-t) \]

\[ y_2(t) = v(t-t_o-5) - v(3-t + t_o) = y_1(t-t_o) \quad \text{(Time invariant)} \]
At time, \( t = 0 \), \( y(0) = x(-5) - x(3) \). Therefore the response at time, \( t = 0 \) depends on the excitation at a later time \( t = 3 \).

(Not causal)

If \( x(t) \) is bounded then \( x(t - 5) \) and \( x(3 - t) \) are bounded and so is \( y(t) \).

(Stable)

18. (D) \( y_1(t) = \frac{1}{4} \left( \frac{t}{2} \right), \quad y_2(t) = k \frac{1}{4} \left( \frac{t}{2} \right) = ky_1(t) \)

(Homogeneous)

\( x_3 = v(t) + w(t) \) then

\( y_3(t) = \left[ \frac{1}{4} \left( \frac{t}{2} \right) + \frac{1}{4} \left( \frac{t}{2} \right) \right] = y_1(t) + y_2(t) \)

(Additive)

Since it is both homogeneous and additive, it is also linear

\( y_1(t) = \frac{1}{4} \left( \frac{t}{2} \right), \quad y_2(t) = \frac{1}{4} \left( \frac{t - t_o}{2} \right) = y(t - t_o) = \frac{1}{4} \left( \frac{t - t_o}{2} \right) \)

(Time variant)

At time \( t = -2 \), \( y(-2) = x(-1) \), therefore, the response at time \( t = -2 \), depends on the excitation at a later time, \( t = -1 \).

(Not causal)

It \( x(t) \) is bounded then \( y(t) \) is bounded.

(Stable)

19. (C) \( y_1(t) = \cos 2\pi t \cdot v(t) \)

\( y_2(t) = k \cos 2\pi t \cdot v(t) = ky_1(t) \)

(Homogeneous)

\( x_3(t) = v(t) + w(t) \)

\( y_3(t) = \cos 2\pi \left[ v(t) + w(t) \right] = y_1(t) + y_2(t) \)

(Additive)

Since it is both homogeneous and additive. It is also linear.

\( y_1(t) = \cos 2\pi t \cdot v(t) \),

\( y_2(t) = \cos 2\pi \left( t - t_o \right) = y(t - t_o) \)

= \cos \left[ 2\pi (t - t_o) \right] v(t - t_o) \)

(Time Variant)

The response at any time \( t = t_o \) depends only on the excitation at that time and not on the excitation at any later time.

(Causal)

If \( x(t) \) is bounded then \( y(t) \) is bounded.

(Stable)

20. (C) \( y_1(t) = |v(t)| \), \( y_2(t) = |kv(t)| = ky_1(t) \)

If \( k \) is negative \( |k|y_1(t) \neq ky_1(t) \)

(Not Homogeneous Not linear)

\( y_1(t) = |v(t)| \), \( y_2(t) = |y(t - t_o)| = y_1(t - t_o) \)

(Time Invariant)

The response at any time \( t = t_o \) depends only on the excitation at that time and not on the excitation at any later time.

(Causal)

If \( x(t) \) is bounded then \( y(t) \) is bounded.

(Stable)

21. (C) All option are linear. So it is not required to check linearity.

Let \( x_1(t) = v(t) \) then \( \frac{d}{dt} y_1(t) - 8y_1(t) = v(t) \)

Let \( x_2(t) = v(t - t_o) \) then \( \frac{d}{dt} y_2(t) - 8y_2(t) = v(t - t_o) \)

The first equation can be written as

\( \frac{d}{dt} (t - t_o) - 8y(t - t_o) = v(t - t_o) \)

This equation is not satisfied if \( y_2(t) = y_1(t - t_o) \) therefore

\( y_2(t) \neq y_1(t - t_o) \)

(Time Variant)

The system can be written as

\( y(t) = \int_{-\infty}^{t} \frac{v(\lambda)}{\lambda} d\lambda + 8 \int_{-\infty}^{t} \frac{y(\lambda)}{\lambda} d\lambda \)

So the response at any time, \( t = t_o \), depends on the excitation at \( t \leq t_o \), and not on any future values.

(Causal)

The Homogeneous solution to the differential equation is of the form \( y(t) = kv_0(t) \). If there is no excitation but the zero excitation, response is not zero. The response will increases without bound as time increases.

(Unstable)

22. (C) \( y_1(t) = \int_{-\infty}^{i\infty} v(\lambda) d\lambda \)

\( y_2(t) = \int_{-\infty}^{i\infty} k v(\lambda) d\lambda = k \int_{-\infty}^{i\infty} \lambda d\lambda = ky_1(t) \)

(Homogeneous)

\( x_3(t) = v(t) + w(t) \)

\( y_3(t) = \int_{-\infty}^{i\infty} [v(\lambda) + w(\lambda)] d\lambda = \int_{-\infty}^{i\infty} v(\lambda) d\lambda + \int_{-\infty}^{i\infty} w(\lambda) d\lambda \)

\( = y_1(t) + y_2(t) \)

(Additive)

Since it is Homogeneous and additive, it is also linear.

\( y_1(t) = \int_{-\infty}^{i\infty} v(\lambda) d\lambda \)

\( y_2(t) = \int_{-\infty}^{i\infty} v(\lambda - t_o) d\lambda = \int_{-\infty}^{i\infty} v(\lambda) d\lambda = y_1(t - t_o) \)

(Time invariant)

The response at any time, \( t = t_o \), depends partially on the excitation at time to \( t_o < t < (t_o + 3) \) which are in future.

(Not causal)

If \( x(t) \) is a constant \( k \), then \( \int_{-\infty}^{t} k d\lambda = k \int_{-\infty}^{t} d\lambda \) and as \( t \to \infty \), \( y(t) \) increases without bound.

(UNSTABLE)
11. \( x[n+2]y[n-2] \)

(A) \[ \begin{array}{cccccccc}
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\
\end{array} \]

(B) \[ \begin{array}{cccccccc}
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\
\end{array} \]

(C) \[ \begin{array}{cccccccc}
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\
\end{array} \]

(D) \[ \begin{array}{cccccccc}
-6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\
\end{array} \]

Statement for Q.12–15:

A discrete-time signal is given. Determine the period of signal and choose correct option.

12. \( x[n] = \cos \frac{\pi n}{9} + \sin \left( \frac{\pi n}{7} + \frac{1}{2} \right) \)

(A) periodic with period \( N = 126 \)
(B) periodic with period \( N = 32 \)
(C) periodic with period \( N = 252 \)
(D) Not periodic

13. \( x[n] = \cos \left( \frac{n}{8} \right) \cos \left( \frac{\pi n}{8} \right) \)

(A) Periodic with period 16\( \pi \)
(B) periodic with period 16\( (\pi + 1) \)
(C) periodic with period 8
(D) Not periodic

14. \( x[n] = \cos \left( \frac{\pi n}{2} \right) - \sin \left( \frac{\pi n}{8} \right) + 3 \cos \left( \frac{\pi n}{4} + \frac{\pi}{3} \right) \)

(A) periodic with period 16
(B) periodic with period 4
(C) periodic with period 2
(D) Not periodic

15. \( x[n] = 2e^{\left( \frac{\pi n}{8} \right)} \)

(A) periodic with 12\( \pi \)
(B) periodic with 12
(C) periodic with 11\( \pi \)
(D) Not periodic

16. The sinusoidal signal has fundamental period \( N = 10 \) samples. The smallest angular frequency, for which \( x[n] \) is periodic, is

(A) \( \frac{1}{10} \) rad/cycle
(B) 10 rad/cycle
(C) 5 rad/cycle
(D) \( \frac{\pi}{5} \) rad/cycle

17. Let \( x[n], -5 \leq n \leq 3 \) and \( h[n], 2 \leq n \leq 6 \) be two finite duration signals. The range of their convolution is

(A) \(-7 \leq n \leq 9\)
(B) \(-3 \leq n \leq 9\)
(C) \(2 \leq n \leq 3\)
(D) \(-5 \leq n \leq 6\)

Statement for Q.18–26:

\( x[n] \) and \( h[n] \) are given in the question. Compute the convolution \( y[n] = x[n] \ast h[n] \) and choose correct option.

18. \( x[n] = \{1, 2, 4\}, \ h[n] = \{1, 1, 1, 1, 1\} \)

(A) \{1, 3, 7, 7, 7, 6, 4\}
(B) \{1, 3, 3, 7, 7, 6, 4\}
(C) \{1, 2, 4\}
(D) \{1, 3, 7\}

19. \( x[n] = \{1, 2, 3, 4, 5\}, \ h[n] = \{1\} \)

(A) \{1, 3, 6, 10, 15\}
(B) \{1, 2, 3, 4, 5\}
(C) \{1, 4, 9, 16, 20\}
(D) \{1, 4, 6, 8, 10\}

20. \( x[n] = \{1, 2, -1\}, \ h[n] = x[n] \)

(A) \{1, 4, 1\}
(B) \{1, 4, 2, -4, 1\}
(C) \{1, 2, -1\}
(D) \{2, 4, -2\}
21. \( x[n] = \{1, -2, 3\}, \ h[n] = \{0, 0, 1, 1, 1\} \)
(A) \(\{1, -2, 4, 1, 1, 1\}\) 
(B) \(\{0, 0, 3\}\) 
(C) \(\{0, 0, 3, 1, 1, 1\}\) 
(D) \(\{0, 0, 1, -1, 2, 2, 1, 3\}\)

22. \( x[n] = \{0, 0, 1, 1, 1, 1\}, \ h[n] = \{1, -2, 3\} \)
(A) \(\{0, 0, 1, -1, 2, 2, 1, 3\}\) 
(B) \(\{0, 0, 1, -1, 2, 2, 1, 3\}\) 
(C) \(\{1, -2, 3, 1, 1, 2, 1, 1\}\) 
(D) \(\{1, -2, 3, 1, 1, 1, 1\}\)

23. \( x[n] = \{1, 1, 0, 1, 1, 1\}, \ h[n] = \{1, -2, -3, 4\} \)
(A) \(\{1, -1, -2, 4, 1, 1\}\) 
(B) \(\{1, -1, -2, 4, 1, 1\}\) 
(C) \(\{1, -1, -5, 2, 3, -5, 1, 4\}\) 
(D) \(\{1, -1, -5, 2, 3, -5, 1, 4\}\)

24. \( x[n] = \{1, 2, 0, 2, 1\}, \ h[n] = x[n] \)
(A) \(\{1, 4, 4, 4, 10, 4, 4, 4, 1\}\) 
(B) \(\{1, 4, 4, 4, 10, 4, 4, 4, 1\}\) 
(C) \(\{1, 4, 4, 10, 4, 4, 4, 1\}\) 
(D) \(\{1, 4, 4, 10, 4, 4, 4, 1\}\)

25. \( x[n] = \{1, 4, -3, 6, 4\}, \ h[n] = \{2, -4, 3\} \)
(A) \(\{2, 4, -19, 36, -25, 2, 12\}\) 
(B) \(\{4, -19, 36, -25\}\) 
(C) \(\{1, 4, -3, 6, 4\}\) 
(D) \(\{1, 4, -3, 6, 4\}\)

26. \( x[n] = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0 & \text{elsewhere} \end{cases} \)
(A) \(\delta[n] - \delta[n - 1] + \delta[n - 4]\) 
(B) \(\delta[n] - 2\delta[n - 1] + 4\delta[n - 4] + \delta[n - 5]\) 
(C) \(\delta[n + 2] - \delta[n + 1] + \delta[n] + 2\delta[n - 3] - \delta[n - 4] + 2\delta[n - 5]\) 
(D) \(\delta[n] + 2\delta[n - 1] + 4\delta[n - 5] + \delta[n - 5]\)

Statement for Q.27–30:
In question \( y[n] \) is the convolution of two signal. Choose correct option for \( y[n] \).

27. \( y[n] = (-1)^n \ast u[2n + 2] \)
(A) \(\frac{4}{6}\) 
(B) \(\frac{4}{6} u[-n + 2]\) 
(C) \(\frac{8}{3} (-1)^n u[-n + 2]\) 
(D) \(\frac{8}{3} (-1)^n\)

28. \( y[n] = \frac{1}{4^n} u[n]^\ast u[n + 2] \)
(A) \(\left(\frac{1}{3} - \frac{4}{12}\right) u[n]\) 
(B) \(\left(\frac{1}{3} - \frac{12}{4^n}\right) u[n + 2]\) 
(C) \(\left(\frac{4}{3} - \frac{121}{124}\right) u[n + 2]\) 
(D) \(\left(\frac{16}{3} - \frac{1}{4^n}\right) u[n + 2]\)

29. \( y[n] = 3^n u[-n + 3]^\ast u[n - 2] \)
(A) \(\left\{\begin{array}{ll} \frac{3^n}{2}, & n \leq 5 \\ \frac{83}{2}, & n \geq 6 \end{array}\right.\) 
(B) \(\left\{\begin{array}{ll} \frac{3^n}{2}, & n \leq 5 \\ \frac{83}{2}, & n \geq 6 \end{array}\right.\) 
(C) \(\left\{\begin{array}{ll} \frac{3^n}{2}, & n \leq 5 \\ \frac{81}{2}, & n \geq 6 \end{array}\right.\) 
(D) \(\left\{\begin{array}{ll} \frac{3^n}{2}, & n \leq 5 \\ \frac{81}{2}, & n \geq 6 \end{array}\right.\)
30. $y[n] = u[n + 3] * u[n - 3]$  
(A) $(n + 1)u[n]$  
(B) $nu[n]$  
(C) $(n - 1)u[n]$  
(D) $u[n]$  

31. The convolution of $x[n] = \cos\left(\frac{\pi}{2} n\right) u[n]$ and $h[n] = u[n - 1]$ is $f[n]u[n - 1]$. The function $f[n]$ is  
(A) \begin{cases} 1, & n = 4m + 1, 4m + 2 \\ 0, & n = 4m, 4m + 3 \end{cases}$  
(B) \begin{cases} 0, & n = 4m + 1, 4m + 2 \\ 1, & n = 4m, 4m + 3 \end{cases}$  
(C) \begin{cases} 1, & n = 4m + 1, 4m + 3 \\ 0, & n = 4m, 4m + 2 \end{cases}$  
(D) \begin{cases} 0, & n = 4m + 1, 4m + 3 \\ 1, & n = 4m, 4m + 2 \end{cases}$

Statement for Q.32–38:

Let P be linearity, Q be time invariance, R be causality and S be stability. In question discrete time input $x[n]$ and output $y[n]$ relationship has been given. In the option properties of system has been given. Choose the option which match the properties for system.

32. $y[n] = \text{rect}(x[n])$  
(A) P, Q, R  
(B) Q, R, S  
(C) R, S, P  
(D) S, P, Q  

33. $y[n] = nx[n]$  
(A) P, Q, R, S  
(B) Q, R, S  
(C) P, R  
(D) S, Q  

34. $y[n] = \sum_{m=-\infty}^{n-1} u[m]$  
(A) P, Q, R, S  
(B) R, S  
(C) P, Q  
(D) Q, S  

35. $y[n] = \sqrt{x[n]}$  
(A) Q, R, S  
(B) R, S, P  
(C) S, P, Q  
(D) P, Q, R  

36. $x[n]$ as shown in fig. P5.2.36  
(A) P, Q, R, S  
(B) Q, R, S  
(C) P, Q  
(D) R, S  

37. $x[n]$ as shown in fig. P5.2.37  
(A) P, Q, R, S  
(B) Q, R, S  
(C) P, R, S  
(D) P, Q, S  

Statement for Q.39–41:

Two discrete time systems $S_1$ and $S_2$ are connected in cascade to form a new system as shown in fig. P5.2.39–41.

39. Consider the following statements  
(a) If $S_1$ and $S_2$ are linear, the $S$ is linear  
(b) If $S_1$ and $S_2$ are nonlinear, then $S$ is nonlinear  
(c) If $S_1$ and $S_2$ are causal, then $S$ is causal  
(d) If $S_1$ and $S_2$ are time invariant, then $S$ is time invariant  
   True statements are:  
   (A) a, b, c  
   (B) b, c, d  
   (C) a, c, d  
   (D) All  

40. Consider the following statements  
(a) If $S_1$ and $S_2$ are linear and time invariant, then interchanging their order does not change the system.  
(b) If $S_1$ and $S_2$ are linear and time varying, then interchanging their order does not change the system.  
   True statement are  
   (A) Both a and b  
   (B) Only a  
   (C) Only b  
   (D) None
41. Consider the statement
(a) If \( S_1 \) and \( S_2 \) are noncausal, the \( S \) is non causal
(b) If \( S_1 \) and/or \( S_2 \) are unstable, the \( S \) is unstable.

True statement are :
(A) Both a and b  (B) Only a
(C) Only b  (D) None

42. The following input output pairs have been observed during the operation of a time invariant system :
\[ x_1[n] = \{1, 0, 2\} \quad \xrightarrow{S} \quad y_1[n] = \{0, 1, 2\} \]
\[ x_2[n] = \{0, 0, 3\} \quad \xrightarrow{S} \quad y_2[n] = \{0, 1, 0, 2\} \]
\[ x_3[n] = \{0, 0, 1\} \quad \xrightarrow{S} \quad y_3[n] = \{1, 2, 1\} \]

The conclusion regarding the linearity of the system is
(A) System is linear
(B) System is not linear
(C) One more observation is required.
(D) Conclusion cannot be drawn from observation.

43. The following input output pair have been observed during the operation of a linear system:
\[ x_1[n] = \{-1, 2, 1\} \quad \xrightarrow{S} \quad y_1[n] = \{1, 2, -1, 0, 1\} \]
\[ x_2[n] = \{1, -1, -1\} \quad \xrightarrow{S} \quad y_2[n] = \{-1, 1, 0, 2\} \]
\[ x_3[n] = \{0, 1, 1\} \quad \xrightarrow{S} \quad y_3[n] = \{1, 2, 1\} \]

The conclusion regarding the time invariance of the system is
(A) System is time-invariant
(B) System is time variant
(C) One more observation is required.
(D) Conclusion cannot be drawn from observation

44. The stable system is
(A) \( y[n] = x[n] + 1.1y[n-1] \)
(B) \( y[n] = x[n] - \frac{1}{2}(y[n-1] + y[n-2]) \)
(C) \( y[n] = x[n] - (1.5y[n-1] + 0.4y[n-2]) \)

45. The system shown in fig. P5.2.45 is
\[ x[n] \rightarrow \frac{1}{2}y[n] \rightarrow D \rightarrow y[n+1] \]

(A) Stable and causal
(B) Stable but not causal
(C) Causal but unstable
(D) Unstable and not causal

46. The impulse response of a LTI system is given as
\[ h[n] = \left(-\frac{1}{2}\right)^n u[n] \]
The step response is
(A) \(\frac{1}{3}\left(2\left(-\frac{1}{2}\right)^n\right) u[n] \)
(B) \(\left(2\left(-\frac{1}{2}\right)^n\right) u[n] \)
(C) \(\frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^n\right) u[n] \)
(D) \(\frac{1}{3}\left(2 - \left(-\frac{1}{2}\right)^n\right) u[n] \)

47. The difference equation representation for a system is
\[ y[n] - \frac{1}{2}y[n-1] = 2x[n], \quad y[-1] = 3 \]
The natural response of system is
(A) \(\frac{3}{2}\left(-\frac{1}{2}\right)^n u[n] \)
(B) \(\frac{3}{2}\left(-\frac{1}{2}\right)^n u[n] \)
(C) \(\frac{3}{2}\left(\frac{1}{2}\right)^n u[n] \)
(D) \(\frac{3}{2}\left(\frac{1}{2}\right)^n u[n] \)

48. The difference equation representation for a system is
If \( y[n] = 0 \) for \( n < 0 \) and \( x[n] = 0 \), then \( y[2] \) will be
(A) 2  (B) -2  (C) -1  (D) 0

49. Consider a discrete-time system \( S \) whose response to a complex exponential input \( e^{jn/2} \) is specified as
\[ S : e^{jn/2} \Rightarrow e^{jn/2} \]
24. (B) \( y[n] = (1, 4, 4, 10, 4, 4, 4, 1) \)

\[
\begin{array}{cccccccc}
1 & 2 & 0 & 2 & 1 \\
1 & 2 & 2 & 0 & 2 & 1 \\
2 & 2 & 4 & 0 & 4 & 2 \\
0 & 0 & 0 & 0 & 0 \\
2 & 2 & 4 & 0 & 4 & 2 \\
1 & 2 & 2 & 0 & 2 & 1 \\
\end{array}
\]

Fig. S5.2.24

25. (A) \( y[n] = (2, 4, -19, 36, -25, 2, 12) \)

\[
\begin{array}{cccccccc}
1 & 4 & -3 & 6 & 4 \\
2 & -4 & 5 & 12 & 8 \\
-1 & -4 & 16 & 12 & 8 \\
3 & 3 & 12 & 9 & 18 & 12 \\
\end{array}
\]

Fig. S5.2.25

26. (B) \( x[n] = (1, 2, 1, 1), h[n] = (1, -1, 0, 0, 1) \)

\[
\begin{array}{cccccccc}
1 & 2 & 1 & 1 \\
1 & 2 & 1 & 1 \\
-1 & -2 & -1 & -1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 2 & 1 \\
\end{array}
\]

Fig. S5.2.26

\[
y[n] = (1, 1, -1, 0, 0, 2, 1, 1) \\
y[n] = \hat{a}[n+2] + \hat{a}[n+1] - \hat{a}[n] + 2\hat{a}[n-3] + \hat{a}[n-4] + \hat{a}[n-5] \\
\]

27. (D) \( y[n] = \sum_{k=-2}^{n} (-1)^k 2^{-k} = 2^n \sum_{k=-2}^{n} \left( -\frac{1}{2} \right)^k \\
= 2^n \left( \frac{1}{2} \right)^{-2} = \frac{8}{3} (-1)^n \\
\)

28. (C) For \( n+2 < 0 \) or \( n < -2 \), \( y[n] = 0 \)

\[
y[n] = \sum_{k=0}^{n+2} \frac{1}{4} \\
\Rightarrow y[n] = \left( \frac{4}{3} \right)(\frac{1}{4})^n u[n+2] \\
\]

29. (D) For \( n-2 \leq 3 \) or \( n \leq 5 \), \( y[n] = \sum_{k=-\infty}^{n} 3^k = \frac{3^n}{6} \\
\)

\[
y[n] = \left\{ \begin{array}{ll}
\frac{3^n}{6}, & n \leq 5 \\
\frac{81}{2}, & n \geq 6 \\
\end{array} \right. \\
\]

30. (A) For \( n-3 < -3 \) or \( n < 0 \), \( y[n] = 0 \)

\[
y[n] = \sum_{k=-3}^{n} 1 = n + 1, \\
y[n] = (n+1)u[n] \\
\]

31. (A) For \( n-1 < 0 \) or \( n < 0 \), \( y[n] = 0 \)

\[
y[n] = \sum_{k=0}^{n} \cos \left( \frac{\pi k}{2} \right) \\
\Rightarrow y[n] = \left\{ \begin{array}{ll}
1, & n = 4m + 1, 4m + 2 \\
0, & n = 4m, 4m + 3 \\
\end{array} \right. \\
\]

32. (B) \( y_1[n] = rect(v[n]), \quad y_2[n] = rect(kv[n]) \)

\[
y_2[n] = k y_1[n] \quad \text{(Not Homogeneous not linear)} \\
y_1[n] = rect(v[n]), \quad y_2[n] = rect(v(n - n_s)) \\
y_1[n - n_s] = rect(v(n - n_s)) = y_2[n] \quad \text{(Time Invariant)} \\
\]

At any discrete time \( n = n_s \), the response depends only on the excitation at that discrete time. \text{(Causal)}

No matter what values the excitation may have the response can only have the values zero or one.

\text{(Stable)}

33. (C) \( y_1[n] = nv[n], \quad y_2[n] = nkv[n] \)

\[
ky_1[n] = y_2[n] \quad \text{(Homogeneous)} \\
\]

Let \( x_1[n] = v[n] \) then \( y_1[n] = nv[n] \)

Let \( x_2[n] = w[n] \) then \( y_2[n] = n w[n] \)

Let \( x_3[n] = v[n] + w[n] \) then

\[
y_3[n] = n(v[n] + w[n]) = nv[n] + nw[n] \\
y_2[n] + y_3[n] \quad \text{(Additive)} \\
\]

Since the system is homogeneous and additive, it is also linear.

\[
y_1[n - n_s] = (n - n_s)v[n - n_s] \neq y_3[n] = nv[n - n_s] \\
\]

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At any discrete time, $n = n_0$, the response depends only on the excitation at that same time. (Causal)

If the excitation is a constant, the response is unbounded as $n$ approaches infinity. (Unstable)

34. (C) $y_1[n] = \sum_{m=-\infty}^{n-1} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n-1} k v[m]$

$y_2[n] = k y_1[n]$ (Homogeneous)

$y_1[n] = \sum_{m=-\infty}^{n-1} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n-1} u[m]$

$y_3[n] = \sum_{m=-\infty}^{n-1} (v[n] + w[m])$

$= \sum_{m=-\infty}^{n-1} v[m] + \sum_{m=-\infty}^{n-1} u[n] = y_1[n] + y_2[n]$ (Additive)

Since the system is homogeneous and additive it is also linear

$y_1[n] = \sum_{m=-\infty}^{n-1} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n-1} v[m - n_0]$

$y_1[n - n_0] = \sum_{m=-\infty}^{m=n-1} v[q - n_0] = y_2[n]$ (Time Invariant)

At any discrete time, $n = n_0$, the response depends on the excitation at the next discrete time in future. (Anti causal)

If the excitation is a constant, the response increases without bound. (Unstable)

35. (A) $y_1[n] = \sqrt{v[n]}$, $y_2 = \sqrt{k v[n]} = \sqrt{k} \sqrt{v[n]}$

$k y_1[n] = k \sqrt{v[n]} \neq y_2[n]$ (Not Homogeneous Not linear)

$y_1[n] = \sqrt{v[n]}$, $y_2[n] = \sqrt{v[n - n_0]}$

$y_1[n - n_0] = \sqrt{v[n - n_0]} = y_2[n]$ (Time Invariant)

At any discrete time $n = n_0$, the response depends only on the excitation at that time (Causal)

If the excitation is bounded, the response is bounded. (Stable)

If the excitation is a constant, the response increases without bound. (Unstable)

36. (B) $y[n] = 2x^2[n]$

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = kv[n]$ then $y_2[n] = 2k^2v^2[n]$

$k y[n] \neq y[n]$ (Not Homogeneous Not linear)

Let $x_1[n] = v[n]$ then $y_1[n] = 2v^2[n]$

Let $x_2[n] = v[n - n_0]$ then $y_2[n] = 2v^2[n - n_0]$

$y_1[n - n_0] = 2v[n - n_0] = y_2[n]$ (Time invariant)

At any discrete time, $n = n_0$, the response depends only on the excitation at that time. (Causal)

If the excitation is bounded, the response is bounded. (Stable)

37. (B) $y_1[n] = 10v[n] - 5$, $y_2[n] = 10k v[n] - 5$

$y_2[n] \neq k y_1[n]$ (Not Homogeneous Not linear)

$y_1[n] = 10v[n] - 5$, $y_2[n] = 10v[n - n_0] - 5$

$y_1[n - n_0] = 10v[n - n_0] - 5$, $= y_2[n]$ (Time Invariant)

At any discrete time, $n = n_0$, the response depends only on the excitation at that discrete time and not on any future excitation. (Causal)

If the excitation is bounded, the response is bounded. (Stable)

38. (B) $y[n] = x[n] + y[n - 1]$, $y[n - 1] = x[n - 1] + y[n - 2]$

$y[n] = x[n] + x[n - 1] + y[n - 2]$, Then by induction

$y[n] = x[n - 1] + x[n - 2] + ... + x[n - k] + ... = \sum x[n - k]$

Let $m = n - k$ then $y[n] = \sum_{m=-\infty}^{n} v[m]$

$y_1[n] = \sum_{m=-\infty}^{n} v[m]$, $y_2[n] = \sum_{m=-\infty}^{n} kv[m] = ky_1[n]$

$y_3[n] = \sum_{m=-\infty}^{n} (v[m] + w[m]) = \sum_{m=-\infty}^{n} v[m] + \sum_{m=-\infty}^{n} w[m]$

$= y_1[n] + y_2[n]$ (Additive)

System is Linear.

$y_1[n] = \sum_{m=-\infty}^{n} v[m]$, $y_2 = \sum_{m=-\infty}^{n} v[n - n_0]$

$y_1[n]$ can be written as

$y_1[n - n_0] = \sum_{m=-\infty}^{m=n-1} v[m] = \sum_{m=-\infty}^{m=n-1} v[q - n_0] = y_2[n]$

(Time Invariant)

At any discrete time $n = n_0$, the response depends only on the excitation at that discrete time and previous discrete time. (Causal)

If the excitation is constant, the response increase without bound. (Unstable)

39. (C) Only statement (b) is false. For example

$S_1 : y[n] = x[n] + b$, and $S_2 : y[n] = x[n] - b$, where $b \neq 0$

$S[x[n]] = S_2 [S_1 [x[n]]] = S_2 [x[n] + b] = x[n]$

Hence $S$ is linear.

40. (B) For example

$S_1 : y[n] = nx[n]$ and $S_2 : y[n] = nx[n + 1]$

If $x[n] = \delta[n]$ then $S_2 [\delta[n]] = S_2 [0] = 0$,
CHAPTER 5.3
THE LAPLACE TRANSFORM

Statement for Q.1-12:
Determine the Laplace transform of given signal.

1. \( x(t) = u(t - 2) \)
   (A) \( \frac{\mathrm{e}^{-2s}}{s} \)
   (B) \( \frac{\mathrm{e}^{-2s}}{s} \)
   (C) \( \frac{\mathrm{e}^{-2s}}{1 + s} \)
   (D) 0

2. \( x(t) = u(t + 2) \)
   (A) \( \frac{1}{s} \)
   (B) \( -\frac{1}{s} \)
   (C) \( \frac{\mathrm{e}^{-2s}}{s} \)
   (D) \( -\frac{\mathrm{e}^{-2s}}{s} \)

3. \( x(t) = \mathrm{e}^{-t}u(t + 1) \)
   (A) \( \frac{1}{s + 2} \)
   (B) \( \frac{\mathrm{e}^{-s}}{s + 2} \)
   (C) \( \frac{\mathrm{e}^{-(s+2)}}{s + 2} \)
   (D) \( -\frac{\mathrm{e}^{-s}}{s + 2} \)

4. \( x(t) = e^{2t}u(-t + 2) \)
   (A) \( \frac{\mathrm{e}^{2(s-2)} - 1}{s - 2} \)
   (B) \( \frac{\mathrm{e}^{-2s}}{s + 2} \)
   (C) \( \frac{1 - \mathrm{e}^{-2(s-2)}}{s - 2} \)
   (D) \( \frac{\mathrm{e}^{-2s}}{s - 2} \)

5. \( x(t) = \sin 5t \)
   (A) \( \frac{5}{s^2 + 5} \)
   (B) \( \frac{s}{s^2 + 5} \)
   (C) \( \frac{5}{s^2 + 25} \)
   (D) \( \frac{s}{s^2 + 25} \)

6. \( x(t) = u(t) - u(t - 2) \)
   (A) \( \frac{\mathrm{e}^{-2s} - 1}{s} \)
   (B) \( \frac{1 - \mathrm{e}^{-2s}}{s} \)
   (C) \( \frac{2}{s} \)
   (D) \( -\frac{2}{s} \)

7. \( x(t) = \frac{d}{dt}(te^{-t}u(t)) \)
   (A) \( \frac{1}{s(s + 1)^2} \)
   (B) \( \frac{s}{(s + 1)^2} \)
   (C) \( \frac{\mathrm{e}^{-s}}{s + 1} \)
   (D) \( \frac{-\mathrm{e}^{-s}}{(s + 1)^2} \)

8. \( x(t) = tu(t) \ast \cos 2\pi t \)
   (A) \( \frac{1}{s(s^2 + 4\pi^2)} \)
   (B) \( \frac{2\pi}{s^2(s^2 + 4\pi^2)} \)
   (C) \( \frac{1}{s^2(s^2 + 4\pi^2)} \)
   (D) \( \frac{s^3}{s^2 + 4\pi^2} \)

9. \( x(t) = t^3u(t) \)
   (A) \( \frac{3}{s^3} \)
   (B) \( -\frac{3}{s^3} \)
   (C) \( \frac{6}{s^3} \)
   (D) \( -\frac{6}{s^4} \)

10. \( x(t) = u(t - 1) \ast \mathrm{e}^{-t}u(t - 1) \)
    (A) \( \frac{\mathrm{e}^{-(s+1)}}{2s + 1} \)
    (B) \( \frac{\mathrm{e}^{-2(s+1)}}{s + 1} \)
    (C) \( \frac{\mathrm{e}^{-(s+2)}}{s + 2} \)
    (D) \( \frac{\mathrm{e}^{-2(s+1)}}{s + 2} \)
11. \( x(t) = \int_{0}^{\infty} e^{-3t} \cos 2\tau \, d\tau \)

(A) \(-\frac{(s + 3)}{s(s + 3)^2 + 4}\)
(B) \(-\frac{(s + 3)}{s(s + 3)^2 + 4}\)
(C) \(\frac{s(s + 3)}{(s + 3)^2 + 4}\)
(D) \(\frac{-s(s + 3)}{(s + 3)^2 + 4}\)

12. \( x(t) = t \frac{d}{dt} [e^{-\varepsilon t} \cos \theta u(t)] \)

(A) \(-\frac{(s^2 + 4s + 2)}{(s^2 + 2s + 2)^2}\)
(B) \(-\frac{(s^2 + 4s + 2)}{(s^2 + 2s + 2)^2}\)
(C) \(\frac{(s^2 + 4s + 2)}{(s^2 + 2s + 2)^2}\)
(D) \(-\frac{(s^2 + 2s + 2)}{(s^2 + 4s + 2)^2}\)

Statement for Q.13–24:

Determine the time signal \( x(t) \) corresponding to given \( X(s) \) and choose correct option.

13. \( X(s) = \frac{s + 3}{s^2 + 3s + 2} \)

(A) \( (2e^{-2t} + e^{-t})u(t) \)
(B) \( (2e^{-t} - e^{-2t})u(t) \)
(C) \( (2e^{-2t} - e^{-t})u(t) \)
(D) \( (2e^{-t} + e^{-2t})u(t) \)

14. \( X(s) = \frac{2s^2 + 10s + 11}{s^2 + 5s + 6} \)

(A) \( 2\delta(t) + (e^{-3t} - e^{-2t})u(t) \)
(B) \( 2\delta(t) + (e^{-2t} - e^{-3t})u(t) \)
(C) \( 2\delta(t) + (e^{-2t} + e^{-3t})u(t) \)
(D) \( 2\delta(t) - (e^{-2t} + e^{-3t})u(t) \)

15. \( X(s) = \frac{2s - 1}{s^2 + 2s + 1} \)

(A) \( (3e^{-t} - 2te^{-t})u(t) \)
(B) \( (3e^{-t} + 2te^{-t})u(t) \)
(C) \( (2e^{-t} - 3te^{-t})u(t) \)
(D) \( (2e^{-t} + 3te^{-t})u(t) \)

16. \( X(s) = \frac{5s + 4}{s^3 + 3s^2 + 2s} \)

(A) \( (2 + e^{-t} + 3e^{-2t})u(t) \)
(B) \( (2 + e^{-t} - 3e^{-3t})u(t) \)
(C) \( (3 + e^{-t} - 3e^{-3t})u(t) \)
(D) \( (3 + e^{-t} + 3e^{-3t})u(t) \)

17. \( X(s) = \frac{s^2 - 3}{(s + 2)(s^2 + 2s + 1)} \)

(A) \( (e^{-2t} - 2te^{-t})u(t) \)
(B) \( (e^{-2t} + 2te^{-t})u(t) \)
(C) \( (e^{-t} - 2te^{-2t})u(t) \)
(D) \( (e^{-t} + 2te^{-2t})u(t) \)

18. \( X(s) = \frac{3s + 2}{s^2 + 2s + 10} \)

(A) \( (3e^{-t} - \frac{1}{3}e^{-2t} + 3t)u(t) \)
(B) \( (3e^{-t} - \frac{1}{3}e^{-2t} + 3t)u(t) \)
(C) \( (3e^{-t} - \frac{1}{3}e^{-2t} + 3t)u(t) \)
(D) \( (3e^{-t} - \frac{1}{3}e^{-2t} + 3t)u(t) \)

19. \( X(s) = \frac{4s^2 + 8s + 10}{(s + 2)(s^2 + 2s + 5)} \)

(A) \( (2e^{-2t} + 2e^{-t} + 2e^{-2t})u(t) \)
(B) \( (2e^{-2t} - 2e^{-t} - 2e^{-2t})u(t) \)
(C) \( (2e^{-2t} - 2e^{-t} + 2e^{-2t})u(t) \)
(D) \( (2e^{-2t} - 2e^{-t} - 2e^{-2t})u(t) \)

20. \( X(s) = \frac{3s^2 + 10s + 10}{(s + 2)(s^2 + 6s + 10)} \)

(A) \( (e^{-2t} + 2e^{-3t} - 2e^{-2t} + 2e^{-3t})u(t) \)
(B) \( (e^{-2t} + 2e^{-3t} - 2e^{-2t} + 2e^{-3t})u(t) \)
(C) \( (e^{-2t} + 2e^{-3t} - 2e^{-2t} + 2e^{-3t})u(t) \)
(D) \( (e^{-2t} + 2e^{-3t} - 2e^{-2t} + 2e^{-3t})u(t) \)

21. \( X(s) = \frac{2s^2 + 11s + 16 + e^{2s}}{(s^2 + 5s + 6)} \)

(A) \( 2\delta(t) + (3e^{-2t} - 2e^{-3t})u(t - 2) \)
(B) \( 2\delta(t) + (2e^{-2t} - e^{-3t} + 3e^{-2t} - 2e^{-3t})u(t) \)
(C) \( 2\delta(t) + (2e^{-2t} - e^{-3t})u(t) + (e^{-2t} - e^{-3t})u(t - 2) \)
(D) \( 2\delta(t) + (2e^{-2t} - e^{-3t})u(t) + (e^{-2t} - e^{-3t})u(t - 2) \)

22. \( X(s) = s \frac{d^2}{ds^2} \left( \frac{1}{s^2 + 9} \right) + \frac{1}{s + 3} \)

(A) \( \left( e^{-3t} - \frac{2t}{3} \right) \cos 3t \)+ t^2 \cos 3t \)
(B) \( \left( e^{-3t} + \frac{2t}{3} \right) \cos 3t \)u(t)
(C) \( \left( e^{-3t} + \frac{2t}{3} \right) \sin 3t + t^2 \cos 3t \)u(t)
(D) \( \left( e^{-3t} - \frac{2t}{3} \right) \sin 3t + 2t \cos 3t \)u(t)
23. \( X(s) = \frac{1}{(2s + 1)^2 + 4} \)

(A) \( e^{-0.5t} \sin t \ u(t) \)  
(B) \( \frac{1}{2} e^{-t} \sin t \ u(t) \)  
(C) \( \frac{1}{4} e^{-0.5t} \sin t \ u(t) \)  
(D) \( e^{-t} \sin t \ u(t) \)

24. \( X(s) = e^{-2s} \frac{d}{ds} \left( \frac{1}{(s + 1)^2} \right) \)

(A) \( -e^{-t}u(1 - t) \)  
(B) \( -te^{-t}u(t - 1) \)  
(C) \( -(t - 2)e^{-t}u(t - 2) \)  
(D) \( te^{-t}u(t - 1) \)

**Statement for Q.25–29:**

Given the transform pair below, determine the time signal \( y(t) \) and choose correct option.

\( \cos 2t \ u(t) \iff X(s) \).

25. \( Y(s) = (s + 1)X(s) \)

(A) \( \cos 2t - 2 \sin 2t)u(t) \)  
(B) \( \left( \cos 2t + \frac{\sin 2t}{2} \right)u(t) \)  
(C) \( \cos 2t + 2 \sin 2t)u(t) \)  
(D) \( \left( \cos 2t - \frac{\sin 2t}{2} \right)u(t) \)

26. \( Y(s) = X(3s) \)

(A) \( \cos \left( \frac{2}{3} t \right)u(t) \)  
(B) \( \frac{1}{3} \cos \left( \frac{2}{3} t \right)u(t) \)  
(C) \( \cos 6t \ u(t) \)  
(D) \( \frac{1}{3} \cos 6t \ u(t) \)

27. \( Y(s) = X(s + 2) \)

(A) \( \cos 2(t - 2) \ u(t) \)  
(B) \( e^{2t} \cos 2t \ u(t) \)  
(C) \( \cos 2(t + 2) \ u(t) \)  
(D) \( e^{-2t} \cos 2t \ u(t) \)

28. \( Y(s) = \frac{X(s)}{s^2} \)

(A) \( 4 \cos 2t \ u(t) \)  
(B) \( \frac{1 - 2 \cos 2t}{4} \ u(t) \)  
(C) \( t^2 \cos 2t \ u(t) \)  
(D) \( \cos 2t \ \frac{t^2}{2} \ u(t) \)

29. \( Y(s) = \frac{d}{ds}[e^{-3s}X(s)] \)

(A) \( t \cos 2(t - 3) \ u(t - 3) \)  
(B) \( t \cos 2(t - 3) \ u(t) \)  
(C) \( -t \cos 2(t - 3) \ u(t - 3) \)  
(D) \( -t \cos 2(t - 3) \ u(t) \)

30. \( y(t) = x(t - 2) \)

(A) \( \frac{2se^{-2s}}{s^2 + 2} \)  
(B) \( \frac{2se^{2s}}{s^2 + 2} \)  
(C) \( \frac{2(s - 2)}{(s - 2)^2 + 1} \)  
(D) \( \frac{2(s + 2)}{(s + 2)^2 + 1} \)

31. \( y(t) = x(t) \ast \frac{dx(t)}{dt} \)

(A) \( \frac{4s^3}{(s^2 + 2)^2} \)  
(B) \( \frac{4}{(s^2 + 2)^2} \)  
(C) \( \frac{-4s^3}{(s^2 + 2)^2} \)  
(D) \( \frac{4}{(s^2 + 2)^2} \)

32. \( y(t) = e^{-t}x(t) \)

(A) \( \frac{2(s + 1)}{(s + 1)^2 + 2} \)  
(B) \( \frac{2(s + 1)}{s^2 + 2s + 2} \)  
(C) \( \frac{2(s + 1)}{s^2 + 2s + 4} \)  
(D) \( \frac{2(s + 1)}{s^2 + 2s} \)

33. \( y(t) = 2tx(t) \)

(A) \( \frac{8 - 4s^2}{(s^2 + 2)^2} \)  
(B) \( \frac{4s^2 - 8}{(s^2 + 2)^2} \)  
(C) \( \frac{4s^2}{s^2 + 1} \)  
(D) \( \frac{s^2}{s^2 + 1} \)

**Statement for Q.30–33:**

Given the transform pair

\( x(t)u(t) \iff \frac{2s}{s^2 + 2} \).

Determine the Laplace transform \( Y(s) \) of the given time signal in question and choose correct option.

34. \( x(t) = e^{-t}u(t + 2) \)

(A) \( \frac{e^{2(s + 1)}}{s + 1} \), \( \text{Re}(s) > -1 \)  
(B) \( \frac{1}{1 + s} \), \( \text{Re}(s) < -1 \)  
(C) \( \frac{e^{2(s + 1)}}{s + 1} \), \( \text{Re}(s) < -1 \)  
(D) \( \frac{1}{1 + s} \), \( \text{Re}(s) > -1 \)
35. \( x(t) = u(-t+3) \)
   (A) \( \frac{1 - e^{-3s}}{s} \), Re \( s > 0 \)
   (B) \( \frac{-e^{-3s}}{s} \), Re \( s < 0 \)
   (C) \( \frac{1 - e^{-3s}}{s} \), Re \( s < 0 \)
   (D) \( \frac{-e^{-3s}}{s} \), Re \( s > 0 \)

36. \( y(t) = \delta(t+1) \)
   (A) \( e^{t} \), Re \( s > 0 \)
   (B) \( e^{t} \), Re \( s < 0 \)
   (C) \( e^{t} \), all \( s \)
   (D) None of above

37. \( x(t) = \sin(t)u(t) \)
   (A) \( \frac{1}{1 + s^2} \), Re \( s < 0 \)
   (B) \( \frac{1}{1 + s^2} \), Re \( s > 0 \)
   (C) \( \frac{-1}{1 + s^2} \), Re \( s < 0 \)
   (D) \( \frac{-1}{1 + s^2} \), Re \( s > 0 \)

38. \( x(t) = e^{-t}u(t) + e^{-t}u(t) + e^{t}u(-t) \)
   (A) \( \frac{6s^2 + 2s - 2}{(2s + 1)(s^2 - 1)} \), Re \( s < -0.5 \)
   (B) \( \frac{6s^2 + 2s - 2}{(2s + 1)(s^2 - 1)} \), -1 < Re \( s > 1 \)
   (C) \( \frac{1}{s + 0.5} + \frac{1}{s + 1} + \frac{1}{s - 1} \), -1 < Re \( s < 1 \)
   (D) \( \frac{1}{s + 0.5} + \frac{1}{s + 1} - \frac{1}{s - 1} \), -0.5 < Re \( s < 1 \)

39. \( x(t) = e^{t} \cos(2t)u(-t) + e^{-t}u(t) + e^{t}u(t) \)
   (A) \( \frac{(1 - s)}{(s^2 - 1) + 4} + \frac{1}{s + 1} + \frac{1}{s - 0.5} \), 0.5 < Re \( s < 1 \)
   (B) \( \frac{(1 - s)}{(s^2 - 1) + 4} + \frac{1}{s + 1} + \frac{1}{s - 0.5} \), -1 < Re \( s < 1 \)
   (C) \( \frac{(s - 1)}{(s^2 - 1) + 4} + \frac{1}{s + 1} + \frac{1}{s - 0.5} \), 0.5 < Re \( s < 1 \)
   (D) \( \frac{(s - 1)}{(s^2 - 1) + 4} + \frac{1}{s + 1} + \frac{1}{s - 0.5} \), -1 < Re \( s < 1 \)

40. \( x(t) = e^{3t}u(t+3) \)
   (A) \( \frac{e^{5s}}{s - 3} \), Re \( s > 3 \)
   (B) \( \frac{e^{5s}}{s - 3} \), Re \( s < 3 \)
   (C) \( \frac{e^{3s-1}}{s - 3} \), Re \( s > 3 \)
   (D) \( \frac{e^{3s-1}}{s - 3} \), Re \( s < 3 \)

41. \( x(t) = \cos(3t)u(-t) * e^{-t}u(t) \)
   (A) \( \frac{-s}{(s + 1)(s^2 + 9)} \), Re \( s > 0 \)
   (B) \( \frac{-s}{(s + 1)(s^2 + 9)} \), -1 < Re \( s < 0 \)
   (C) \( \frac{s}{(s + 1)(s^2 + 9)} \), -1 < Re \( s < 0 \)
   (D) \( \frac{s}{(s + 1)(s^2 + 9)} \), Re \( s > 0 \)

42. \( x(t) = e^{t} \sin(2t + 2)u(t + 2) \)
   (A) \( \frac{e^{2(t+2)}}{(s - 1)^2 + 4} \), Re \( s > 1 \)
   (B) \( \frac{e^{2(t+2)}}{(s - 1)^2 + 4} \), Re \( s < 1 \)
   (C) \( \frac{e^{(t-2)}}{(s - 1)^2 + 4} \), Re \( s > 1 \)
   (D) \( \frac{e^{(t-2)}}{(s - 1)^2 + 4} \), Re \( s < 1 \)

43. \( x(t) = e^{t} \frac{d}{dt}[e^{-2t}u(-t)] \)
   (A) \( \frac{1 - s}{s + 1} \), Re \( s < -1 \)
   (B) \( \frac{1 - s}{s + 1} \), Re \( s > -1 \)
   (C) \( \frac{s - 1}{s + 1} \), Re \( s < -1 \)
   (D) \( \frac{s - 1}{s + 1} \), Re \( s > -1 \)

**Statement for Q.44–49:**

Determine the corresponding time signal for given bilateral Laplace transform.

44. \( X(s) = \frac{e^{5s}}{s + 2} \) with ROC: Re \( s < -2 \)
   (A) \( e^{-2(s-5)}u(t+5) \)

45.
32. A stable system has input \( x(t) \) and output \( y(t) = e^{-2t} \cos t u(t) \). The impulse response of the system is
(A) \( \delta(t) - (e^{-2t} \cos t + e^{-2t} \sin t)u(t) \)
(B) \( \delta(t) - (e^{-2t} \cos t + e^{-2t} \sin t)u(t - 2) \)
(C) \( \delta(t) - (e^{-2t} \cos t + e^{-2t} \sin t)u(t) \)
(D) \( \delta(t) - (e^{-2t} \cos t + e^{-2t} \sin t)u(t + 2) \)

33. The relationship between the input \( x(t) \) and output \( y(t) \) of a causal system is described by the differential equation
\[
\frac{dy(t)}{dt} + 10y(t) = 10x(t)
\]
The impulse response of the system is
(A) \(-10e^{-10t}u(-t + 10)\)
(B) \(10e^{-10t}u(t)\)
(C) \(10e^{-10t}u(-t + 10)\)
(D) \(-10e^{-10t}u(t)\)

34. The relationship between the input \( x(t) \) and output \( y(t) \) of a causal system is defined as
\[
\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = -4x(t) + 5 \frac{dx(t)}{dt}.
\]
The impulse response of the system is
(A) \(3e^{-t}u(t) + 2e^{2t}u(-t)\)
(B) \((3e^{-t} + 2e^{2t})u(t)\)
(C) \(3e^{-t}u(t) - 2e^{2t}u(-t)\)
(D) \((3e^{-t} - 2e^{2t})u(-t)\)

35. The transform function \( H(s) \) of a causal system is
\[
H(s) = \frac{2s^2 + 2s - 2}{s^3 - 1}
\]
The impulse response is
(A) \(2\delta(t) - (e^{-t} + e^t)u(-t)\)
(B) \(2\delta(t) - (e^{-t} + e^t)u(t)\)
(C) \(2\delta(t) + e^{-t}u(t) - e^t u(-t)\)
(D) \(2\delta(t) + (e^{-t} + e^t)u(t)\)

36. The transform function \( H(s) \) of a stable system is
\[
H(s) = \frac{2s - 1}{s^2 + 2s + 1}
\]
The impulse response is
(A) \(2u(-t + 1) - 3tu(-t + 1)\)
(B) \((3e^{-t} - 2e^{-2t})u(t)\)
(C) \(2u(t + 1) - 3tu(t + 1)\)
(D) \((2e^{-t} - 3e^{-2t})u(t)\)

37. The transform function \( H(s) \) of a stable system is
\[
H(s) = \frac{s^2 + 5s - 9}{(s + 1)(s^2 - 2s + 10)}
\]
The impulse response is
(A) \(-e^{-t}u(t) + (e^t \sin 3t + 2e^t \cos 3t)u(t)\)
(B) \(-e^{-t}u(t) - (e^t \sin 3t + 2e^t \cos 3t)u(-t)\)
(C) \(-e^{-t}u(t) - (e^t \sin 3t + 2e^t \cos 3t)u(t)\)
(D) \(-e^{-t}u(t) + (e^t \sin 3t + 2e^t \cos 3t)u(-t)\)
1. (B) \( X(s) = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty e^{-2t}dt = \frac{e^{-2s}}{s} \)

2. (A) \( X(s) = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty u(t+2)e^{-st}dt = e^{-3s}dt = \frac{1}{s+2} \)

3. (A) \( X(s) = \int_0^\infty e^{-\omega t}dt = \frac{1}{s+2} \)

4. (C) \( X(s) = \int_0^\infty x(t)e^{-st}dt = \int_0^\infty e^{-t}(t+2)e^{-st}dt = \frac{1}{s^2} - \frac{1}{s} \)

5. (C) \( X(s) = \int_0^\infty \frac{e^{\omega t} - e^{-\omega t}}{2j}e^{-st}dt = \frac{5}{s^2 + 25} \)

6. (B) \( X(s) = \int_0^\infty e^{-st}dt = \frac{1}{s} - \frac{e^{-2s}}{s} \)

7. (B) \( p(t) = te^{-t}u(t) \quad \xrightarrow{L} \quad P(s) = \frac{1}{(s+1)^2} \)

8. (A) \( p(t) = tu(t) \quad \xrightarrow{L} \quad P(s) = \frac{1}{s^2} \)

9. (A) \( q(t) = \cos 2\pi t u(t) \quad \xrightarrow{L} \quad Q(s) = \frac{s}{s^2 + 4\pi^2} \)

10. (D) \( p(t) = u(t) \quad \xrightarrow{L} \quad P(s) = \frac{1}{s} \)

11. (B) \( p(t) = e^{-3t}u(t-1) \quad \xrightarrow{L} \quad \frac{1}{s+3} \)

12. (A) \( p(t) = e^{-t}u(t) \quad \xrightarrow{L} \quad P(s) = \frac{s+1}{(s+1)^2 + 1} \)

13. (B) \( x(t) = [2e^{-t} - e^{-2t}]u(t) \)

14. (A) \( x(t) = 2 - \frac{1}{(s+2)(s+3)} = 2 - \frac{1}{s+2} + \frac{1}{s+3} \)

15. (C) \( x(t) = 2\delta(t) + (e^{-3t} - e^{-2t})u(t) \)

16. (B) \( x(t) = \left[ sX(s) \right]_{s=0} = 2, \quad B = \left[ (s+1)X(s) \right]_{s=0} = 1 \)

17. (C) \( x(t) = \left[ \frac{s^2 - 3}{(s+2)(s^2 + 2s + 1)} \right] \)

\[ x(t) = \frac{A}{s+2} + \frac{B}{s+1} + \frac{C}{(s+1)^2} \]
A = (s + 2)X(s) \bigg|_{s=2} = 1, \ C = (s + 1)^2 X(s) \bigg|_{s=2} = -2
A + B = 1 \Rightarrow B = 0
x(t) = [e^{-2t} - t^2]u(t)

18. (A) \quad X(s) = \frac{3s^2 + 2}{(s^2 + 2s + 10) = \frac{3(s+1)^2}{(s+1)^2 + 3^2} - \frac{1}{(s+1)^2 + 3^2}}
x(t) = \left[3e^{-t} \cos 3t - \frac{1}{3} e^{-t} \sin 3t\right]u(t)

19. (C) \quad X(s) = \frac{4s^2 + 8s + 10}{(s + 2)(s^2 + 2s + 5)}
= \frac{A}{(s + 2)} + \frac{B(s + 1) + C}{(s + 3)^2 + 1}
A = (s + 2)X(s) \bigg|_{s=2} = 2
A + B = 4 \Rightarrow B = 2
5A + 2B + 2C = 10 \Rightarrow C = -2
x(t) = [2e^{-2t} + 2e^{-3t} \cos 2t - e^{-2t} \sin 2t]u(t)

20. (B) \quad X(s) = \frac{3s^2 + 10s + 10}{(s + 2)(s^2 + 6s + 10)}
= \frac{A}{(s + 2)} + \frac{B(s + 3) + C}{(s + 3)^2 + 1}
A = (s + 2)X(s) \bigg|_{s=2} = 1, \ A + B = 3 \Rightarrow B = 2
10A + 6B + 2C = 10 \Rightarrow C = -6
x(t) = [e^{-2t} + 2e^{-3t} \cos t - 6e^{-2t} \sin t]u(t)

21. (D) \quad X(s) = \frac{2s^2 + 11s + 16 + e^{-2s}}{(s^2 + 5s + 6)}
= 2 + \frac{A}{(s + 2)} + \frac{B}{(s + 3)} + \frac{e^{-2s}}{(s + 2)} - \frac{e^{-2s}}{(s + 3)}
A = (s + 2)X(s) \bigg|_{s=2} = 2
B = (s + 3)(2s^2 + 11s + 16) \bigg|_{s=3} = -1
x(t) = 2\delta(t) + [2e^{-2t} - e^{-3t}]u(t) + [e^{-2(t-1)} - e^{-3(t-2)}]u(t - 2)

22. (C) \quad P(s) = \frac{1}{s^2 + 9} \quad \xrightarrow{L} \quad p(t) = \frac{1}{3} \sin 3t u(t)
Q(s) = \frac{d^2}{ds^2} P(s) \quad \xrightarrow{L} \quad q(t) = (-1)^2 t^2 p(t) = \frac{t^2}{3} \sin 3t u(t)
R(s) = sQ(s) \quad \xrightarrow{L} \quad r(t) = \frac{d}{dt} q(t) - q(0^-)
= 2t \sin 3t u(t) + t^2 \cos 3t u(t)
V(s) = \frac{1}{s + 3} \quad \xrightarrow{L} \quad v(t) = e^{-3t} u(t)
x(t) = v(t) + r(t) = \left[\frac{2t}{3} \sin 3t u(t) + t^2 \cos 3t u(t) + e^{-3t}\right]u(t)

23. (C) \quad P\left(\frac{s}{a}\right) \quad \xrightarrow{L} \quad \frac{1}{a^2} \sin 2t u(t)
\quad \quad = \frac{1}{(s + 1)^2 + 4} \quad \xrightarrow{L} \quad \frac{1}{2} e^{-t} \sin 2t u(t)
x(t) \quad \xrightarrow{L} \quad \frac{1}{4} e^{-\frac{3}{2}t} \sin t u(t)

24. (C) \quad P(s) = \frac{1}{(s + 1)^2} \quad \xrightarrow{L} \quad p(t) = te^{-t} u(t)
Q(s) = \frac{d}{ds} P(s) \quad \xrightarrow{L} \quad q(t) = -tp(t) = -t^2 e^{-t} u(t)
X(s) = e^{-2}\cdot Q(s) \quad \xrightarrow{L} \quad x(t) = q(t - 2)
\Rightarrow x(t) = -(t - 2) e^{(t-2)} u(t - 2)

25. (A) sX(s) + X(s) \quad \xrightarrow{L} \quad \frac{dx(t)}{dt} + x(t)
Y(s) \quad \xrightarrow{L} \quad \frac{1}{3} \sin \left(\frac{2}{3} t\right) u(t)

26. (B) \quad X\left(\frac{s}{a}\right) \quad \xrightarrow{L} \quad ax(at)
X(3s) \quad \xrightarrow{L} \quad \frac{1}{3} \cos \left(\frac{2}{3} t\right) u(t)

27. (D) \quad X(s + 2) \quad \xrightarrow{L} \quad e^{2t} x(t)

28. (B) \quad P(s) = \frac{X(s)}{s} \quad \xrightarrow{L} \quad \int x(\tau) d\tau
\quad \xrightarrow{L} \quad \frac{1}{2} \sin \frac{2\pi}{2} = \frac{1 - \cos 2t}{4} u(t),

29. (C) \quad P(s) = e^{-s} X(s) \quad \xrightarrow{L} \quad p(t) = x(t - 3)
= \cos 2(t - 3) u(t - 3)
Q(s) = \frac{d}{ds} P(s) \quad \xrightarrow{L} \quad q(t) = -p(t)
= -\cos 2(t - 3) u(t - 3).

30. (A) \quad x(t - 2) \quad \xrightarrow{L} \quad e^{-2t} X(s), \quad Y(s) = \frac{2s e^{-2s}}{s^3 + 2}

31. (A) \quad p(t) = \frac{d}{dt} x(t) \quad \xrightarrow{L} \quad P(s) = sX(s)
Y(t) = x(t) \ast p(t) \quad \xrightarrow{L} \quad Y(s) = P(s) X(s) = s^2 (X(s))^2

32. (A) \quad e^{-t} x(t) \quad \xrightarrow{L} \quad X(s + 1) = \frac{2(s + 1)}{(s + 1)^2 + 2}
33. (B) 2tx(t)  \xrightarrow{L} -2 \frac{d}{ds} X(s) = \frac{4s^2 - 8}{(s^2 + 2)^2} \\

34. (A) X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \\
= \int_{-\infty}^{0} e^{-t}e^{-st}dt + \int_{0}^{\infty} e^{-t}e^{-st}dt = \frac{e^{2t+1}}{s+1}, \quad \text{Re}(s) > -1 \\

35. (B) X(s) = \int_{-\infty}^{\infty} u(t-3)e^{-st}dt = \frac{3}{s} e^{-3s} dt = \frac{e^{-3s}}{s} \quad \text{Re}(s) < 0 \\

36. (C) Y(s) = \int_{-\infty}^{\infty} (t+1)e^{-st}dt = e^s, \quad \text{All } s \\

37. (B) X(s) = \int_{0}^{\infty} (e^s - e^{-\beta})e^{-st}dt \\
= \frac{1}{2j} \int_{0}^{\infty} e^{(j-\beta)} e^{-st}dt - \frac{1}{2j} \int_{0}^{\infty} e^{(j+\beta)} e^{-st}dt = \frac{1}{1+s^2}, \quad \text{Re}(s) > 0 \\

38. (D) X(s) = \int_{0}^{\infty} e^{-t}e^{-st}dt + \int_{0}^{\infty} e^{-t}e^{-st}dt + \int_{-\infty}^{0} e^{t}e^{-st}dt \\
= \frac{1}{s+0.5} + \frac{1}{s+1} - \frac{1}{s-1} \\
\text{Re}(s) > -0.5, \quad \text{Re}(s) > -1, \quad \text{Re}(s) > 1 \\
\Rightarrow -0.5 < \text{Re}(s) < 1 \\

39. (A) X(s) = \int_{-\infty}^{\infty} e^{i(\beta - \gamma)}e^{-st}dt + \int_{0}^{\infty} e^{i(\beta - \gamma)}e^{-st}dt + \int_{-\infty}^{0} e^{i(\beta - \gamma)}e^{-st}dt \\
\text{Re}(s) < 0, \quad \text{Re}(s) > 0, \quad \text{Re}(s) > 0.5 \\
\text{Therefore} \quad 0.5 < \text{Re}(s) < 1 \\
X(s) = -\frac{s-1}{(s-1)^2 + 4} + \frac{1}{s+1} + \frac{1}{s-0.5} \\

40. (C) x(t) = e^{-3}e^{-3(t-3)}u(t+3) \\
p(t) = e^u(t) \quad \xrightarrow{L} P(s) = \frac{1}{s-3} \\
q(t) = p(t+3) \quad \xrightarrow{L} Q(s) = e^{3}P(s) = \frac{e^{3s}}{s-3} \\
X(s) = \frac{e^{3(s-3)}}{s-3}, \quad \text{Re}(s) > 3 \\

41. (B) p(t) * q(t) \quad \xrightarrow{L} P(s)Q(s) \\
X(s) = -\frac{s}{s^2 + 9 + \frac{1}{s+1}} \\
\text{Re}(s) > -1, \quad \text{Re}(s) < 0 \\
\Rightarrow -1 < \text{Re}(s) < 0 \\

42. (A) x(t) = e^{-t}e^{i-\frac{1}{2}} \sin(2t + 4)\ u(t + 2) \\
p(t+2) \quad \xrightarrow{L} e^{2}P(s) \\
X(s) = \frac{e^{-2}e^{-2}}{(s-1)^2 + 4}, \quad \text{Re}(s) > 1 \\

43. (A) p(t) = e^{-21u} (-t) \quad \xrightarrow{L} P(s) = \frac{1}{s+2} \quad \text{Re}(s) < -2 \\
q(t) = \frac{d}{dt} p(t) \quad \xrightarrow{L} Q(s) = sP(s) \\
x(t) = e^t q(t) \quad \xrightarrow{L} X(s) = Q(s-1) = \frac{1-s}{1+s} \\
\text{Re}(s) < -1 \text{ thus left-sided} . \\

44. (C) Left-sided \\
P(s) = \frac{1}{s+2} \quad \xrightarrow{L} \ p(t) = -e^{-21u}(-t) \\
X(s) = e^uP(s) \quad \xrightarrow{L} \ x(t) = p(t + 5) \\
\Rightarrow \ x(t) = -e^{-21u}(-t + 5) \\

45. (A) Right-sided \\
P(s) = \frac{1}{(s-3)} \quad \xrightarrow{L} \ p(t) = e^{-}u(t) \\
X(s) = \frac{1}{(s-3)} \quad \xrightarrow{L} \ x(t) = t^2 e^{-}u(t) \\

46. (D) Left-sided \\
x(t) = -u(-t) + u(-t + 1) + \delta(t + 2) \\

47. (C) Right-sided, \ P(s) = \frac{1}{s} \quad \xrightarrow{L} \ p(t) = u(t) \\
Q(s) = e^{-3}P(s) \quad \xrightarrow{L} q(t) = p(t - 3) = u(t - 3) \\
R(s) = \frac{d}{ds} Q(s) \quad \xrightarrow{L} r(t) = -q(t) = -u(t - 3) \\
V(s) = \frac{1}{s} R(s) \quad \xrightarrow{L} v(t) = \int r(t)\,dt \\
\Rightarrow v(t) = \frac{1}{3} t^3 = -\frac{1}{2} (t^2 - 9) \\
X(s) = \frac{1}{s} v(s) \quad \xrightarrow{L} x(t) = -\frac{1}{2} (t^2 - 9) \\

48. (B) X(s) = \frac{-3 - 2}{s^2 + 3s + 2} = \frac{-3}{s+1} + \frac{2}{s+2} \\
\text{Left-sided, } x(t) = 3e^{-t}u(-t) - 2e^{-2t}u(-t) \\

49. (A) X(s) = \frac{5}{(s+1)^2} \quad \xrightarrow{L} \ x(t) = -5u(-t) + te^{-t}u(-t) \\
\text{Left-sided, } x(t) = -5u(-t) + te^{-t}u(-t)
50. (D) \( x(0^+) = \lim_{s \to -}\frac{s}{s^2 + 5s - 2} = 0 \)

51. (A) \( x(0^+) = \lim_{s \to -}\frac{s^2 + 2s}{s^2 + 2s - 3} = 1 \)

52. (D) \( x(0^+) = \lim_{s \to a}\frac{s^2 + 3s}{s^2 + 2s - 2} = 1 \)

53. (A) \( x(\infty) = \lim_{s \to 0}\frac{2s^3 + 3s}{s^2 + 5s + 1} = 0 \)

54. (C) \( x(\infty) = \lim_{s \to 0}\frac{s + 2}{s^2 + 3s + 1} = 2 \)

55. (B) \( x(\infty) = \lim_{s \to 0}\frac{e^{-3s}(2s^3 + 1)}{s^2 + 5s + 4} = \frac{1}{4} \)

56. (C) \( sY(s) - y(0^-) + 10Y(s) = 10(s) \)
\( y(0^-) = 1, \quad X(s) = \frac{1}{s} \)
\( Y(s) = \frac{10}{s(s + 1)} + \frac{1}{s + 1} = \frac{1}{s} \)
\( \Rightarrow y(t) = u(t) \)

57. (C) \( s^2Y(s) - 2s + 2sY(s) - 2 + 5Y(s) = 1 \)
\( (s^2 + 2s + 5)Y(s) = 3 + 2s \)
\( Y(s) = \frac{2s + 3}{s^2 + 2s + 5} = \frac{2s + 1}{(s + 1)^2 + 2^2} + \frac{1}{(s + 1)^2 + 2^2} \)
\( \Rightarrow y(t) = 2e^{-t}\sin t \cos t u(t) + \frac{1}{2}e^{-t}\sin t u(t) \)

58. (B) \( s^3Y(s) + 4s^2Y(s) + 3sY(s) = \frac{10}{s + 2} \)
\( Y(s) = \frac{10}{s(s + 1)(s + 2)(s + 3)} = \frac{A}{s} + \frac{B}{s + 1} + \frac{C}{s + 2} + \frac{D}{s + 3} \)
\( A = sY(s) \bigg|_{s=0} = \frac{5}{3}, \quad B = (s + 1)Y(s) \bigg|_{s=-1} = -5, \)
\( C = (s + 2)Y(s) \bigg|_{s=-2} = 5, \quad D = (s + 3)Y(s) \bigg|_{s=-3} = \frac{5}{3} \)
\( \Rightarrow y(t) = \left[ \frac{5}{3} - 5e^{-t} + 5e^{-2t} + \frac{5}{3}e^{-3t} \right] u(t) \)

59. (D) For a causal system \( h(t) = 0 \) for \( t < 0 \)
\( H(s) = 2 + \frac{1}{s + 1} + \frac{1}{s - 1} \)
\( \Rightarrow h(t) = 2\delta(t) + (e^{-t} + e^t)u(t) \)

60. (D) \( H(s) = \frac{2}{s + 1} - \frac{3}{(s + 1)^2}, \) System is stable
CHAPTER 5.4

THE Z-TRANSFORM

Statement for Q.1-12:

Determine the z-transform and choose correct option.

1. \(x[n] = \delta(n - k), k > 0\)
   - (A) \(z^k, \ z > 0\)
   - (B) \(z^{-k}, \ z > 0\)
   - (C) \(z^k, \ z \neq 0\)
   - (D) \(z^{-k}, \ z \neq 0\)

2. \(x[n] = \delta(n + k), k > 0\)
   - (A) \(z^{-k}, \ z \neq 0\)
   - (B) \(z^k, \ z \neq 0\)
   - (C) \(z^k, \ \text{all } z\)
   - (D) \(z^{-k}, \ \text{all } z\)

3. \(x[n] = u[n]\)
   - (A) \(\frac{1}{1 - z^{-1}}, \ |z| > 1\)
   - (B) \(\frac{1}{1 - z^{-1}}, \ |z| < 1\)
   - (C) \(\frac{z}{1 - z^{-1}}, \ |z| < 1\)
   - (D) \(\frac{z}{1 - z^{-1}}, \ |z| > 1\)

4. \(x[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n-5])\)
   - (A) \(\frac{z^5 - 0.25^5}{z^5(z - 0.25)}, \ z > 0.25\)
   - (B) \(\frac{z^5 - 0.25^5}{z^5(z - 0.25)}, \ z > 0\)
   - (C) \(\frac{z^5 - 0.25^5}{z^5(z - 0.25)}, \ z < 0.25\)
   - (D) \(\frac{z^5 - 0.25^5}{z^5(z - 0.25)}, \ \text{all } z\)

5. \(x[n] = \left(\frac{1}{4}\right)^n u[-n]\)
   - (A) \(\frac{4z}{4z - 1}, \ |z| > \frac{1}{4}\)
   - (B) \(\frac{4z}{4z - 1}, \ |z| < \frac{1}{4}\)
   - (C) \(\frac{1}{1 - 4z}, \ |z| > \frac{1}{4}\)
   - (D) \(\frac{1}{1 - 4z}, \ |z| < \frac{1}{4}\)

6. \(x[n] = \left(\frac{2}{3}\right)^n\)
   - (A) \(-\frac{5z}{(2z - 3)(3z - 2)}, \ -\frac{3}{2} < z < -\frac{2}{3}\)
   - (B) \(-\frac{5z}{(2z - 3)(3z - 2)}, \ \frac{2}{3} < |z| < \frac{3}{2}\)
   - (C) \(\frac{5z}{(2z - 3)(3z - 2)}, \ \frac{2}{3} < |z| < \frac{3}{2}\)
   - (D) \(\frac{5z}{(2z - 3)(3z - 2)}, \ -\frac{3}{2} < z < -\frac{2}{3}\)

7. \(x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[-n - 1]\)
   - (A) \(\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - \frac{1}{4} z^{-1}}, \ \frac{1}{4} < |z| < \frac{1}{2}\)
   - (B) \(\frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{1}{1 - \frac{1}{4} z^{-1}}, \ \frac{1}{4} < |z| < \frac{1}{2}\)
   - (C) \(\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - \frac{1}{4} z^{-1}}, \ |z| > \frac{1}{2}\)
   - (D) None of the above
33. \( X(z) = \frac{1}{1 - 4z^{-2}}, \ |z| < \frac{1}{2} \)

(A) \(-\sum_{k=0}^{\infty} 2^{2(k+1)}z^{-n-2(k+1)}\)

(B) \(-\sum_{k=0}^{\infty} 2^{2(k+1)}z^{-n+2(k+1)}\)

(C) \(-\sum_{k=0}^{\infty} 2^{2(k+1)}z^{n+2(k+1)}\)

(D) \(-\sum_{k=0}^{\infty} 2^{2(k+1)}z^{n-2(k+1)}\)

34. \( X(z) = \ln(1 + z^{-1}), \ |z| > 0 \)

(A) \(\frac{(-1)^{k-1}}{k} z^{-n+1}\)

(B) \(\frac{(-1)^{k-1}}{k} z^{n+1}\)

(C) \(\frac{(-1)^{k}}{k} z^{-n+1}\)

(D) \(\frac{(-1)^{k}}{k} z^{n+1}\)

35. If z-transform is given by

\( X(z) = \cos(z^{-3}), \ |z| > 0, \)

The value of \(x[12]\) is

(A) \(-\frac{1}{24}\)

(B) \(\frac{1}{24}\)

(C) \(-\frac{1}{6}\)

(D) \(\frac{1}{6}\)

36. \( X(z) \) of a system is specified by a pole zero pattern in fig. P.5.4.36.

Consider three different solution of \(x[n]\)

\( x_1[n] = 2^n - \left(\frac{1}{3}\right)^n u[n] \)

\( x_2[n] = -2^n u[n-1] - \frac{1}{3^n} u[n] \)

\( x_3[n] = -2^n u[n-1] + \frac{1}{3^n} u[-n-1] \)

Correct solution is

(A) \(x_1[n]\)

(B) \(x_2[n]\)

(C) \(x_3[n]\)

(D) All three

37. Consider three different signal

\( x_1[n] = \left[ 2^n - \left(\frac{1}{2}\right)^n \right] u[n] \)

\( x_2[n] = -2^n u[-n-1] + \frac{1}{2^n} u[-n-1] \)

\( x_3[n] = -2^n u[-n-1] - \frac{1}{2^n} u[n] \)

Fig. P.5.4.37 shows the three different region. Choose the correct option for the ROC of signal

\( R_1 \quad R_2 \quad R_3 \)

(A) \(x_1[n] \quad x_2[n] \quad x_3[n]\)

(B) \(x_2[n] \quad x_3[n] \quad x_1[n]\)

(C) \(x_1[n] \quad x_3[n] \quad x_2[n]\)

(D) \(x_3[n] \quad x_2[n] \quad x_1[n]\)

38. Given

\( X(z) = \left\{ \frac{1 + \frac{7}{6}z^{-1}}{1 - \frac{1}{2}z^{-1} - \left(\frac{1}{3}\right)^n} \right\} \)

For three different ROC consider there different solution of signal \(x[n]\):

(a) \(|z| > \frac{1}{2}, \ x[n] = \left[ \frac{1}{2^n - \left(\frac{1}{3}\right)^n} \right] u[n]\)

(b) \(|z| < \frac{1}{3}, \ x[n] = \left[ -\frac{1}{2^n - \left(\frac{1}{3}\right)^n} + \left(\frac{1}{3}\right)^n \right] u[n-1]\)

(c) \(\frac{1}{3} < |z| < \frac{1}{2}, \ x[n] = -\frac{1}{2^n - \left(\frac{1}{3}\right)^n} u[-n-1] - \left(\frac{1}{3}\right)^n u[n]\)

Correct solutions are

(A) (a) and (b)

(B) (a) and (c)

(C) (b) and (c)

(D) (a), (b), (c)
39. \( X(z) \) has poles at \( z = 1/2 \) and \( z = -1 \). If \( x[1]=1 \) \( x[-1]=1 \), and the ROC includes the point \( z = 3/4 \). The time signal \( x[n] \) is
(A) \( \frac{1}{2^{n-1}} u[n] - (-1)^n u[-n-1] \)
(B) \( \frac{1}{2^n} u[n] - (-1)^n u[-n-1] \)
(C) \( \frac{1}{2^{n-1}} u[n] + u[-n+1] \)
(D) \( \frac{1}{2^n} u[n] + u[-n+1] \)

40. \( x[n] \) is right-sided, \( X(z) \) has a signal pole, and \( x[0]=2 , x[2]=1/2 \). \( x[n] \) is
(A) \( \frac{u[-n]}{2^{n+1}} \)
(B) \( \frac{u[n]}{2^{n+1}} \)
(C) \( \frac{u[-n]}{2^{n+1}} \)
(D) \( \frac{u[n]}{2^{n+1}} \)

41. The \( z \)-transform function of a stable system is given as
\[
H(z) = \frac{2 - \frac{3}{2} z^{-1}}{(1 - 2z^{-1})(1 + \frac{1}{2} z^{-1})}
\]

The impulse response \( h[n] \) is
(A) \( 2^n u[-n+1] - \left(\frac{1}{2}\right)^n u[n] \)
(B) \( 2^n u[-n-1] + \left(\frac{1}{2}\right)^n u[n] \)
(C) \( -2^n u[-n-1] + \left(\frac{1}{2}\right)^n u[n] \)
(D) \( 2^n u[n] - \left(\frac{1}{2}\right)^n u[n] \)

42. Let \( x[n] = \delta[n-2] + \delta[n+2] \). The unilateral \( z \)-transform is
(A) \( z^{-2} \)
(B) \( z^2 \)
(C) \( -2z^{-2} \)
(D) \( 2z^2 \)

43. The unilateral \( z \)-transform of signal \( x[n] = u[n+4] \) is
(A) \( 1 + z + z^2 + 3z + z^4 \)
(B) \( \frac{1}{1 - z} \)
(C) \( 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} \)
(D) \( \frac{1}{1 - z^{-1}} \)

44. The transfer function of a causal system is given as
\[
H(z) = \frac{5z^2}{z^2 - z - 6}
\]

The impulse response is
(A) \( (3^n + (-1)^n 2^{n-1})u[n] \)
(B) \( (3^{n+1} + 2(-2)^n)u[n] \)
(C) \( (3^{n-1} + (-1)^n 2^{n+1})u[n] \)
(D) \( (3^{n-1} - 2^{n-1})u[n] \)

45. A causal system has input
\[
x[n] = \delta[n] + \frac{1}{4} \delta[n-1] - \frac{1}{8} \delta[n-2]
\]
and output
\[
y[n] = \delta[n] - \frac{3}{4} \delta[n-1].
\]

The impulse response of this system is
(A) \( \frac{1}{3} \left[ 5 \left(\frac{1}{2}\right)^n - 2 \left(\frac{1}{4}\right)^n \right] u[n] \)
(B) \( \frac{1}{3} \left[ 5 \left(\frac{1}{2}\right)^n + 2 \left(\frac{1}{4}\right)^n \right] u[n] \)
(C) \( \frac{1}{3} \left[ 5 \left(\frac{1}{2}\right)^n - 2 \left(\frac{1}{4}\right)^n \right] u[n] \)
(D) \( \frac{1}{3} \left[ 5 \left(\frac{1}{2}\right)^n + 2 \left(\frac{1}{4}\right)^n \right] u[n] \)

46. A causal system has input \( x[n] = (-3)^n u[n] \) and output
\[
y[n] = \left[ 4(2)^n - \left(\frac{1}{2}\right)^n \right] u[n].
\]

The impulse response of this system is
(A) \( \left[ 7 \left(\frac{1}{2}\right)^n - 10 \left(\frac{1}{2}\right) \right] u[n] \)
(B) \( \left[ 7(2)^n - 10 \left(\frac{1}{2}\right)^n \right] u[n] \)
(C) \( \left[ 10 \left(\frac{1}{2}\right)^2 - 7(2)^n \right] u[n] \)
(D) \( \left[ 10(2)^n - 7 \left(\frac{1}{2}\right)^n \right] u[n] \)

47. A system has impulse response
\[
h[n] = \frac{1}{2^n} u[n]
\]

The output \( y[n] \) to the input \( x[n] \) is given by
\[
y[n] = 2 \delta[n - 4]. \quad \text{The input } x[n] \text{ is}
(A) 2\delta[-n - 4] - \delta[-n - 5]
(B) 2\delta[n + 4] - \delta[n + 5]
(C) 2\delta[-n + 4] - \delta[-n + 5]
(D) 2\delta[n - 4] - \delta[n - 5]
48. A system is described by the difference equation
\[ y[n] - \frac{1}{2} y[n-1] = 2x[n-1] \]

The impulse response of the system is
(A) \[ \frac{-1}{2^{n-2}} u[n-1] \]
(B) \[ \frac{1}{2^{n-2}} u[n+1] \]
(C) \[ \frac{1}{2^{n-2}} u[n-2] \]
(D) \[ \frac{-1}{2^{n-2}} u[n-2] \]

49. A system is described by the difference equation

The impulse response of system is
(A) \[ \alpha[n] - 2\alpha[n+2] + 4\alpha[n+4] - 6\alpha[n+6] \]
(B) \[ \alpha[n] + 2\alpha[n-2] - 4\alpha[n-4] + 6\alpha[n-6] \]
(C) \[ \alpha[n] - \alpha[n-2] + \alpha[n-4] - \alpha[n-6] \]
(D) \[ \alpha[n] - \alpha[n+2] + \alpha[n+4] - \alpha[n+6] \]

50. The impulse response of a system is given by
\[ h[n] = \frac{3}{4^n} u[n-1] \]

The difference equation representation for this system is
(A) \[ 4y[n] - y[n-1] = 3x[n-1] \]
(B) \[ 4y[n] - y[n+1] = 3x[n+1] \]
(C) \[ 4y[n] + y[n-1] = -3x[n-1] \]
(D) \[ 4y[n] + y[n+1] = 3x[n+1] \]

51. The impulse response of a system is given by
\[ h[n] = \alpha[n] - \alpha[n-5] \]

The difference equation representation for this system is
(A) \[ y[n] = x[n] - x[n-5] \]
(B) \[ y[n] = x[n] - x[n+5] \]
(C) \[ y[n] = x[n] + 5x[n-5] \]
(D) \[ y[n] = x[n] - 5x[n+5] \]

52. The transfer function of a system is given by
\[ H(z) = \frac{z(3z-2)}{z^2 - z - \frac{1}{4}} \]

The system is
(A) Causal and Stable
(B) Causal, Stable and minimum phase
(C) Minimum phase
(D) None of the above

53. The z-transform of a signal \( x[n] \) is given by
\[ X(z) = \frac{3}{1 - \frac{10}{3}z^{-1} + z^{-2}} \]

If \( X(z) \) converges on the unit circle, \( x[n] \) is
(A) \[ \frac{-1}{3^{n-1}} 8u[n] - \frac{3^{n-1}}{8} u[-n-1] \]
(B) \[ \frac{1}{3^{n-1}} 8u[n] - \frac{3^{n-1}}{8} u[-n-1] \]
(C) \[ \frac{1}{3^{n-1}} 8u[n] - \frac{3^{n+1}}{8} u[-n] \]
(D) \[ \frac{1}{3^{n-1}8} u[n] - \frac{3^{n+1}}{8} u[-n] \]

54. The transfer function of a system is given as
\[ H(z) = \frac{2\left(z + \frac{1}{2}\right)}{\left(z - 1\right)\left(2z - \frac{1}{2}\right)} \]

The \( h[n] \) is
(A) Stable
(B) Causal
(C) Stable and Causal
(D) None of the above

55. The transfer function of a system is given as
\[ H(z) = \frac{2\left(z + \frac{1}{2}\right)}{\left(z - 1\right)\left(2z - \frac{1}{2}\right)} \]

Consider the two statements
Statement (i) : System is causal and stable.
Statement (ii) : Inverse system is causal and stable.

The correct option is
(A) (i) is true
(B) (ii) is true
(C) Both (i) and (ii) are true
(D) Both are false

56. The impulse response of a system is given by
\[ h[n] = 10\left(-\frac{1}{2}\right)^n u[n] - 9\left(-\frac{1}{4}\right)^n u[n] \]

For this system two statement are
Statement (i) : System is causal and stable
Statement (ii) : Inverse system is causal and stable.
The correct option is
(A) (i) is true  (B) (ii) is true
(C) Both are true  (D) Both are false

57. The system
\[ y[n] = cy[n-1] - 0.12y[n-2] + x[n-1] + x[n-2] \]
is stable if
(A) \( c < 1.12 \)  (B) \( c > 1.12 \)
(C) \( |c| < 1.12 \)  (D) \( |c| > 1.12 \)

58. Consider the following three systems
\[ y_1[n] = 0.2y[n-1] + x[n] - 0.3x[n-1] + 0.02x[n-2] \]
\[ y_2[n] = x[n] - 0.1x[n-1] \]
\[ y_3[n] = 0.5y[n-1] + 0.4x[n] - 0.3x[n-1] \]
The equivalent system are
(A) \( y_1[n] \) and \( y_2[n] \)  (B) \( y_2[n] \) and \( y_3[n] \)
(C) \( y_3[n] \) and \( y_1[n] \)  (D) all

59. The \( z \)-transform of a causal system is given as
\[ X(z) = \frac{2 - 15z^{-1}}{1 - 15z^{-1} + 0.5z^{-2}} \]
The \( x[0] \) is
(A) \(-1.5\)  (B) \(2\)
(C) \(1.5\)  (D) \(0\)

60. The \( z \)-transform of an anti causal system is
\[ X(z) = \frac{12 - 21z}{3 - 7z + 12z^2} \]
The value of \( x[0] \) is
(A) \(-\frac{7}{4}\)  (B) \(0\)
(C) \(4\)  (D) Does not exist

61. Given the \( z \)-transforms
\[ X(z) = \frac{z(8z - 7)}{4z^2 - 7z + 3} \]
The limit of \( x[\infty] \) is
(A) \(1\)  (B) \(2\)
(C) \(\infty\)  (D) \(0\)

62. The impulse response of the system shown in fig. P5.4.62 is
![Fig. P5.4.62](image)
(A) \( 2^{\left(\frac{n}{2}\right)} (1 + (-1)^n)u[n] + \frac{1}{2} \delta[n] \)
(B) \( \frac{2^n}{2} (1 + (-1)^n)u[n] + \frac{1}{2} \delta[n] \)
(C) \( 2^{\left(\frac{n}{2}\right)} (1 + (-1)^n)u[n] - \frac{1}{2} \delta[n] \)
(D) \( \frac{2^n}{2} [1 + (-1)^n]u[n] - \frac{1}{2} \delta[n] \)

63. The system diagram for the transfer function
\[ H(z) = \frac{z}{z^2 + z + 1} \]
is shown in fig. P5.4.63. This system diagram is a
![Fig. P5.4.63](image)
(A) Correct solution  (B) Not correct solution
(C) Correct and unique solution  (D) Correct but not unique solution

***************
24. (A) \( X(z) = \frac{z^2 - 3z}{z^2 + 3z^{-1} - 1 + \frac{3}{2}z^{-1} - z^{-2}} = \frac{1 - 3z^{-1}}{1 + \frac{3}{2}z^{-1} - z^{-2}} \)
   
   \[
   = \frac{2}{1 + 2z^{-1}} - \frac{1}{1 - z^{-1}}, \text{ ROC } \frac{1}{2} < |z| < 2
   \]
   
   \( x[n] = -2(2)^n u[-n - 1] - \frac{1}{2^n} u[n] \)

25. (A) \( x[n] \) is right sided

\[
X(z) = \frac{z - \frac{1}{z^{-1}}}{1 - \frac{4}{16}z^{-1}} = \frac{49}{32} + \frac{47}{32}z^{-1}
\]

\( x[n] = [\frac{49}{32}(-4)^n + \frac{47}{32}4^n]u[n] \)

26. (C) \( x[n] \) is right sided

\[
X(z) = \left(2 + \frac{1}{1 + z^{-1}} - \frac{1}{1 - z^{-1}}\right)z^2
\]

\( x[n] = 2\delta[n + 2] + (-1)^n - u[n + 2] \)

27. (A) \( \bar{a}[n] + 2\bar{a}[n - 6] + 4\bar{a}[n - 8] \)

28. (B) \( x[n] \) is right sided, \( x[n] = \sum_{k=5}^{\infty} \frac{1}{k} \bar{a}[n - k] \)

29. (D) \( x[n] \) is right sided signal

\[
X(z) = 1 + 3z^{-1} + 3z^{-2} + z^{-3}
\]

\( x[n] = \bar{a}[n] + 3\bar{a}[n - 1] + 3\bar{a}[n - 2] + \bar{a}[n - 3] \)

30. (A)

\[
x[n] = \bar{a}[n + 6] + \bar{a}[n + 2] + 3\bar{a}[n] + 2\delta[n - 3] + \bar{a}[n - 4]
\]

31. (B) \( X(z) = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots \)

\[
x[n] = \bar{a}[n] + \frac{\bar{a}[n + 1]}{2!} + \frac{\bar{a}[n + 2]}{3!} + \ldots
\]

\[
+ \frac{\bar{a}[n] + \bar{a}[n + 1]}{2!} + \frac{\bar{a}[n + 2]}{3!} + \ldots
\]

\[
x[n] = \delta[n] + \frac{1}{n}!
\]

32. (A) \( X(z) = 1 + \frac{z^2}{4} + \left(\frac{z^2}{4}\right)^2 + \ldots \) \( = \sum_{k=0}^{\infty} \left(\frac{1}{4}z^2\right)^k \)

\( x[n] = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k \bar{a}[n - 2k] \)

33. (C) \( X(z) = -4z^2 \sum_{k=0}^{\infty} (2z)^{2k} = -\frac{4z^{2(k+1)} - 2z^{2(k-1)}}{1 - 4z^{-2}} \)

\( x[n] = -\sum_{k=0}^{\infty} 2^{2(k+1)}\delta[n + 2(k + 1)] \)

34. (A) \( \ln(1 + \alpha) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} (\alpha)^k \)

\( X(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} (z^{-1})^k \)

\( x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} \delta[n - 1] \)

35. (B) \( \cos \alpha = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \alpha^{2k} \)

\( X(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (z^{-3})^{2k} \)

\( x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \delta[n - 6k] \)

\( n = 12 \Rightarrow 12 - 6k = 0, k = 2, \)

\( x[12] = \frac{(-1)^2}{4!} = \frac{1}{24} \)

36. (D) All gives the same z transform with different ROC. So all are the solution.

37. (C) \( x_t[n] \) is right-sided signal

\( z_1 > 2, z_1 > \frac{1}{2} \) gives \( z_1 > 2 \)

\( x_t[n] \) is left-sided signal

\( z_2 < 2, z_2 < \frac{1}{2} \) gives \( z_2 < \frac{1}{2} \)

\( x_t[n] \) is double sided signal

\( z_3 > \frac{1}{2} \) and \( z_3 < 2 \) gives \( \frac{1}{2} < z_3 < 2 \)

38. (B) \( X(z) = \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{-1}{1 + \frac{1}{2}z^{-1}} \)

\( |z| > \frac{1}{2} \) (Right-sided) \( x[n] = \frac{2}{2^n} u[n] - \left(\frac{1}{3}\right)^n u[n] \)

\( |z| < \frac{1}{3} \) (Left-sided) \( x[n] = \left[\frac{-2}{2^n} + \left(\frac{1}{3}\right)^n\right] u[-n - 1] \)
\[ \frac{1}{3} < |z| < \frac{1}{2} \text{ (Two-sided)} \quad x[n] = -\frac{2}{2^z} u[-n-1] - \left( -\frac{1}{3} \right)^n u[n] \]

So (b) is wrong.

39. (A) Since the ROC includes the \( z = \frac{3}{4} \), ROC is

\[ \frac{1}{2} < |z| < 1, \]

\[ X(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + z^{-1}} \]

\[ \Rightarrow x[n] = A \left(\frac{1}{2}\right)^n u[n] \cdot B (-1)^n u[-n-1] \]

\[ 1 = \frac{A}{2} \Rightarrow A = 2, \]

\[ x[-1] = 1 = (-1)B(-1) \Rightarrow B = 1 \]

\[ \Rightarrow x[n] = \frac{1}{2^{n+1}} u[n] - (-1)^n u[-n-1] \]

40. (B) \( x[n] = C p^n u[n] \), \( x[0] = 2 = C \)

\[ x[2] = \frac{1}{2} = 2p^2 \Rightarrow p = \frac{1}{2}, \]

\[ x[n] = 2 \left(\frac{1}{2}\right)^n u(n) \]

41. (B) \( H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 + \frac{1}{2}z^{-1}} \)

\( h[n] \) is stable, so ROC includes \( |z| = 1 \)

ROC : \( \frac{1}{2} < |z| < 2 \),

\[ h[n] = (2)^n u[-n-1] + \left( -\frac{1}{2} \right)^n u[n] \]

42. (A) \( X^*(z) = \sum_{n=0} x[n]z^{-n} = \delta(n-2)z^{-n} = z^2 \)

43. (D) \( X^*(z) = \sum_{n=0} \sum_{m=0} x[n]z^{-m} = \sum_{n=0} z^{-m} = \frac{1}{1 - z^{-1}} \)

44. (B) \( H(z) = \frac{3}{1 - 3z^{-1}} + \frac{2}{1 + 2z^{-1}} \)

\( h[n] \) is causal so ROC is \( |z| > 3 \),

\[ \Rightarrow h[n] = [3^{n-1} + 2(-2)^n] u[n] \]

45. (A) \( X(z) = 1 + \frac{z^{-1}}{4} - \frac{z^{-2}}{8}, \quad Y(z) = 1 - \frac{3z^{-1}}{4} \)

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{2}{3}}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{5}{3}}{1 + \frac{1}{2}z^{-1}}, \]

\[ \Rightarrow h[n] = \frac{1}{3} \left[ \delta \left( -\frac{1}{2} \right)^n - 2 \left( \frac{1}{4} \right)^n \right] u[n] \]

46. (D) \( X(z) = \frac{1}{1 + 3z^{-1}} \)

\[ Y(z) = \frac{4}{1 - 2z^{-1}} - \frac{1}{1 - \frac{1}{2}z^{-1}} = \frac{3}{(1 - 2z^{-1}) \left( 1 - \frac{1}{2}z^{-1} \right)} \]

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{10}{1 - 2z^{-1}} + \frac{-7}{1 - \frac{1}{2}z^{-1}} \]

\[ \Rightarrow h[n] = \left[ 10(2)^n - 7 \left( \frac{1}{2} \right)^n \right] u(n) \]

47. (D) \( H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \quad Y(z) = 2z^{-4} \)

\[ X(z) = \frac{Y(z)}{H(z)} = 2z^{-4} - z^{-5} \]

\[ \Rightarrow x[n] = 2[\delta - 4] - \delta(n-5) \]

48. (A) \( Y(z) \left[ 1 - \frac{z^{-1}}{2} \right] = 2z^{-1}X(z) \)

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{2z^{-1}}{1 - \frac{z^{-1}}{2}} \]

\[ \Rightarrow h[n] = 2 \left( \frac{1}{2} \right)^{n-1} u[n-1] \]

49. (C) \( H(z) = \frac{Y(z)}{X(z)} = (1 - z^{-2} + z^{-4} - z^{-6}) \)

\[ \Rightarrow h[n] = \delta(n) - \delta(n-2) + \delta(n-4) - \delta(n-6) \]

50. (A) \( h[n] = \frac{3}{4} \left( \frac{1}{4} \right)^{n-1} u[n-1] \)

\[ H(z) = \frac{Y(z)}{X(z)} = \frac{3}{4} \left( \frac{1}{4} \right)^z \]

\[ Y(z) = \frac{1}{4} z^{-1} Y(z) = \frac{3}{4} z^{-1} X(z) \]

\[ \Rightarrow y[n] - \frac{1}{4} y[n-1] = \frac{3}{4} x[n-1] \]
51. (A) \( H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-5} \)

\( y[n] = x[n] - x[n - 5] \)

52. (D) Zero at : \( z = 0, \frac{2}{3} \) poles at \( z = \frac{1 \pm \sqrt{2}}{2} \)

(i) Not all poles are inside \( |z|=1 \), the system is not causal and stable.

(ii) Not are poles and zero are inside \( |z|=1 \), the system is not minimum phase.

53. (A) \( X(z) = -\frac{3}{8} - \frac{27}{1 - \frac{1}{3}z^{-1}} + \frac{8}{1 - 3z^{-1}} \)

Since \( X(z) \) converges on \( |z|=1 \). So ROC must include this circle.

ROC : \( \frac{1}{3} < |z| < 3 \),

\( x[n] = -\frac{1}{3^{n-1}} 8 u[n] - \frac{3^{n-3}}{8} u[-n - 1] \)

54. (C) \( h[n] = 16n \left( \frac{1}{4} \right)^n u[n] \). So system is both stable and causal. ROC includes \( z = 1 \).

55. (C) Pole of system at : \( z = -\frac{1}{2}, \frac{1}{3} \)

Pole of inverse system at : \( z = -\frac{1}{2} \)

For this system and inverse system all poles are inside \( |z|=1 \). So both system are both causal and stable.

56. (A) \( H(z) = \frac{10}{1 + \frac{1}{2}z^{-1}} - \frac{9}{1 + \frac{1}{4}z^{-1}} \)

\( = \frac{1 - 2z^{-1}}{\left( 1 + \frac{1}{2}z^{-1} \right) \left( 1 + \frac{1}{4}z^{-1} \right)} \)

Pole of this system are inside \( |z|=1 \). So the system is stable and causal.

For the inverse system not all pole are inside \( |z|=1 \). So inverse system is not stable and causal.

57. (C) \( |a_2| < 1, a_1 = -c < 1 + 0.12, |c| < 1.12 \)

58. (A) \( Y_1(z) = 1 - 0.1z^{-1}, Y_2(z) = 1 - 0.1z^{-1} \)

\( Y_3(z) = \frac{0.4 - 0.3z^{-1}}{1 - 0.5z^{-1}} \)

So \( y_1 \) and \( y_2 \) are equivalent.

59. (B) Causal signal \( x[0] = \lim_{z \to \infty} X(z) = 2 \)

60. (C) Anti causal signal, \( x[0] = \lim_{z \to \infty} X(z) = 4 \)

61. (A) The function has poles at \( z = 1, \frac{3}{4} \). Thus final value theorem applies.

\[ \lim_{n \to \infty} x[n] = \lim_{z \to 1} (z - 1) X(z) = (z - 1) \frac{z(2z - \frac{7}{4})}{(z - 1)(z - \frac{3}{4})} = 1 \]

62. (C) \( [2Y(z) + X(z)] z^{-2} = Y(z) \)

\( H(z) = \frac{z^{-2}}{1 - 2z^{-2}} \)

\( h[n] = -\frac{1}{2} + \frac{1}{4} \left( \frac{1}{\sqrt{2}} \right)^n + \frac{1}{4} \left( \frac{1}{\sqrt{2}} \right)^{-n} u[n] \)

\( = -\frac{1}{2} a[n] + \frac{1}{4} (\sqrt{2}^n + (\sqrt{2})^{-n}) u[n] \)

63. (D) \( Y(z) = X(z) z^{-1} - [Y(z) z^{-1} + Y(z) z^{-2}] \)

\( Y(z) = \frac{z^{-1}}{1 + z^{-1} + z^{-2}} = \frac{z}{z^2 + z + 1} \)

So this is a solution but not unique. Many other correct diagrams can be drawn.

***********
CHAPTER 5.6
THE DISCRETE-TIME FOURIER TRANSFORM

Statement for Q.1–9:
Determine the discrete-time Fourier Transform for the given signal and choose correct option.

1. \( x[n] = \begin{cases} 1, & |n| \leq 2 \\ 0, & \text{otherwise} \end{cases} \)
   (A) \( \sin 5\Omega \over \sin \Omega \)  
   (B) \( \sin 4\Omega \over \sin \Omega \)  
   (C) \( \sin 2.5\Omega \over \sin \Omega \)  
   (D) None of the above

2. \( x[n] = \left( \frac{3}{4} \right)^n u[n - 4] \)
   (A) \( \frac{3}{2} e^{-\frac{3}{4}n} \over 1 - \frac{3}{4} e^{-\frac{3}{4}n} \)  
   (B) \( \frac{3}{4} e^{\frac{3}{4}n} \over 1 + \frac{3}{4} e^{\frac{3}{4}n} \)  
   (C) \( \frac{3}{4} e^{-\frac{3}{4}n} \over 1 + \frac{3}{4} e^{-\frac{3}{4}n} \)  
   (D) None of the above

3. \( x[n] = \left( \frac{1}{2} \right)^n u[-n - 1] \)
   (A) \( e^{\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}} \)  
   (B) \( \frac{e^{\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}}}{2 - e^{-\frac{2\Omega}{n}}} \)  
   (C) \( e^{\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}} \)  
   (D) None of the above

4. \( x[n] = u[n-2] - u[n-6] \)
   (A) \( e^{3\Omega} + e^{3\Omega} + e^{4\Omega} + e^{5\Omega} \)  
   (B) \( e^{-2\Omega}(1 - e^{-3\Omega}) \over 1 - e^{-\frac{2\Omega}{n}} \)  
   (C) \( e^{-2\Omega} + e^{-3\Omega} + e^{-4\Omega} + e^{-5\Omega} \)  
   (D) None of the above

5. \( x[n] = 2\Omega[4 - 2n] \)
   (A) \( 2e^{-\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}} \)  
   (B) \( 2e^{\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}} \)  
   (C) \( 2^2 e^{-\frac{2\Omega}{2 - e^{-\frac{2\Omega}{n}}}} \)  
   (D) None of the above

6. \( x[n] = u[n] \)
   (A) \( \pi\delta(\Omega) - \frac{1}{1 + e^{-\frac{2\Omega}{n}}} \)  
   (B) \( \frac{1}{1 - e^{-\frac{2\Omega}{n}}} \)  
   (C) \( \pi\delta(\Omega) + \frac{1}{1 - e^{-\frac{2\Omega}{n}}} \)  
   (D) None of the above

7. \( x[n] = [-2, -1, 0, 1, 2] \)
   (A) \( 2j(2\sin 2\Omega + \sin \Omega) \)  
   (B) \( 2(2\cos 2\Omega - \cos \Omega) \)  
   (C) \( -2j(2\sin 2\Omega + \sin \Omega) \)  
   (D) \(-2(2\cos 2\Omega - \cos \Omega) \)  

8. \( x[n] = \sin \left( \frac{\pi}{2} n \right) \)
   (A) \( \pi(\delta(\Omega - \pi/2) - \delta(\Omega + \pi/2)) \)  
   (B) \( \frac{j}{2}(\delta(\Omega + \pi/2) - \delta(\Omega - \pi/2)) \)  
   (C) \( 2\pi(\delta(\Omega - \pi/2) - \delta(\Omega + \pi/2)) \)  
   (D) \( j\pi(\delta(\Omega + \pi/2) - \delta(\Omega - \pi/2)) \)
Statement for Q.10–21:

Determine the signal having the Fourier transform given in question.

10. \[ X(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1 \]
(A) \((n-1)a^n u[n]\) \quad (B) \((n+1)a^n u[n]\)
(C) \(na^n u[n]\) \quad (D) None of the above

11. \[ X(e^{j\omega}) = 8 \cos^2 \omega \]
(A) \((\delta[n+2] + 2\delta[n] + \delta[n-2])\)
(B) \(2(\delta[n+2] + 2\delta[n] + \delta[n-2])\)
(C) \(-4(\delta[n+2] + \delta[n] + \delta[n-2])\)
(D) \(\frac{1}{2}(\delta[n+2] + \delta[n] + \delta[n-2])\)

12. \[ X(e^{j\omega}) = \begin{cases} 2j, & 0 < \omega \leq \pi \\ -2j, & -\pi < \omega \leq 0 \end{cases} \]
(A) \(-\frac{4}{\pi n} \sin^2 \left(\frac{\pi n}{2}\right)\) \quad (B) \(\frac{4}{\pi n} \sin^2 \left(\frac{\pi n}{2}\right)\)
(C) \(\frac{8}{\pi n} \sin^2 \left(\frac{\pi n}{2}\right)\) \quad (D) \(-\frac{8}{\pi n} \sin^2 \left(\frac{\pi n}{2}\right)\)

13. \[ X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \leq |\omega| < \frac{3\pi}{4} \\ 0, & 0 \leq |\omega| < \frac{\pi}{4} \quad \frac{3\pi}{4} \leq |\omega| \leq \pi \end{cases} \]
(A) \(\frac{2}{n} \left( \sin \left(\frac{3\pi n}{4}\right) - \sin \left(\frac{\pi n}{4}\right) \right)\)
(B) \(\frac{1}{n} \left( \sin \left(\frac{3\pi n}{4}\right) - \sin \left(\frac{\pi n}{4}\right) \right)\)
(C) \(\frac{2}{n} \left( \cos \left(\frac{3\pi n}{4}\right) + \cos \left(\frac{\pi n}{4}\right) \right)\)
(D) \(\frac{1}{n} \left( \cos \left(\frac{3\pi n}{4}\right) + \cos \left(\frac{\pi n}{4}\right) \right)\)

14. \[ X(e^{j\omega}) = e^{j\omega} \quad \text{for} \quad -\pi \leq \omega \leq \pi \]
(A) \(\pi\delta[n - 1/2]\) \quad (B) \(2\pi\delta[n - 1/2]\)
(C) \(\frac{(-1)^{n+1}}{\pi(n - \frac{1}{2})}\) \quad (D) None of the above

15. \[ X(e^{j\omega}) = \cos 2\omega + j\sin 2\omega \]
(A) \(2\pi\delta[n + 2]\) \quad (B) \(\delta[n + 2]\)
(C) \(0\) \quad (D) None of the above

16. \[ X(e^{j\omega}) = j4 \sin 4\omega - 1 \]
(A) \(4\pi\delta[n + 4] - 4\pi\delta[n - 4] - 2\pi\delta[n]\)
(B) \(2\delta[n + 4] - 2\delta[n - 4] - \delta[n]\)
(C) \(\delta[n + 4] - \delta[n - 4] - \delta[n]\)
(D) None of the above

17. \[ X(e^{j\omega}) = \frac{2}{-e^{-j2\omega} + e^{j\omega} + 6} \]
(A) \(\frac{5}{2} + \left(\frac{2}{3}\right)u[n]\)
(B) \(2^{-n} \left(1 - \left(-\frac{2}{3}\right)^{n+1}\right)u[n]\)
(C) \(\frac{2^{-n}}{5} \left((-1)^n + \left(\frac{2}{3}\right)^{n+1}\right)u[n]\)
(D) None of the above

18. \[ X(e^{j\omega}) = \frac{2 + \frac{1}{2}e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}} \]
(A) \(2^{-n-1}[1 + (-2)^{-n}]u[n]\)
(B) \(2^{-n}[1 + (-2)^{-n}]u[n]\)
(C) \(2^{-n-1}[(-1)^n + 2^{-n}]u[n]\)
(D) \(2^{-n}[(1)^n + 2^{-n}]u[n]\)

19. \[ X(e^{j\omega}) = \frac{2e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega}} \]
(A) \(2^{n-1}[1 + (-1)^n]u[n]\)
(B) \(2^{n-1}[(1)^n + (-1)^n]u[n]\)
(C) \(2^{n-1}[1 + (-1)^n]u[n]\)
(D) \(2^{n-1}[(-1)^n + 2^{-n}]u[n]\)

20. \[ X(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega}}{1 - \frac{1}{4} e^{j\omega} - \frac{1}{8} e^{-j\omega}} \]
(A) \(\left(\frac{2}{9}\left(\frac{1}{2}\right)^n + \frac{7}{9}\left(-\frac{1}{4}\right)^n\right)u[n]\)
(B) \(\left(\frac{2}{9}\left(\frac{1}{2}\right)^n + \frac{7}{9}\left(\frac{1}{4}\right)^n\right)u[n]\)
(C) \(\left(\frac{2}{9}\left(\frac{1}{2}\right)^n - \frac{7}{9}\left(\frac{1}{4}\right)^n\right)u[n]\)
(D) None of the above
20. (A) \( X(e^{j\alpha}) = \frac{1 - \frac{1}{3} e^{-j\alpha}}{1 - \frac{1}{4} e^{-j\alpha} - \frac{7}{9} e^{-2j\alpha}} \)

\( x[n] = \frac{2}{9} \left( \frac{1}{2} \right)^n u[n] + \frac{7}{9} \left( -\frac{1}{4} \right)^n u[n] \)

21. (C) \( X(e^{j\alpha}) = \frac{(b-a)e^{j\alpha}}{1 - (a+b)e^{-j\alpha} + ab e^{-2j\alpha}} \)

\( x[n] = b^n u[n] + a^n u[-n-1] \).

22. (D) The signal must be real and odd. Only signal (h) is real and odd.

23. (A) The signal must be real and even. Only signal (c) and (e) are real and even.

24. (A) \( Y(e^{j\alpha}) = e^{j\alpha} X(e^{j\alpha}), \quad y[n] = x[n + \alpha] \)

If \( Y(e^{j\alpha}) \) is real, then \( y[n] \) is real and even (if \( x[n] \) is real). Therefore \( x[n + \alpha] \) is even and \( x[n] \) has to be symmetric about \( \alpha \). This is true for signal (a), (c), (e), (f) and (g).

25. (D) \( \int_{-\infty}^{\infty} X(e^{j\alpha}) d\alpha = 2\pi x[0] \),

\( x[0] = 0 \) is for signal (c), (f), (g) and (h).

26. (D) \( X(e^{j\alpha}) \) is always periodic with period \( 2\pi \). Therefore all signals satisfy the condition.

27. (D) \( X(e^{j\alpha}) = \sum_{n=-\infty}^{\infty} x[n] \), This condition is satisfied only if the samples of the signal add up to zero. This is true for signal (b) and (h).

28. (A) \( X(e^{j\alpha}) = \sum_{n=-\infty}^{\infty} x[n] = 6 \)

29. (A) \( y[n] = x[n + 2] \) is an even signal. Therefore \( Y(e^{j\alpha}) \) is real and even.

\( Y(e^{j\alpha}) = e^{j2\Omega} X(e^{j\alpha}) \Rightarrow X(e^{j\alpha}) = e^{-j2\Omega} Y(e^{j\alpha}) \),

Since \( Y(e^{j\alpha}) \) is real. This imply arg \( Y(e^{j\alpha}) = 0 \)

Thus \( \text{arg}(X(e^{j\alpha})) = -2\Omega \)

30. (C) \( \int_{-\infty}^{\infty} X(e^{j\alpha}) d\Omega = 2\pi x[0] = 4\pi \)

31. (A) \( X(e^{j\alpha}) = \sum_{n=-\infty}^{\infty} (-1)^n x[n] = 2 \)

32. (C) \( \text{Ev}[x[n]] \xleftarrow{\text{DTFT}} \text{Re}[X(e^{j\alpha})] \)

\( \text{Ev}[x[n]] = \frac{1}{2} \left[ \sum_{n=-\infty}^{\infty} x[n] + x[-n] \right] \)

33. (D) \( \int_{-\infty}^{\infty} |X(e^{j\alpha})|^2 = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28\pi \)

34. (C) \( nx[n] \xleftarrow{\text{DTFT}} \frac{j}{2\pi} \int_{-\infty}^{\infty} \frac{dX(e^{j\alpha})}{d\Omega} \)

\( \int_{-\infty}^{\infty} \left| \frac{dX(e^{j\alpha})}{d\Omega} \right| = 2\pi \sum_{n=-\infty}^{\infty} |n||x[n]| = 316\pi \)

35. (A) \( Y(e^{j\alpha}) = e^{-j4\Omega} X(e^{j\alpha}) \)

\( y[n] = x[n-4] = (n-4) \left( -\frac{3}{4} \right)^{|n-4|} \)

36. (C) Since \( x[n] \) is real and odd, \( X(e^{j\alpha}) \) is purely imaginary. Thus \( y[n] = 0 \).

37. (D) \( X_s(e^{j\alpha}) = X(e^{j2\Omega}) \)

\( X(e^{j2\Omega}) \xleftarrow{\text{DTFT}} x_s[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & \text{otherwise} \end{cases} \)

\( y[n] = \begin{cases} -jn & \left( \frac{3}{4} \right)^{|n|}, & n \text{ even} \\ 0, & \text{otherwise} \end{cases} \)
38. (B) \( Y(e^{j\Omega}) = X(e^{j\Omega}) \ast X(e^{j(\Omega-\omega)}) \)
\[ y[n] = 2\pi n x_1[n], \quad x_1[n] = e^{j\Omega/2} x[n], \]
\[ y[n] = 2\pi n x_1[n] (3/4) \]

39. (C) \( Y(e^{j\Omega}) = \frac{d}{d\Omega} X(e^{j\Omega}) \)
\[ y[n] = -j n x[n] = -j n (3/4) \]

40. (B) \( Y(e^{j\Omega}) = X(e^{j\Omega}) + X(e^{-j\Omega}) \)
\[ y[n] = x[n] + x[-n] = 0 \]

41. (C) For a real signal \( x[n] \)
\[ \text{od}[x[n]] \overset{\text{DTFT}}{\leftrightarrow} j \text{Im}[X(e^{j\Omega})] \]
\[ j \text{Im}[X(e^{j\Omega})] = j \sin \Omega - j \sin 2\Omega, \]
\[ = \frac{1}{2} (e^{j\Omega} - e^{-j\Omega} + e^{-2j\Omega}) \]
Therefore \( \text{od}[x[n]] = \mathcal{F}^{-1}[j \text{Im}[X(e^{j\Omega})]] \)
\[ = \frac{1}{2} (\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]) \]
\[ \text{od}[x[n]] = \frac{x[n] - x[-n]}{2} \]

Since \( x[n] = 0 \) for \( n > 0 \),
\( x[n] = 2 \text{od}[x[n]] = \delta[n+1] - \delta[n+2] \) For \( n < 0 \)
Using Parseval’s relation
\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\Omega})|^2 d\Omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 \]
\[ = \frac{1}{2} \sum_{n=0}^{\infty} |x[n]|^2 = (x[0])^2 + 2 \]
\( x[0] = \pm 1, \quad \text{But } x[0] = 0 \), Hence \( x[0] = 1 \)
\( x[n] = \delta[n] + \delta[n+1] - \delta[n+2] \)

42. (C) \( \left(\frac{1}{4}\right)^n u[n] \overset{\text{DTFT}}{\leftrightarrow} \frac{1}{1 - \frac{1}{4} e^{-j\Omega}} \left(\frac{1}{4}\right)^n \)
\[ n \left(\frac{1}{2}\right)^n u[n] \overset{\text{DTFT}}{\leftrightarrow} j \frac{d}{d\Omega} \left(\frac{1}{1 - \frac{1}{4} e^{-j\Omega}}\right) = \frac{1}{4} e^{-j\Omega} \]
\[ \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n = \sum_{n=0}^{\infty} x[n] = X(e^{j\Omega}) = \frac{4}{9} \]

43. (A) For all pass system \( |H(e^{j\Omega})| = 1 \) for all \( \Omega \)
\[ H(e^{j\Omega}) = \frac{b + e^{-j\Omega}}{1 - a e^{-j\Omega}}, \quad |b + e^{-j\Omega}| = |1 - a e^{-j\Omega}| \]
\[ 1 + b^2 + 2b \cos \Omega = 1 + a^2 - 2a \cos \Omega \]

This is possible only if \( b = -a \).

44. (A) For \( x[n] = \delta[n], \quad X(e^{j\Omega}) = 1, \quad dX(e^{j\Omega}) = 0 \)
\[ h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\Omega}) e^{j\Omega} d\Omega = \frac{1}{2\pi} \left[ \frac{1}{e^{j\Omega}} - \frac{1}{e^{-j\Omega}} \right] \]
\[ = \frac{1}{2\pi} \left[ \frac{e^{j(\Omega-1)}}{e^{j-1}} \right] d\Omega = \frac{\sin \pi n}{\pi(n-1)} \]

45. (B) \[ H(e^{j\Omega}) = H_1(e^{j\Omega}) + H_2(e^{j\Omega}) \]
\[ \frac{12 + 5e^{j\Omega} + 1}{12 - 7e^{j\Omega} + e^{-j\Omega}} = \frac{1}{3} e^{-j\Omega} + \frac{1}{3} e^{j\Omega} \]
\[ H_2(e^{j\Omega}) = \frac{1}{3} e^{j\Omega}, \quad h_2[n] = \left(\frac{1}{3}\right)^n u[n] \]

46. (D) \[ H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} \]
\[ (\frac{2}{3})^n u[n] \overset{\text{DTFT}}{\leftrightarrow} \frac{1}{1 - \frac{2}{3} e^{-j\Omega}} \]
\[ n (\frac{2}{3})^n u[n] \overset{\text{DTFT}}{\leftrightarrow} j \frac{d}{d\Omega} \left(\frac{1}{1 - \frac{2}{3} e^{-j\Omega}}\right) = \frac{2}{3} e^{-j\Omega} \]
\[ H(e^{j\Omega}) = \frac{2}{3} e^{-j\Omega} \]

47. (B) \[ H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{2}{3} e^{-j\Omega} \]
\[ \left(1 - \frac{2}{3} e^{-j\Omega}\right) Y(e^{j\Omega}) = \frac{2}{3} e^{-j\Omega} X(e^{j\Omega}) \]
\[ y[n] = \frac{2}{3} y[n-1] = \frac{2}{3} x[n-1] \]
\[ 3y[n] - 2y[n-1] = 2x[n-1] \]

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CHAPTER 5.7
THE CONTINUOUS-TIME FOURIER SERIES

Statement for Q.1-5:

Determine the Fourier series coefficient for given periodic signal $x(t)$.

1. $x(t)$ as shown in fig. P5.7.1

(A) 1  
(B) $\cos\left(\frac{\pi}{2}k\right)$  
(C) $\sin\left(\frac{\pi}{2}k\right)$  
(D) 2

2. $x(t)$ as shown in fig. P5.7.2

(A) $\frac{A}{jnk} \sin\left(\frac{\pi}{2}k\right)$  
(B) $\frac{A}{jnk} \cos\left(\frac{\pi}{2}k\right)$  
(C) $\frac{A}{nk} \sin\left(\frac{\pi}{2}k\right)$  
(D) $\frac{A}{nk} \cos\left(\frac{\pi}{2}k\right)$

3. $x(t)$ as shown in fig. P5.7.3

(A) $\frac{A}{2\pi k}\left(e^{-\frac{j4\pi k}{T}} - 1\right)$  
(B) $j\frac{A}{2\pi k}\left(e^{-\frac{j4\pi k}{T}} - 1\right)$  
(C) $-j\frac{A}{2\pi k}\left(e^{-\frac{j4\pi k}{T}} - 1\right)$  
(D) $-\frac{A}{2\pi k}\left(e^{-\frac{j4\pi k}{T}} - 1\right)$

4. $x(t)$ as shown in fig. P5.7.4

(A) $\frac{A}{k\pi}\left(1 - (-1)^{k}\right)$  
(B) $\frac{A}{j\pi}\left(1 - (-1)^{k}\right)$  
(C) $\frac{A}{j\pi}\left(-1 - (-1)^{k}\right)$  
(D) $\frac{A}{j\pi}\left(-1 - (-1)^{k}\right)$

5. $x(t) = \sin^2 t$

(A) $-\frac{1}{4} \delta[k - 1] + \frac{1}{2} \delta[k] - \frac{1}{4} \delta[k + 1]$  
(B) $-\frac{1}{4} \delta[k - 2] + \frac{1}{2} \delta[k] - \frac{1}{4} \delta[k + 2]$  
(C) $-\frac{1}{2} \delta[k - 1] + \delta[k] - \frac{1}{2} \delta[k + 1]$  
(D) $\frac{1}{2} \delta[k - 2] + \delta[k] - \frac{1}{2} \delta[k + 2]$
Statement for Q.6-11:

In the question, the FS coefficient of time-domain signal have been given. Determine the corresponding time domain signal and choose correct option.

7. \[ X[k] = j\alpha(k - 1) - j\alpha(k + 1) + \beta(k + 3) + \beta(k - 3), \omega_c = 2\pi \]
   (A) \( 2(\cos 3\pi t - \sin \pi t) \)
   (B) \( -2(\cos 3\pi t - \sin \pi t) \)
   (C) \( 2(\cos 6\pi t - \sin 2\pi t) \)
   (D) \( -2(\cos 6\pi t - \sin 2\pi t) \)

8. \[ X[k] = \left(-\frac{1}{3}\right)^k, \omega_c = 1 \]
   (A) \( \frac{5}{5 + 3\sin t} \)
   (B) \( \frac{5}{4 + 3\sin t} \)
   (C) \( \frac{5}{4 + 3\cos t} \)
   (D) \( \frac{4}{5 + 3\cos t} \)

9. \[ X[k] \text{ as shown in fig. P5.7.9, } \omega_c = \pi \]

10. \[ X[k] \text{ as shown in fig. P5.7.10, } \omega_c = 2\pi \]

Statement for Q.12-16:

Consider a continuous time periodic signal \( x(t) \) with fundamental period \( T \) and Fourier series coefficients \( X[k] \). Determine the Fourier series coefficient of the signal \( y(t) \) given in question and choose correct option.

11. \[ X[k] \text{ as depicted in fig. P5.7.11, } \omega_c = \pi \]
   (A) \( 3\cos 3\pi t + 2\cos 2\pi t + \cos \pi t \)
   (B) \( 3\sin 3\pi t + 2\sin 2\pi t + \sin \pi t \)
   (C) \( 6\sin 3\pi t + 4\sin 2\pi t + 2\sin \pi t \)
   (D) \( 6\cos 3\pi t + 4\cos 2\pi t + 2\cos \pi t \)

12. \[ y(t) = x(t - t_o) + x(t + t_o) \]
   (A) \( 2\cos \left(\frac{2\pi}{T}k t_o\right)X[k] \)
   (B) \( 2\sin \left(\frac{2\pi}{T}k t_o\right)X[k] \)
   (C) \( e^{-i\pi}X[k] + e^{-i\pi}X[-k] \)
   (D) \( e^{-i\pi}X[k] + e^{-i\pi}X[-k] \)

13. \[ y(t) = \text{Ev}\{x(t)\} \]
   (A) \( \frac{X[k] + X[-k]}{2} \)
   (B) \( \frac{X[k] - X[-k]}{2} \)
   (C) \( \frac{X[k] + X^*[m]}{2} \)
   (D) \( \frac{X[k] + X^*[m]}{2} \)

14. \[ y(t) = \text{Re}\{x(t)\} \]
   (A) \( \frac{X[k] + X[-k]}{2} \)
   (B) \( \frac{X[k] - X[-k]}{2} \)
   (C) \( \frac{X[k] + X^*[m]}{2} \)
   (D) \( \frac{X[k] + X^*[m]}{2} \)
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15. \( y(t) = \frac{d^2x(t)}{dt^2} \)
   \( (A) \left( \frac{2\pi k}{T} \right)^2 X[k] \)
   \( (B) -\left( \frac{2\pi k}{T} \right)^2 X[k] \)
   \( (C) j \left( \frac{2\pi k}{T} \right)^2 X[k] \)
   \( (D) -j \left( \frac{2\pi k}{T} \right)^2 X[k] \)

16. \( y(t) = x(4t - 1) \)
   \( (A) \frac{8\pi}{T} X[k] \)
   \( (B) \frac{4\pi}{T} X[k] \)
   \( (C) e^{-j\left( \frac{8\pi}{T} \right)} X[k] \)
   \( (D) e^{j\left( \frac{8\pi}{T} \right)} X[k] \)

17. Consider a continuous-time signal
   \( x(t) = 4 \cos 100\pi t \sin 1000\pi t \)
   with fundamental period \( T = \frac{1}{50} \). The nonzero FS coefficient for this function are
   \( (A) X[-4], X[4], X[-7], X[7] \)
   \( (B) X[-1], X[1], X[-10], X[10] \)
   \( (C) X[-3], X[3], X[-4], X[4] \)
   \( (D) X[-9], X[9], X[-11], X[11] \)

18. A real valued continuous-time signal \( x(t) \) has a fundamental period \( T = 8 \). The nonzero Fourier series coefficients for \( x(t) \) are
   The signal \( x(t) \) would be
   \( (A) 4 \cos \left( \frac{\pi}{4} t \right) + 4j \sin \left( \frac{3\pi}{4} t \right) \)
   \( (B) 4 \cos \left( \frac{\pi}{4} t \right) - 4j \cos \left( \frac{3\pi}{4} t \right) \)
   \( (C) 8 \cos \left( \frac{\pi}{4} t \right) + 8 \cos \left( \frac{3\pi}{4} t + \frac{\pi}{2} \right) \)
   \( (D) \text{None of the above} \)

19. The continuous-time periodic signal is given as
   \( x(t) = 4 + \cos \left( \frac{2\pi}{3} t \right) + 6 \sin \left( \frac{5\pi}{3} t \right) \)
   The nonzero Fourier coefficients are
   \( (A) X[0], X[-1], X[1], X[-5], X[5] \)
   \( (B) X[0], X[-2], X[2], X[-10], X[10] \)
   \( (C) X[0], X[-4], X[4], X[-10], X[10] \)
   \( (D) \text{None of the above} \)

Statement for Q.20-21:

Let \( x_1(t) \) and \( x_2(t) \) be continuous time periodic signal with fundamental frequency \( \omega_1 \) and \( \omega_2 \), Fourier series coefficients \( X_1[k] \) and \( X_2[k] \) respectively. Given that \( x_3(t) = x_1(t - 1) + x_2(1-t) \)

20. The relation between \( \omega_1 \) and \( \omega_2 \) is
   \( (A) \omega_2 = \frac{\omega_1}{2} \)
   \( (B) \omega_2 = \omega_1^2 \)
   \( (C) \omega_2 = \omega_1 \)
   \( (D) \omega_2 = \sqrt{\omega_1} \)

21. The Fourier coefficient \( X_3[k] \) will be
   \( (A) (X_1[k] - jX_2[-k])e^{-j\omega_1 k} \)
   \( (B) (X_1[-k] - jX_2[k])e^{-j\omega_1 k} \)
   \( (C) (X_1[k] + jX_2[-k])e^{-j\omega_1 k} \)
   \( (D) \text{None of the above} \)

Statement for Q.22-23:

Consider three continuous-time periodic signals whose Fourier series representation are as follows.
\( x_1(t) = \sum_{k=-100}^{100} \frac{1}{k} e^{-jk \frac{2\pi}{50}} \)
\( x_2(t) = \sum_{k=-100}^{100} \cos k\pi e^{-jk \frac{2\pi}{50}} \)
\( x_3(t) = \sum_{k=-100}^{100} j \sin \left( \frac{k\pi}{2} \right) e^{-jk \frac{2\pi}{50}} \)

22. The even signals are
   \( (A) x_2(t) \) only
   \( (B) x_2(t) \) and \( x_3(t) \)
   \( (C) x_1(t) \) and \( x_3(t) \)
   \( (D) x_1(t) \) only

23. The real valued signals are
   \( (A) x_1(t) \) and \( x_2(t) \)
   \( (B) x_2(t) \) and \( x_3(t) \)
   \( (C) x_3(t) \) and \( x_1(t) \)
   \( (D) x_1(t) \) and \( x_3(t) \)

24. Suppose the periodic signal \( x(t) \) has fundamental period \( T \) and Fourier coefficients \( X[k] \). Let \( Y[k] \) be the Fourier coefficient of \( y(t) \) where \( y(t) = dx(t)/dt \). The Fourier coefficient \( X[k] \) will be
   \( (A) \frac{TY[k]}{j2\pi k}, k \neq 0 \)
   \( (B) \frac{TY[k]}{j2\pi k} \)
   \( (C) \frac{TY[k]}{jk}, k \neq 0 \)
   \( (D) \frac{TY[k]}{jk} \)
25. Suppose we have given the following information about a signal \( x(t) \):
   1. \( x(t) \) is real and odd.
   2. \( x(t) \) is periodic with \( T=2 \)
   3. Fourier coefficients \( X[k]=0 \) for \( |k|>1 \)
   4. \( \frac{1}{2} \int_{0}^{1} x(t)^2 \, dt = 1 \)

   The signal, that satisfy these condition, is
   (A) \( \sqrt{2} \sin \pi t \) and unique
   (B) \( \sqrt{2} \sin \pi t \) but not unique
   (C) 2 sin \( \pi t \) and unique
   (D) 2 sin \( \pi t \) but not unique

26. Consider a continuous-time LTI system whose frequency response is
\[
H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} \, dt = \frac{\sin 4\omega}{\omega}
\]

   The input to this system is a periodic signal
   \[
x(t) = \begin{cases} 
2, & 0 \leq t \leq 4 \\
-2, & 4 < t \leq 8 
\end{cases}
\]

   with period \( T=8 \). The output \( y(t) \) will be
   (A) \( 1 + \sin^2 \left(\frac{\pi t}{4}\right) \)
   (B) \( 1 + \cos^2 \left(\frac{\pi t}{4}\right) \)
   (C) \( 1 + \sin \left(\frac{\pi t}{4}\right) + \cos \left(\frac{\pi t}{4}\right) \)
   (D) 0

27. Consider a continuous-time ideal low pass filter having the frequency response
   \[
H(j\omega) = \begin{cases} 1, & |\omega| \leq 80 \\
0, & |\omega| > 80 
\end{cases}
\]

   When the input to this filter is a signal \( x(t) \) with fundamental frequency \( \omega_0 = 12 \) and Fourier series coefficients \( X[k] \) it is found that \( x(t) \leftrightarrow \omega \rightarrow y(t) = x(t) \).

   The largest value of \( |k| \), for which \( X[k] \) is nonzero, is
   (A) 6
   (B) 80
   (C) 7
   (D) 12

28. A continuous-time periodic signal has a fundamental period \( T=8 \). The nonzero Fourier series coefficients are as,
   \[
X[1] = X'[\{-1\}] = j, \quad X[5] = X[-5] = 2,
\]

   The signal will be
   (A) \( 4 \cos \left(\frac{\pi t}{4}\right) - 2 \sin \left(\frac{\pi t}{4}\right) \)
   (B) \( 2 \cos \left(\frac{\pi t}{4}\right) + 4 \sin \left(\frac{\pi t}{4}\right) \)
   (C) \( 2 \cos \left(\frac{\pi t}{4}\right) + 2 \sin \left(\frac{\pi t}{4}\right) \)
   (D) None of the above

Statement for Q.29-31:

Consider the following three continuous-time signals with a fundamental period of \( T=1 \)
\[
x(t) = \cos 2\pi t, \quad y(t) = \sin 2\pi t, \quad z(t) = x(t)y(t)
\]

29. The Fourier series coefficient \( X[k] \) of \( x(t) \) are
   (A) \( \frac{1}{2} (\bar{a} [k+1] + \bar{a} [k-1]) \)
   (B) \( \frac{1}{2} (\bar{a} [k+1] - \bar{a} [k-1]) \)
   (C) \( \frac{1}{2} (\bar{a} [k-1] - \bar{a} [k+1]) \)
   (D) None of the above

30. The Fourier series coefficient of \( y(t) \), \( Y[k] \) will be
   (A) \( \frac{1}{2} (\bar{a} [k+1] + \bar{a} [k+1]) \)
   (B) \( \frac{1}{2} (\bar{a} [k+1] - \bar{a} [k-1]) \)
   (C) \( \frac{1}{2} (\bar{a} [k-1] - \bar{a} [k+1]) \)
   (D) None of the above

31. The Fourier series coefficient of \( z(t) \), \( Z[k] \) will be
   (A) \( \frac{1}{2} (\bar{a} [k-2] - \bar{a} [k+2]) \)
   (B) \( \frac{1}{2} (\bar{a} [k-2] - \bar{a} [k+2]) \)
   (C) \( \frac{1}{2} (\bar{a} [k+2] - \bar{a} [k-2]) \)
   (D) None of the above

32. Consider a periodic signal \( x(t) \) whose Fourier series coefficients are
   \[
   X[k] = \begin{cases} 
2, & k = 0 \\
\frac{1}{2}, & \text{otherwise}
\end{cases}
\]

   Consider the statements
   1. \( x(t) \) is real.
   2. \( x(t) \) is even
   3. \( \frac{dx(t)}{dt} \) is even

   The true statements are
   (A) 1 and 2
   (B) only 2
   (C) only 1
   (D) 1 and 3
Statement for Q.33-36:

A waveform for one period is depicted in figure in question. Determine the trigonometric Fourier series and choose correct option.

33. [Diagram of a waveform with a triangular shape]

(A) \( \frac{2}{\pi} \left( \cos t + \frac{1}{2} \cos 2t + \frac{1}{3} \cos 3t + \frac{1}{4} \cos 4t + \ldots \right) \)

(B) \( \frac{2}{\pi} \left( \sin t - \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t - \frac{1}{4} \sin 4t + \ldots \right) \)

(C) \( \frac{2}{\pi} \left( \sin t + \cos t - \frac{1}{2} \sin 2t - \frac{1}{3} \cos 2t + \frac{1}{4} \sin 3t + \ldots \right) \)

(D) \( \frac{2}{\pi} \left( \sin t + \cos t + \frac{1}{3} \sin 3t - \frac{1}{4} \cos 4t + \ldots \right) \)

34. [Diagram of a rectangular waveform]

(A) \( \frac{A}{2} + \frac{4A}{\pi} \left( \sin t - \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t + \ldots \right) \)

(B) \( \frac{A}{2} + \frac{4A}{\pi} \left( \cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t + \ldots \right) \)

(C) \( \frac{4A}{\pi} \left( \sin t + \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \ldots \right) \)

(D) \( \frac{4A}{\pi} \left( \cos t + \frac{1}{2} \cos 2t + \frac{1}{3} \cos 3t + \ldots \right) \)

35. [Diagram of a waveform with a triangular shape]

(A) \( \frac{A}{2} + \frac{2A}{\pi} \left( \sin t - \frac{1}{3} \sin 3t + \frac{1}{5} \sin 5t + \ldots \right) \)

(B) \( \frac{A}{2} + \frac{2A}{\pi} \left( \cos t - \frac{1}{2} \cos 2t + \frac{1}{3} \cos 3t + \ldots \right) \)

(C) \( \frac{A}{2} + \frac{2A}{\pi} \left( \cos t - \frac{1}{3} \cos 3t + \frac{1}{5} \cos 5t + \ldots \right) \)

(D) \( \frac{A}{2} + \frac{2A}{\pi} \left( \sin t + \cos t + \frac{1}{3} \sin 3t + \frac{1}{5} \cos 5t + \ldots \right) \)
1. (D) \( X[k] = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \hat{x}(t)e^{-j2\pi k \frac{t}{T}} dt = \frac{A}{T} \)
\( A = 10 \), \( T = 5 \), \( X[k] = 2 \)

2. (C) \( X[k] = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j2\pi k \frac{t}{T}} dt = \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{T}{4}} A e^{-j2\pi k \frac{t}{T}} dt \)
\( = \frac{A}{T} \left[ \frac{1}{2\pi k} e^{-j2\pi k \frac{T}{4}} \right] = A \sin \left( \frac{\pi k}{2} \right) \)

3. (B) \( T = 2\pi \), \( \omega_o = \frac{2\pi}{2\pi} = 1 \), \( x(t) = \begin{cases} A, & 0 < t < \frac{4\pi}{3} \\ 0, & \frac{4\pi}{3} < t < 2\pi \end{cases} \)
\( X[k] = \frac{1}{2\pi} \int_{0}^{2\pi} x(t)e^{-j2\pi k \frac{t}{T}} dt = \frac{1}{2\pi} \int_{0}^{\frac{4\pi}{3}} A e^{-j2\pi k \frac{t}{T}} dt = \frac{jA}{2\pi k} \left[ e^{-j\frac{4\pi k}{3}} - 1 \right] \)

4. (C) \( T = 2 \), \( \omega_o = \frac{2\pi}{2\pi} = \pi \), \( x(t) = \begin{cases} -A, & -1 < t < 0 \\ A, & 0 < t < 1 \end{cases} \)
\( X[k] = \frac{1}{2} \int_{-1}^{1} x(t)e^{-j2\pi k \frac{t}{T}} dt = \frac{1}{2} \left( \int_{-1}^{0} -A e^{-j2\pi k \frac{t}{T}} dt + \int_{0}^{1} A e^{-j2\pi k \frac{t}{T}} dt \right) \)
\( = A \left( \frac{1}{2} e^{j\pi k} + \frac{1}{2} e^{-j\pi k} \right) = A \left( 1 - (-1)^k \right) \)

5. (A) \( \sin^2 t = \left( \frac{e^{j2\pi} - e^{-j2\pi}}{2j} \right)^2 = \frac{1}{4} (e^{2\pi} - 2 + e^{-2\pi}) \)

The fundamental period of \( \sin^2 (t) \) is \( \pi \) and \( \omega_o = \frac{2\pi}{\pi} = 2 \),
\( X[k] = -\frac{1}{4} \delta[k-1] + \frac{1}{2} \delta[k] - \frac{1}{4} \delta[k+1] \)

7. (C) \( x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k t} = j e^{j\pi t} - j e^{-j\pi t} + e^{j2\pi t} + e^{-j2\pi t} \)
\( = -2 \sin 2\pi t + 2 \cos 6\pi t \)

8. (D) \( x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k t} = \sum_{k=-\infty}^{\infty} \left( \frac{1}{3} \right)^k e^{j2\pi k t} + \sum_{k=0}^{\infty} \left( \frac{1}{3} \right)^k e^{j2\pi k t} \)
\( = -\frac{1}{3} e^{-j\pi t} + \frac{1}{1 + \frac{1}{3} e^{-j2\pi t}} + \frac{1}{1 + \frac{1}{3} e^{j2\pi t}} = \frac{4}{5 + 3 \cos t} \)

9. (D) \( x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k t} \)
\( = 2 e^{-j\frac{\pi}{4} e^{-j\frac{\pi}{4} t}} + e^{j\frac{\pi}{4} e^{-j\frac{\pi}{4} t}} + e^{j\frac{\pi}{4} e^{j\frac{\pi}{4} t}} + 2 e^{j\frac{\pi}{4} e^{j\frac{\pi}{4} t}} \)
\( = 2(e^{-j\frac{\pi}{4} e^{j\frac{\pi}{4} t}} + e^{j\frac{\pi}{4} e^{-j\frac{\pi}{4} t}} + e^{j\frac{\pi}{4} e^{j\frac{\pi}{4} t}} + e^{-j\frac{\pi}{4} e^{-j\frac{\pi}{4} t}}) \)
\( = 4 \cos (4\pi t + \pi/4) + 2 \cos (3\pi t - \pi/4) \)

10. (A) \( X[k] = e^{-j\pi k}, -4 \leq k \leq 4 \)
\( x(t) = \sum_{k=-4}^{4} e^{-j\pi k} e^{j2\pi k t} = \sum_{k=-4}^{4} e^{-j\pi k} (t-1) = \frac{\sin 9\pi t}{\sin \pi t} \)

11. (D) \( X[k] = k, -3 \leq k \leq 3 \)
\( x(t) = \sum_{k=-3}^{3} X[k] e^{j2\pi k t} = 3e^{-j3\pi t} + 2e^{-j2\pi t} + e^{-j\pi t} + 2e^{j\pi t} + 2e^{j2\pi t} + 3e^{j3\pi t} \)
\( = 6 \cos 3\pi t + 4 \cos 2\pi t + 2 \cos \pi t \)

12. (A) \( x(t-t_o) \) is also periodic with \( T \). The Fourier series coefficients \( X[k] \) of \( x(t-t_o) \) are \( X[k] = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t-t_o)e^{-j2\pi k \frac{t}{T}} dt = \frac{e^{-j\pi k t_o}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j2\pi k \frac{t}{T}} dt \)
\( = e^{-j\pi k t_o} X[k] \)

Similarly, the FS coefficients of \( x(t+t_o) \) are \( X[k] = e^{j\pi k t_o} X[k] \)
The FS coefficients of \( x(t-t_o) + x(t+t_o) \) are \( Y[k] = X[k] + X[k] = e^{-j\pi k t_o} X[k] + e^{j\pi k t_o} X[k] \)
\( = 2 \cos (\omega_o k t_o) X[k] \)

13. (A) \( \text{Ev}[x(t)] = \frac{x(t) + x(-t)}{2} \)
The FS coefficients of \( x(t) \) are \( X[k] = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j2\pi k t/o} dt \)
\( = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{j2\pi k t/o} dt = X[-k] \)

Therefore, the FS coefficients of \( \text{Ev}[x(t)] \) are \( Y[k] = \frac{X[k] + X[-k]}{2} \)

14. (C) \( \text{Re}[x(t)] = \frac{x(t) + x^*(t)}{2} \)
The FS coefficient of \( x^*(t) \) is \( X[k]^* = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^*(t)e^{j2\pi k t} dt = X[k]^* \)
\( X[k] = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{j2\pi k t} dt = X[-k] \)
\( X[k]^* = X^*[k] \)
\( Y[k] = \frac{X[k] + X^*[k]}{2} \)

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Statement for Q.15-20:

Consider a periodic signal \( x[n] \) with period \( N \) and FS coefficients \( X[k] \). Determine the FS coefficients \( Y[k] \) of the signal \( y[n] \) given in question.

15. \( y[n] = x[n-n_s] \)
   (A) \( e^{jn} \frac{2\pi}{N} \) \( X[k] \)  
   (B) \( e^{jn} \frac{2\pi}{N} \) \( X[k] \)  
   (C) \( kX[k] \)  
   (D) \( -kX[k] \)

16. \( y[n] = x[n] - x[n-2] \)
   (A) \( \sin \left( \frac{4\pi}{N} k \right) X[k] \)  
   (B) \( \cos \left( \frac{4\pi}{N} k \right) X[k] \)  
   (C) \( 1 - e^{jn} \frac{2\pi}{N} k \) \( X[k] \)  
   (D) \( 1 - e^{jn} \frac{2\pi}{N} k \) \( X[k] \)

17. \( y[n] = x[n] + x[n+N/2] \), (assume that \( N \) is even)
   (A) \( 2X[2k-1] \), for \( 0 \leq k \leq \frac{N}{2} - 1 \)  
   (B) \( 2X[2k-1] \), for \( 0 \leq k \leq \frac{N}{2} \)  
   (C) \( 2X[2k] \), for \( 0 \leq k \leq \frac{N}{2} - 1 \)  
   (D) \( 2X[2k] \), for \( 0 \leq k \leq \frac{N}{2} \)

18. \( y[n] = x[n] - x[n-N/2] \), (assume that \( N \) is even)
   (A) \( 1 - (-1)^k \frac{2\pi}{N} kX[k] \)  
   (B) \( 1 - (-1)^k \frac{2\pi}{N} kX[k] \)  
   (C) \( 1 - (-1)^k \frac{2\pi}{N} kX[k] \)  
   (D) \( 1 - (-1)^k \frac{2\pi}{N} kX[k] \)

19. \( y[n] = x'[n] \)
   (A) \( X'[k] \)  
   (B) \( X'[k] \)  
   (C) \( X'[k] \)  
   (D) \( X'[k] \)

20. \( y[n] = (-1)^n x[n] \), (assume that \( N \) is even)
   (A) \( X[k + \frac{N}{2}] \)  
   (B) \( X[k + \frac{N}{2}] \)  
   (C) \( X[k + \frac{N}{2} + 1] \)  
   (D) \( X[k + \frac{N}{2} - 1] \)

Statement for Q.21-23:

Consider a discrete-time periodic signal
\[
x[n] = \begin{cases}  
1, & 0 \leq n \leq 7 \\
0, & 8 \leq n \leq 9 
\end{cases}
\]
with period \( N = 10 \). Also \( y[n] = x[n] - x[n-1] \)

21. The fundamental period of \( y[n] \) is
   (A) 9  
   (B) 10  
   (C) 11  
   (D) None of the above

22. The FS coefficients of \( y[n] \) are
   (A) \( \frac{1}{10} \left( 1 - e^{-jn} \frac{2\pi}{N} k \right) \)  
   (B) \( \frac{1}{10} \left( 1 - e^{-jn} \frac{2\pi}{N} k \right) \)  
   (C) \( \frac{1}{10} \left( 1 - e^{-jn} \frac{2\pi}{N} k \right) \)  
   (D) \( \frac{1}{10} \left( 1 - e^{-jn} \frac{2\pi}{N} k \right) \)

23. The FS coefficients of \( x[n] \) are
   (A) \( \frac{j}{2} e^{-jn} \frac{2\pi}{N} \) cosec \( \frac{\pi k}{10} \) \( Y[k] \), \( k \neq 0 \) 
   (B) \( \frac{j}{2} e^{-jn} \frac{2\pi}{N} \) cosec \( \frac{\pi k}{10} \) \( Y[k] \), \( k \neq 0 \) 
   (C) \( \frac{1}{2} e^{-jn} \frac{2\pi}{N} \) sec \( \frac{\pi k}{10} \) \( Y[k] \) 
   (D) \( \frac{1}{2} e^{-jn} \frac{2\pi}{N} \) sec \( \frac{\pi k}{10} \) \( Y[k] \)
Statement for Q.24-27:

Consider a discrete-time signal with Fourier representation.

\[ x[n] \xrightarrow{\text{DFTS}} X[k] \]

In question the FS coefficient \( Y[k] \) is given. Determine the corresponding signal \( y[n] \) and choose correct option.

24. \( Y[k] = X[k-5] + X[k+5] \)
   
   (A) \( 2 \sin \left( \frac{\pi}{5} n \right) x[n] \)
   
   (B) \( 2 \cos \left( \frac{\pi}{5} n \right) x[n] \)
   
   (C) \( 2 \sin \left( \frac{\pi}{2} n \right) x[n] \)
   
   (D) \( 2 \cos \left( \frac{\pi}{2} n \right) x[n] \)

25. \( Y[k] = \cos \left( \frac{n\pi}{5} \right) X[k] \)
   
   (A) \( \frac{1}{2} (x[n+5] + x[n+5]) \)
   
   (B) \( \frac{1}{2} (x[n+2] + x[n-2]) \)
   
   (C) \( \frac{1}{2} (x[n+10] + x[n+10]) \)
   
   (D) None of the above

26. \( Y[k] = X[k]^* X[k] \)
   
   (A) \( \frac{(x[n])^2}{2\pi} \)
   
   (B) \( j2\pi(x[n])^2 \)
   
   (C) \( (x[n])^2 \)
   
   (D) \( 2\pi(x[n])^2 \)

27. \( Y[k] = \text{Re}\{X[k]\} \)
   
   (A) \( \frac{x[n] + x[-n]}{2} \)
   
   (B) \( \frac{x[n] - x[-n]}{2} \)
   
   (C) \( \frac{x[n] - x[-n]}{2\pi} \)
   
   (D) \( \frac{x[n] + x[-n]}{2\pi} \)

28. Consider a sequence \( x[n] \) with following facts:
   
   1. \( x[n] \) is periodic with \( N = 6 \)
   
   2. \( \sum_{n=0}^{5} x[n] = 2 \)
   
   3. \( \sum_{n=-2}^{7} (-1)^n x[n] = 1 \)
   
   4. \( x[n] \) has the minimum power per period among the set of signals satisfying the preceding three conditions.

   The sequence would be...
   
   (A) \( \left\{ \ldots, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots \right\} \uparrow \)
   
   (B) \( \left\{ \ldots, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \right\} \uparrow \)
   
   (C) \( \left\{ \ldots, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots \right\} \uparrow \)
   
   (D) \( \left\{ \ldots, 1, 2, 3, 4, \ldots \right\} \uparrow \)

29. A real and odd periodic signal \( x[n] \) has fundamental period \( N = 7 \) and FS coefficients \( X[k] \). Given that \( X[15] = j, \ X[16] = 2j, \ X[17] = 3j \). The values of \( X[0], X[-1], X[-2], \) and \( X[-3] \) will be

   (A) \( 0, j, 2j, 3j \)
   
   (B) \( 1, 1, 2, 3 \)
   
   (C) \( 1, -1, -2, -3 \)
   
   (D) \( 0, -j, -2j, -3j \)

30. Consider a signal \( x[n] \) with following facts:

   1. \( x[n] \) is a real and even signal.
   
   2. The period of \( x[n] \) is \( N = 10 \)
   
   3. \( X[11] = 5 \)

   \[ \frac{1}{10} \sum_{n=0}^{9} |X[k]|^2 = 50 \]

   The signal \( x[n] \) is

   (A) \( 5 \cos \left( \frac{\pi}{10} n \right) \)
   
   (B) \( 5 \sin \left( \frac{\pi}{10} n \right) \)
   
   (C) \( 10 \cos \left( \frac{\pi}{5} n \right) \)
   
   (D) \( 10 \sin \left( \frac{\pi}{5} n \right) \)

31. Each of two sequence \( x[n] \) and \( y[n] \) has a period \( N = 4 \). The FS coefficients are

   \( X[0] = X[3] = \frac{1}{2} \), \( X[1] = \frac{1}{2} \), \( X[2] = 1 \) and

   \( Y[0], Y[1], Y[2], Y[3] = 1 \)

   The FS coefficient \( Z[k] \) for the signal \( z[n] = x[n] y[n] \) will be

   (A) \( 6 \)
   
   (B) \( 6|k| \)
   
   (C) \( 6|k| \)
   
   (D) \( e^{\frac{\pi i}{2}} \)

32. Consider a discrete-time periodic signal

   \[ x[n] = \frac{\sin \left( \frac{11\pi n}{20} \right)}{\sin \left( \frac{\pi n}{20} \right)} \]

   with a fundamental period \( N = 20 \). The Fourier series coefficients of this function are

   (A) \( \frac{1}{20} \left( u[k+5] - u[k-6] \right), \ |k| \leq 10 \)
   
   (B) \( \frac{1}{20} \left( u[k+5] - u[k-5] \right), \ |k| \leq 10 \)
   
   (C) \( u[k+5] - u[k+6], \ |k| \leq 10 \)
   
   (D) \( u[k+5] - u[k-6], \ |k| \leq 10 \)
11. (D) \( N = 7, \Omega_o = \frac{2\pi}{7} \),
\[
x[n] = \sum_{k=-3}^{k=3} X[k]e^{j\frac{2\pi}{7}kn} = 2e^{j\frac{2\pi}{7}kn} - 1 + 2e^{j\frac{4\pi}{7}kn} = 4 \cos\left(\frac{2\pi}{7}n\right) - 1
\]

12. (C) \( N = 12, \Omega_o = \frac{\pi}{6} \), \( X[k] = e^{j\frac{\pi}{6}kN} \)
\[
x[n] = \sum_{k=-6}^{k=6} \frac{e^{-j\frac{\pi}{6}kN} - e^{j\frac{\pi}{6}kN}}{2} = \sum_{k=-6}^{k=6} \frac{e^{-j\frac{\pi}{6}kN} - e^{j\frac{\pi}{6}kN}}{2} = 4 \sin\left(\frac{\pi}{12}n\right)
\]

13. (A) \( N = 19, \Omega_o = \frac{2\pi}{19} \)
\[
X[k] = \cos\left(\frac{10\pi}{19}k\right) + 2j \sin\left(\frac{10\pi}{19}k\right)
\]
\[
= \frac{1}{2}\left( e^{-j\frac{2\pi}{19}k} + e^{j\frac{2\pi}{19}k}\right) + \frac{1}{2}\left( e^{-j\frac{2\pi}{19}k} + e^{j\frac{2\pi}{19}k}\right)
\]
By inspection
\[
x[n] = \frac{19}{2}(\delta(n+5) + \delta(n-5)) + 19(\delta(n+2) - \delta(n-2)),
\]
Where \( |n| < 9 \)

14. (A) \( N = 21, \Omega_o = \frac{2\pi}{21} \)
\[
X[k] = \cos\left(\frac{8\pi}{21}k\right) = \frac{1}{2}\left( e^{-j\frac{4\pi}{21}k} + e^{j\frac{4\pi}{21}k}\right)
\]
Since \( X[k] = \frac{1}{N} \sum_{n=-N}^{N} x[n]e^{-j\Omega_nk} \), By inspection
\[
x[n] = \begin{cases} 
\frac{21}{2}, & n = \pm 4 \\
0, & \text{otherwise } n \in \{-10, -9, ..., 9, 10\}\end{cases}
\]

15. (B) \( Y[k] = \frac{1}{N} \sum_{n=-N}^{N} x[n]e^{-j\frac{2\pi}{N}kn} \)
\[
= \frac{1}{N} e^{-j\frac{2\pi}{N}kn} \sum_{n=-N}^{N} x[n]e^{-j\frac{2\pi}{N}kn} = e^{-j\frac{2\pi}{N}kn}X[k]
\]

16. (C) \( Y[k] = X[k] - e^{-j\frac{2\pi}{N}kn}X[k] \Rightarrow \frac{Y[k]}{1-e^{-j\frac{2\pi}{N}kn}} = X[k] \)

17. (C) Note that \( y[n] = x[n] + x[n + N/2] \) has a period of \( N/2 \) and \( N \) has been assumed to be even,
\[
Y[k] = 2 \sum_{k=0}^{N-1} (x[n] + x[n + N/2])e^{-j\frac{2\pi}{N}kn} = 2X[2k] \text{ for } 0 \leq k \leq (N/2 - 1)
\]

18. (B) \( y[n] = x[n] - x[n - N/2] \)
\[
Y[k] = \left\{ \begin{array}{l}
0, \quad k \text{ even} \\
2X[k], \quad k \text{ odd}
\end{array} \right.
\]

19. (C) \( y[n] = x'[\cdot n] \)
\[
Y[k] = \frac{1}{N} \sum_{n=-N}^{N-1} x'[\cdot n]e^{-j\frac{2\pi}{N}kn} = X'[k]
\]

20. (A) With \( N \) even
\[
y[n] = (-1)^n x[n] = e^{j\pi n}x[n] = e^{j\frac{2\pi}{N}N/2} x[n]
\]
\[
Y[k] = \frac{1}{N} \sum_{n=-N}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} = \frac{1}{N} \sum_{n=-N}^{N-1} e^{j\frac{2\pi}{N}k} X[k]
\]

21. (B) \( y[n] \) is shown is fig. S5.8.21. It has fundamental

![Fig. S5.8.21](gn.png)

period of 10.

22. (B) \( Y[k] = \frac{1}{10} \sum_{n=-10}^{n=9} y[n]e^{-j\frac{2\pi}{10}kn} \)
\[
= \frac{1}{10} \left( 1 - e^{-j\frac{2\pi}{10}kn} \right) = \frac{1}{10} \left( 1 - e^{-j\frac{\pi}{5}k} \right)
\]

23. (A) \( y[n] = x[n] - x[n - 1] \)
\[
Y[k] = X[k] - e^{-j\frac{2\pi}{10}kn}X[k] \Rightarrow X[k] = \frac{Y[k]}{1-e^{-j\frac{2\pi}{10}kn}}
\]
\[ X[k] = e^{j \frac{\pi}{10} k} Y[k] = e^{j \frac{\pi}{10} k} Y[k] - e^{-j \frac{\pi}{10} k} Y[k] = 2j \sin \left( \frac{n k}{10} \right) \]
\[ \frac{-j}{2} e^{-j \frac{\pi}{10} k} \cos \left( \frac{n}{10} k \right) Y[k] \]

24. (D) \( \Omega_o = \frac{\pi}{10} \), \( Y[k] = X[k - 5] + X[k + 5] \)
\[ y[n] = \left( e^{j \frac{\pi}{10} n} + e^{-j \frac{\pi}{10} n} \right) x[n] = 2 \cos \left( \frac{\pi}{2} n \right) x[n] \]

25. (B) \( Y[k] = \cos \left( \frac{\pi}{5} k \right) X[k] = \left( e^{j \frac{k}{5}} + e^{-j \frac{k}{5}} \right) \]
\[ \frac{1}{2} \left( e^{j \frac{2k}{10}} + e^{-j \frac{2k}{10}} \right) X[k] \]
\[ y[n] = \frac{1}{2} \left( x[n - 2] + x[n + 2] \right) \]

26. (C) \( Y[k] = X[k]^* X[k] \) \( y[n] = x[n] x[n] = (x[n])^2 \)

27. (A) \( Y[k] = \text{Re}[X[k]] \) \( y[n] = \text{Ev}[x[n]] = \frac{x[n] + x[-n]}{2} \)

28. (A) \( N = 6 \), \( \Omega_o = \frac{2\pi}{6} \), \( \frac{\pi}{10} \)

From fact 2, \( \sum_{n=0}^{5} x[n] = 2 \)
\[ \Rightarrow 1 = \frac{1}{6} \sum_{n=0}^{5} e^{j \frac{n \pi}{10} x[n]} = \frac{1}{3} \Rightarrow X[0] = \frac{1}{3} \]

From fact 3, \( \sum_{n=0}^{5} (-1)^n x[n] = 1 \)
\[ \Rightarrow 1 = \frac{1}{6} \sum_{n=0}^{5} e^{j \frac{n \pi}{10} x[n]} = \frac{1}{6} \Rightarrow X[3] = \frac{1}{6} \]

By Parseval’s relation, the average power in \( x[n] \) is \( P = \sum_{k=0}^{N} |X[k]|^2 \)


Therefore \( x[n] = \left\{ \begin{array}{l} 1, 0 \leq n \leq 5 \\ -1, 0 < n < 10 \end{array} \right\} \)


The signal real and odd, the FS coefficient \( X[k] \) will be purely imaginary and odd. Therefore \( X[0] = 0 \)

Therefore (D) is correct option.

Since \( x[n] \) is real and even \( X[k] \) is also real and even. Therefore \( X[1] = X[-1] = 5 \).

Using Parseval’s relation \( \sum_{k=0}^{8} |X[k]|^2 = 50 = \sum_{k=1}^{8} |X[k]|^2 \)
\[ |X[-1]|^2 + |X[1]|^2 + |X[0]|^2 + \sum_{k=2}^{8} |X[k]|^2 = 50 \]
\[ |X[0]|^2 + \sum_{k=2}^{8} |X[k]|^2 = 0 \]

Therefore \( X[k] = 0 \) for \( k = 0, 2, 3, \ldots, 8 \).

\[ x[n] = \sum_{k=1}^{8} X[k] |e^{j \frac{2\pi k}{k}}| = \sum_{k=1}^{8} X[k] |e^{j \frac{2\pi k}{10}}| = \sum_{k=1}^{8} X[k] |e^{j \frac{2\pi k}{10}}| = 10 \cos \left( \frac{\pi}{5} n \right) \]

31. (A) \( z[n] = x[n] y[n] \) \( \xrightarrow{DTFS} \) \( \sum_{k=-N}^{N} X[l] Y[k - l] \)
\[ Z[k] = \sum_{l=0}^{N} X[l] Y[k - l] \]
\[ = Y[k] + 2Y[k - 1] + 2Y[k - 2] + Y[k - 3] \]
Since \( Y[k] \) is 1 for all values of \( k \). Thus \( Z[k] = 6 \), for all \( k \).

32. (A) \( N = 20 \) We know that
\[ \begin{array}{l}
1, \quad 1 \leq n \leq 5 \\
0, \quad 5 < n \leq 10
\end{array} \]

Using duality
\[ \frac{\sin \left( \frac{11\pi k}{20} \right)}{\sin \left( \frac{\pi k}{20} \right)} \]

**********
1. The equivalent transfer function of three parallel blocks

\[ G_1(s) = \frac{1}{s + 1}, \quad G_2(s) = \frac{1}{s + 4}, \quad \text{and} \quad G_3(s) = \frac{s + 3}{s + 5} \]

is

(A) \[ \frac{(s^3 + 10s^2 + 34s + 37)}{(s + 1)(s + 4)(s + 5)} \]

(B) \[ \frac{(s + 3)}{(s + 1)(s + 4)(s + 5)} \]

(C) \[ \frac{-(s^3 + 10s^2 + 34s + 37)}{(s + 1)(s + 4)(s + 5)} \]

(D) \[ \frac{-(s + 3)}{(s + 1)(s + 4)(s + 5)} \]

2. The block having transfer function

\[ G_1(s) = \frac{1}{s + 2}, \quad G_2(s) = \frac{1}{s + 5}, \quad G_3(s) = \frac{s + 1}{s + 3} \]

are cascaded. The equivalent transfer function is

(A) \[ \frac{(s^3 + 10s^2 + 37s^2 + 31)}{(s + 2)(s + 3)(s + 5)} \]

(B) \[ \frac{s + 1}{(s + 2)(s + 3)(s + 5)} \]

(C) \[ \frac{(s^3 + 10s^2 + 37s^2 + 31)}{(s + 2)(s + 3)(s + 5)} \]

(D) \[ \frac{-(s + 1)}{(s + 2)(s + 3)(s + 5)} \]

3. For a negative feedback system shown in fig. P.6.1.3

\[ G(s) = \frac{s + 1}{s(s + 2)} \quad \text{and} \quad H(s) = \frac{s + 3}{s + 4} \]

The equivalent transfer function is

(A) \[ \frac{G_1G_2}{1 + H_1G_1G_2G_3} \]

(B) \[ \frac{G_2G_3}{G_1(1 + H_1G_1G_2G_3)} \]

(C) \[ \frac{G_1G_2}{1 + H_1G_1G_2G_3} \]

(D) \[ \frac{G_1G_2}{G_1(1 + H_1G_1G_2G_3)} \]
5. Consider the system shown in fig. P.6.1.5.

The input output relationship of this system is

\[ R(s) \rightarrow G_1 G_2 \rightarrow C(s) \]

\[ R(s) \rightarrow G_1 + G_2 \rightarrow C(s) \]

(A) \( G_1 G_2 \) \( 1 + G_1 G_2 H \)

(B) \( G_1 G_2 \) \( 1 + G_1 H \)

(C) \( G_1 G_2 \) \( 1 + G_2 H \)

(D) None of the above

6. A feedback control system shown in fig. P.6.1.6 is subjected to noise \( N(s) \).

\[ R(s) \rightarrow G_1 G_2 \rightarrow C(s) \]

\[ R(s) \rightarrow G_1 + G_2 \rightarrow C(s) \]

(A) \( G \) \( G H \)

(B) \( G \) \( H \)

(C) \( G \) \( H \)

(D) None of the above

7. The closed loop gain of the system shown in fig. P.6.1.8 is

\[ R \rightarrow G \rightarrow C \]

(A) \( -2 \)

(B) \( 6 \)

(C) \( -6 \)

(D) \( 2 \)

8. The block diagrams shown in fig. P.6.1.9 are equivalent if \( G \) is equal to

\[ R(s) \rightarrow G \rightarrow C(s) \]

(A) \( s + 1 \)

(B) \( 2 \)

(C) \( s + 2 \)

(D) \( 1 \)

9. Consider the systems shown in fig. P.6.1.10. If the forward path gain is reduced by 10% in each system, then the variation in \( C_1 \) and \( C_2 \) will be respectively

\[ R_1 \rightarrow 16 \rightarrow 3 \rightarrow C_1 \]

\[ R_2 \rightarrow 10 \rightarrow C_2 \]

(A) 10% and 1%

(B) 2% and 10%

(C) 10% and 0%

(D) 5% and 1%

10. The transfer function \( \frac{C}{R} \) of the system shown in fig. P.6.1.11 is

\[ R \rightarrow \frac{1}{H} \rightarrow H_2 \rightarrow G_1 \rightarrow C \]

(A) \( \frac{G_1 G_2}{1 + G_1 G_2 H + G_2 H_1} \)

(B) \( \frac{G_1 G_2}{1 + G_1 G_2 + H_1 H_2} \)

(C) \( \frac{G_1 G_2}{1 - G_1 H_1 - G_2 H_2 + G_1 G_2 H_1 H_2} \)

(D) \( \frac{G_1 G_2}{1 + G_1 H_1 + G_2 H_2 + G_1 G_2 H_1 H_2} \)
12. In the signal flow graph shown in fig. P.6.1.12, the sum of loop gain of non-touching loops is

(A) \( \frac{G_{1}H_{2}}{H_{1}(1+G_{1}G_{2}H_{2})} \)  
(B) \( \frac{G_{1}G_{2}H_{2}}{H_{1}(1+G_{1}G_{2}H_{2})} \)  
(C) \( \frac{G_{2}G_{1}}{1+H_{1}H_{2}G_{1}G_{2}} \)  
(D) \( \frac{G_{1}G_{2}}{H_{1}(1+G_{1}G_{2}H_{2})} \)

13. For the SFG shown in fig. P.6.1.14, the graph determinant \( \Delta \) is

(A) \( t_{22}t_{23} + t_{44} \)  
(B) \( t_{22}t_{23} + t_{34}t_{43} \)  
(C) \( t_{24}t_{32} + t_{44} \)  
(D) \( t_{24}t_{32} + t_{34}t_{43} + t_{44} \)

14. The sum of the gains of the feedback paths in the signal flow graph shown in fig. P.6.1.13 is

(A) \( 1 - bc - fg - bcfg + cij \)  
(B) \( 1 - bc - fg - cij + bcfg \)  
(C) \( 1 + bc + fg + cij - bcfg \)  
(D) \( 1 + bc + fg + bcfg - cij \)

15. A closed-loop system is shown in fig. P.6.1.15. The noise transfer function \( \frac{C_{n}(s)}{N(s)} \) is approximately

\[ R(s) \frac{1}{G_{1}(s)H_{1}(s)} \]

For \( |G_{1}(s)H_{1}(s)H_{2}(s)| << 1 \)

\[ \frac{1}{-H_{1}(s)} \]

For \( |G_{1}(s)H_{1}(s)H_{2}(s)| >> 1 \)

\[ \frac{1}{H_{1}(s)H_{2}(s)} \]

For \( |G_{1}(s)H_{1}(s)H_{2}(s)| << 1 \)

16. The overall transfer function \( \frac{C}{R} \) of the system shown in fig. P.6.1.16 will be

(A) \( G \)  
(B) \( \frac{G}{1 + H_{2}} \)  
(C) \( \frac{G}{(1 + H_{1})(1 + H_{2})} \)  
(D) \( \frac{G}{1 + H_{1} + H_{2}} \)

17. Consider the signal flow graphs shown in fig. P.6.1.17. The transfer 2 is of the graph

(A) \( a \)  
(B) \( b \)  
(C) \( b \) and \( c \)  
(D) \( a \), \( b \) and \( c \)
18. Consider the List I and List II

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Signal Flow Graph)</td>
<td>(Transfer Function)</td>
</tr>
<tr>
<td>P. 1. $a/x_{1}$</td>
<td>1. $a + b$</td>
</tr>
<tr>
<td>Q. $x_{2} a/b$, $x_{3}$</td>
<td>2. $ab$</td>
</tr>
<tr>
<td>S. $x_{1} a/b$, $x_{2}$</td>
<td>3. $a/(1-ab)$</td>
</tr>
<tr>
<td>R. $x_{1} a/b$, $x_{2}$</td>
<td>4. $a/(1-b)$</td>
</tr>
</tbody>
</table>

The correct match is

(A) 2 1 3 4
(B) 2 1 4 3
(C) 1 2 4 3
(D) 1 2 3 4

19. For the signal flow graph shown in fig. P6.1.19 an equivalent graph is

20. Consider the block diagram shown in figure P6.1.20

For this system the signal flow graph is

21. The block diagram of a system is shown in fig. P6.1.21. The closed loop transfer function of this system is

(A) \( \frac{G_{1}G_{2}G_{3}}{1 + G_{1}G_{2}G_{3}H_{1}} \)
(B) \( \frac{G_{1}G_{2}G_{3}}{1 + G_{1}G_{2}G_{3}H_{1}H_{2}} \)
(C) \( \frac{G_{1}G_{2}G_{3}}{1 + G_{1}G_{2}H_{1} + G_{2}G_{3}H_{2}} \)
(D) \( \frac{G_{1}G_{2}}{1 + G_{1}G_{2}H_{1} + G_{1}G_{2}H_{2} + G_{2}G_{3}H_{4}} \)

22. For the system shown in fig. P6.1.22 transfer function \( C(s)/R(s) \) is

For this system the signal flow graph is
23. In the signal flow graph shown in fig. P.6.1.23 the transfer function is

\[ \frac{G_3}{1 - H_1G_2 - H_2G_3 - G_1G_2H_2} \]

(A) \( \frac{G_3}{1 - H_1G_2 - H_2G_3 - G_1G_2H_2} \)

(B) \( \frac{G_3 + G_1G_2}{1 + H_1G_2 + H_2G_3 + G_1G_2H_2} \)

(C) \( \frac{G_3}{1 + H_1G_2 + H_2G_3 + G_1G_2H_2} \)

(D) \( \frac{G_3}{1 - H_1G_2 - H_2G_3 - G_1G_2H_2} \)

24. In the signal flow graph shown in fig. P.6.1.24 the gain \( C/R \) is

\[ \frac{G}{R} \]

(A) \( \frac{44}{23} \)

(B) \( \frac{29}{19} \)

(C) \( \frac{44}{19} \)

(D) \( \frac{29}{11} \)

25. The gain \( C(s)/R(s) \) of the signal flow graph shown in fig. P.6.1.25 is

\[ \frac{G}{R} \]

(A) \( \frac{G_3}{G_1G_2 + G_2G_3} \)

(B) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3} \)

(C) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3 - G_4} \)

(D) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3 - G_4} \)

26. The transfer function of the system shown in fig. P.6.1.26 is

\[ \frac{G_1G_2}{1 - G_1G_2H_4 - G_1G_2H_2} \]

(A) \( \frac{G_1G_2}{1 - G_1G_2H_4 - G_1G_2H_2} \)

(B) \( \frac{G_1G_2}{1 - G_1G_2H_4 - G_1G_2H_2} \)

(C) \( \frac{G_1G_2}{1 - G_1G_2H_4 - G_1G_2H_2} \)

(D) \( \frac{G_1G_2}{1 - G_1G_2H_4 - G_1G_2H_2} \)

27. For the block diagram shown in fig. P.6.1.27 transfer function \( C(s)/R(s) \) is

\[ \frac{G}{R} \]

(A) \( \frac{G_3}{G_1G_2} \)

(B) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3G_6 + G_2G_3G_6G_7} \)

(C) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3G_6G_7} \)

(D) \( \frac{G_3}{1 + G_1G_2 + G_2G_3 + G_1G_2G_3G_6G_7} \)

28. For the block diagram shown in fig. P.6.1.28 the numerator of transfer function is

\[ \frac{G}{R} \]

(A) \( G_4[G_4 + G_3 + G_3(G_2 + G_2)] \)

(B) \( G_4[G_2 + G_3 + G_3(G_2 + G_4)] \)
(C) $G_6[G_1+G_2+G_3(G_4+G_5)]$

(D) None of the above

29. For the block diagram shown in fig. P.6.1.29 the transfer function $C(s)/R(s)$ is
(A) $\frac{50(s-2)}{s^3 + s^2 + 150s - 100}$
(B) $\frac{50(s-2)}{s^3 + s^2 + 150s}$
(C) $\frac{50s}{s^3 + s^2 + 150s - 100}$
(D) $\frac{50}{s^2 + s + 150}$

30. For the SFG shown in fig. P.6.1.30 the transfer function $\frac{C}{R}$ is
\[\frac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]

31. Consider the SFG shown in fig. P.6.1.31. The $\Delta$ for this graph is
\[\frac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1 + G_2 + G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]
\[\frac{G_1G_2G_3}{1 + G_1H_1 + G_2H_2 + G_3H_3} \]

32. The transfer function of the system shown in fig. P.6.1.32 is
(A) $\frac{G_2G_3 + G_1G_1}{1 - G_1H_1 + G_2G_2}$
(B) $\frac{G_2G_3 + G_1G_1}{1 + G_3H_1 - G_4G_3}$
(C) $\frac{G_2G_3 + G_1G_1}{1 - G_4H_1 + G_2G_2}$
(D) $\frac{G_2G_3 + G_1G_1}{1 - G_4H_1 - G_2G_2}$

33. The closed loop transfer function of the system shown in fig. P.6.1.33 is
\[\frac{G_1G_2G_3 + G_2G_4G_4 + G_4G_4}{1 + G_1G_2G_3 + G_2G_4G_4 + G_4G_4} \]
\[\frac{G_1G_2G_3 + G_2G_4G_4 + G_4G_4}{1 + G_1G_2G_3 + G_2G_4G_4 + G_4G_4} \]
\[\frac{G_1G_2G_3 + G_2G_4G_4 + G_4G_4}{1 + G_1G_2G_3 + G_2G_4G_4 + G_4G_4} \]
\[\frac{G_1G_2G_3 + G_2G_4G_4 + G_4G_4}{1 + G_1G_2G_3 + G_2G_4G_4 + G_4G_4} \]

**Statement for Q.34-37:**

A block diagram of feedback control system is shown in fig. P.6.1.34-37

34. The transfer function $\frac{C}{R}$ is
(A) $\frac{G}{1-2G^2}$
(B) $\frac{G(1-G)}{1-2G^2}$
(C) $\frac{G(1-2G)}{1-G^2}$
(D) $\frac{G}{1-G^2}$
35. The transfer function \( \frac{C_1}{R_2} \) is
(A) \( \frac{G}{1 - 2G^2} \)  
(B) \( \frac{G}{1 - G^2} \)  
(C) \( \frac{G^2}{1 - 2G^2} \)  
(D) \( \frac{G^2}{1 - G^2} \)  

36. The transfer function \( \frac{C_2}{R_1} \) is
(A) \( \frac{G(1 + G)}{1 - 2G^2} \)  
(B) \( \frac{G}{1 - 2G^2} \)  
(C) \( \frac{G^2}{1 - G^2} \)  
(D) \( \frac{G}{1 - G^2} \)  

37. The transfer function \( \frac{C_2}{R_2} \) is
(A) \( \frac{G(1 + G)}{1 - 2G^2} \)  
(B) \( \frac{G}{1 - 2G^2} \)  
(C) \( \frac{G}{1 + G} \)  
(D) \( \frac{G}{1 - G^2} \)  

**Statement for Q.38-39:**

A signal flow graph is shown in fig. P.6.1.38-39.

![Signal Flow Graph](image1)

38. The transfer function \( \frac{Y_2}{Y_1} \) is
(A) \( \frac{1}{\Delta} \)  
(B) \( \frac{1 + G_2H_2}{\Delta} \)  
(C) \( \frac{G_1G_2G_3}{\Delta} \)  
(D) None of the above  

39. The transfer function \( \frac{Y_5}{Y_2} \) is
(A) \( \frac{G_1G_2G_3 + G_4G_5}{\Delta} \)  
(B) \( \frac{G_1G_2G_3 + G_4G_5}{\Delta} \)  
(C) \( \frac{G_1G_2G_3 + G_4G_5}{G_1G_2G_3} \)  
(D) \( \frac{G_1G_2G_3 + G_4G_5}{1 + G_2H_2} \)  

**Statement for Q.40-41:**

A block diagram is shown in fig. P.6.1.40-41.

![Block Diagram](image2)

40. The transfer function for this system is

![Diagram](image3)

(A) \( \frac{2s(2s + 1)}{2s^2 + 3s + 5} \)  
(B) \( \frac{2s(2s + 1)}{2s^2 + 13s + 5} \)  
(C) \( \frac{2s(2s + 1)}{4s^2 + 13s + 5} \)  
(D) \( \frac{2s(2s + 1)}{4s^2 + 3s + 5} \)  

41. The pole of this system are

(A) \( -0.75 \pm j1.39 \)  
(B) \(-0.41, -6.09 \)  
(C) \(-0.5, -1.67 \)  
(D) \(-0.25 \pm j0.88 \)  

********
SOLUTIONS

1. (A) \( G_c(s) = G_1(s) + G_2(s) + G_3(s) \)
\[
= \frac{1}{s+1} + \frac{1}{s+4} + \frac{s + 3}{s + 5}
\]
\[
= \frac{s^2 + 9s + 20 + s^2 + 6s + 5 + s^2 + 5s^2 + 4s + 3s^2 + 15s + 12}{(s + 1)(s + 4)(s + 5)}
\]
\[
= \frac{s^3 + 10s^2 + 34s + 37}{(s + 1)(s + 4)(s + 5)}
\]

2. (B) \( G_c(s) = G_1(s)G_2(s)G_3(s) = \frac{(s + 1)}{(s + 2)(s + 5)(s + 3)} \)

3. (C) \( \frac{C(s)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)} \)
\[
= \frac{s + 1}{s(s + 2)} \frac{1}{1 + \frac{(s + 1)(s + 4)}{(s + 4)(s + 5)}}
\]
\[
= \frac{s + 1}{s + 4} \frac{1}{s + 7s^2 + 12s + 3}
\]

4. (B) Multiply \( G_2 \) and \( G_4 \) and apply feedback formula and then again multiply with \( \frac{1}{G_1} \).

\( T(s) = \frac{G_2G_3}{G_1(1 + G_2G_4H_1)} \)

5. (D) \( T(s) = G_2(1 + G_1) + 1 = 1 + G_1 + G_1G_2 \)

6. (A) Open-loop gain = \( G_2 \)

Feed back gain = \( HG_1 \)

\( T_b(s) = \frac{G_2}{1 + G_1G_2H} \)

7. (D) Apply the feedback formula to both loop and then multiply

\( T(s) = \left( \frac{G_1}{1 + G_1H_1} \right) \left( \frac{G_2}{1 + G_2H_2} \right) \)
\[
= \frac{G_1G_2}{1 + G_1H_1 + G_2H_2 + G_1G_2H_1H_2}
\]

8. (C) For positive feedback \( C = \frac{6}{1 - \frac{6}{5}} = -6 \)

9. (D) For system (b) closed loop transfer function

\( G + 1 = \frac{G + s + 1}{s + 1}, \quad G + s + 1 = \frac{s + 2}{s + 1}, \quad \text{Hence} \ G = 1 \)

10. (A) In open loop system change will be 10% in \( C_i \) also but in closed loop system change will be less

\( C = \frac{10}{10 + 1} = \frac{10}{11} \)

11. (A) Apply the feedback formula and then multiply by \( \frac{1}{H_1} \).

\[
\frac{C}{R} = \frac{(H_2G_1)}{1 + H_2G_1} H_2G_1 = H_1(1 + G_2H_2H_1)
\]

12. (A) There cannot be common subscript because subscript refers to node number. If subscript is common, that means that node is in both loop.

13. (D) \( L_a = -bc, \quad L_b = -fg, \quad L_c = jgc, \quad L_d = bcfg \)

\( \Delta = 1 - (-bc + fg + cij) + bcfg = 1 + bc + fg - cij + bcfg \)

14. (A) In this graph there are three feedback loop. \( abef \) is not a feedback path because between path \( x_2 \) is a summing node.

15. (B) By putting \( R(s) = 0 \)

\( P_1 = -H_2G_1, \quad L_1 = -G_1H_2H_1, \quad \Delta_1 = 1, \quad T_n(s) = -\frac{H_2G_1}{1 + G_1H_2H_1} \)

if \( |G_1H_2H_1| >> 1, \quad T_n(s) = \frac{-H_2G_1}{G_1H_1H_1} = \frac{-1}{H_1} \)

16. (C) \( P_1 = G, \quad L_1 = -H_1, \quad L_2 = -H_2, \quad L_3L_4 = H_1H_2, \quad \Delta_1 = 1 \)

\( T(s) = \frac{G}{1 + H_1 + H_2 + H_1H_2} = \frac{G}{(1 + H_1)(1 + H_2)} \)

17. (B) \( G_a = 1, \quad G_b = 1 + 1 = 2, \quad G_c = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1 \)

There are no loop in any graph. So option (B) is correct.

18. (B)

P. \( P_1 = ab, \quad \Delta = 1, \quad L = 0, \quad T = ab \)

Q. \( P_2 = a, \quad P_3 = b, \quad \Delta = 1, \quad L = 0, \quad T = a + b \)

R. \( P_4 = a, \quad L_4 = b, \quad \Delta = 1 - b, \quad \Delta = 1, \quad T = \frac{a}{a - b} \)

S. \( P_5 = a, \quad L_5 = ab, \quad \Delta = 1 - ab, \quad \Delta = 1, \quad T = \frac{a}{a - b} \)

19. (A) Between \( e_1 \) and \( e_2 \), there are two parallel path. Combining them gives \( t_s + t_a \). Between \( e_2 \) and \( e_4 \) there is a path given by total gain \( t_a t_s \). So remove node \( e_1 \) and place gain \( t_s t_a \) of the branch \( e_2 e_4 \). Hence option (A) is correct.
20. (A) Option (A) is correct. Best method is to check the signal flow graph. In block diagram there is feedback from 4 to 1 of gain \(-H_1 H_2\). The signal flow graph of option (A) has feedback from 4 to 1 of gain \(-H_1 H_2\).

21. (C) Consider the block diagram as SFG. There are two feedback loop \(-G_1 G_2 H_1\) and \(-G_2 G_1 H_2\) and one forward path \(G_1 G_2 G_3\). So (D) is correct option.

22. (B) Consider the block diagram as a SFG. Two forward path \(G_1 G_2\) and \(G_3\) and three loops

\(-G_1 G_2 H_1\) , \(-G_2 H_1\) , \(-G_2 H_2\).

There are no nontouching loop. So (B) is correct.

23. (C) \(P_1 = 5 \times 3 \times 2 = 30\), \(\Delta = 1 - (3 \times 3) = 10\)

\[\Delta_1 = 1, \quad \frac{C}{R} = \frac{30}{10} = 3\]

24. (A) \(P_1 = 2 \times 3 \times 4 = 24\), \(P_2 = 1 \times 5 \times 1 = 5\)

\(L_1 = -2\), \(L_2 = -3\), \(L_3 = -4\), \(L_4 = -5\), \(L_5 L_6 = 8, \Delta = 1 - (-2 - 3 - 4 - 5) + 8 = 23\),

\(\Delta_1 = 1, \Delta_2 = 1 - (-3) = 4,\)

\[\frac{C}{R} = \frac{24 + 5 \times 4}{24} = \frac{44}{24} = 23\]

25. (B) \(P_1 = G_1 G_2\), \(P_2 = G_1 G_2\)

\(L_1 = -G_3 G_2 H_1\), \(L_2 = -G_1 G_2 H_1\), \(L_3 = G_4\), \(\Delta_2 = 1\)

There are no nontouching loop.

\[T_s = \frac{P_1 \Delta_1 + P_2 \Delta_2}{1 - (L_1 + L_2 + L_3)} = \frac{G_1 G_2 + G_1 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_1 - G_4}\]

26. (C) \(P_1 = G_1 G_2\), \(P_2 = G_1 G_2\)

\(L_1 = -G_3 G_2 H_1\), \(L_2 = -G_1 G_2 H_2\), \(L_3 = G_4\), \(\Delta_1 = \Delta_2 = 1\)

\[C(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_2 - G_3 G_1 H_2}\]

27. (B) There is one forward path \(G_s G_2\).

Four loops \(-G_s G_1\), \(-G_1 G_2 G_3\), \(-G_1 G_2 G_3 G_7\),

and \(-G_2 G_1 G_2 G_6 G_7\).

28. (A) SFG:

\[
P_1 = G_2 G_3 G_6, P_2 = G_3 G_4 G_6, P_3 = G_4 G_6, P_4 = G_1 G_6\]

If any path is deleted, there would not be any loop.

Hence \(\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1\)

\[
C(s) = \frac{G_1 G_2 + G_3 G_6 + G_1 G_2 G_6 + G_1 G_3 G_6}{\Delta}
\]

29. (A)

\[
P_1 = \frac{1}{s^2} \left(\frac{50}{s + 1}\right) \cdot \frac{s}{s^2}, P_2 = \frac{1}{s^2} \left(\frac{50}{s + 1}\right) \cdot (-2) = -\frac{100}{s^2(s + 1)}\]

\[
L_1 = \frac{50}{s^2}, L_2 = \frac{-2}{s^2} = -\frac{100}{s^2(s + 1)}\]

\[
L_3 = \frac{50}{s^2}, L_4 = \frac{-2}{s^2} = -\frac{100}{s^2(s + 1)}\]

\(\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1\)

\[
C(s) = \frac{P_1 + P_2}{\Delta} = \frac{50(s - 2)}{s^3 + s^2 + 150s - 100}\]

30. (D) \(P_1 = G_1 G_2 G_3\)

\(L_1 = -G_1 H_1\), \(L_2 = -G_2 H_2\), \(L_3 = -G_3 H_3\)

\[
L_4 L_5 = G_1 G_2 H_4 H_3\]

\[
\Delta = 1 - (-G_1 H_1 - G_2 H_2 - G_3 H_3) + G_4 G_2 H_4 H_3\]

\[
\Delta = 1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_1 G_3 H_3 H_3\]

\(\Delta_1 = 1\)

\[
C(s) = \frac{G_1 G_2 G_3}{1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_1 G_3 H_3 H_3}\]
CHAPTER

6.2

STABILITY

1. Consider the system shown in fig. P6.2.1. The range of \( K \) for the stable system is

\[
\begin{align*}
1 \quad \text{(A)} & \quad K < \frac{1}{2} \\
2 \quad \text{(B)} & \quad \frac{1}{2} < K < 1 \\
3 \quad \text{(C)} & \quad -1 < K < 1 \\
4 \quad \text{(D)} & \quad \text{Unstable}
\end{align*}
\]

2. The forward transfer function of a \( \text{ufb} \) system is

\[
G(s) = \frac{K(s^2 + 1)}{(s + 1)(s + 2)}
\]

The system is stable for

\[
\begin{align*}
\text{(A)} & \quad K < -1 \\
\text{(B)} & \quad K > -1 \\
\text{(C)} & \quad K < -2 \\
\text{(D)} & \quad K > -2
\end{align*}
\]

3. The open-loop transfer function with \( \text{ufb} \) are given below for different systems. The unstable system is

\[
\begin{align*}
\text{(A)} & \quad \frac{2}{s + 2} \\
\text{(B)} & \quad \frac{2}{s^2(s + 2)} \\
\text{(C)} & \quad \frac{2}{s(s + 2)} \\
\text{(D)} & \quad \frac{2(s + 1)}{s(s + 2)}
\end{align*}
\]

4. Consider a \( \text{ufb} \) system with forward-path transfer function

\[
G(s) = \frac{K(s + 3)(s + 5)}{(s - 2)(s - 4)}
\]

The range of \( K \) to ensure stability is

\[
\begin{align*}
\text{(A)} & \quad K > \frac{6}{8} \\
\text{(B)} & \quad K < -1 \text{ or } K > \frac{3}{4} \\
\text{(C)} & \quad K < -1 \\
\text{(D)} & \quad -1 < K < \frac{3}{4}
\end{align*}
\]

5. Consider a \( \text{ufb} \) system with forward-path transfer function

\[
G(s) = \frac{K(s + 3)}{s^2(s + 2)}
\]

The system is stable for the range of \( K \)

\[
\begin{align*}
\text{(A)} & \quad K > 0 \\
\text{(B)} & \quad K < 0 \\
\text{(C)} & \quad K > 1 \\
\text{(D)} & \quad \text{Always unstable}
\end{align*}
\]

6. The open-loop transfer function of a \( \text{ufb} \) control system is

\[
G(s) = \frac{K(s + 2)}{(s + 1)(s - 7)}
\]

For \( K > 6 \), the stability characteristic of the open-loop and closed-loop configurations of the system are respectively

\[
\begin{align*}
\text{(A)} & \quad \text{stable and unstable} \\
\text{(B)} & \quad \text{stable and stable} \\
\text{(C)} & \quad \text{unstable and stable} \\
\text{(D)} & \quad \text{unstable and unstable}
\end{align*}
\]

7. The forward-path transfer function of a \( \text{ufb} \) system is

\[
G(s) = \frac{K(s^2 - 4)}{s^2 + 3}
\]

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For the system to be stable the range of $K$ is
(A) $K > -1$  
(B) $K < \frac{3}{4}$
(C) $-1 < K < \frac{3}{4}$  
(D) marginal stable

3. A u/fb system have the forward-path transfer function

$$G(s) = \frac{K(s + 6)}{s(s + 1)(s + 3)}$$

The system is stable for
(A) $K < 6$  
(B) $-6 < K < 0$
(C) $0 < K < 6$  
(D) $K > 6$

4. The feedback control system shown in the fig. P6.2.8.

is stable for all positive value of $K$, if
(A) $T = 0$  
(B) $T < 0$
(C) $T > 1$  
(D) $0 < T < 1$

10. Consider a u/fb system with forward-path transfer function

$$G(s) = \frac{K}{(s + 15)(s + 27)(s + 38)}$$

The system will oscillate for the value of $K$ equal to
(A) 23690  
(B) 2369
(C) 144690  
(D) 14469

11. The forward-path transfer function of a u/fb system is

$$G(s) = \frac{K(s - 2)(s + 4)(s + 5)}{(s^2 + 3)}$$

For system to be stable, the range of $K$ is
(A) $K > \frac{1}{54}$  
(B) $K < \frac{3}{40}$
(C) $\frac{1}{54} < K < \frac{3}{40}$  
(D) Unstable

12. The closed loop system shown in fig. P6.2.12 become marginally stable if the constant $K$ is chosen to be
(A) 30  
(B) $-30$
(C) 10  
(D) $-10$

13. The open-loop transfer function of a u/fb system is

$$G(s) = \frac{K(s + 10)(s + 20)}{s^2(s + 2)}$$

The closed loop system will be stable if the value of $K$ is
(A) 2  
(B) 3
(C) 4  
(D) 5

Statement for Q.14–15:
A feedback system is shown in fig. P6.14-15.

14. The closed loop transfer function for this system is

(A) $\frac{s^5 + s^4 + 2s^3 + (K + 2)s^2 + (K + 2)s + K}{s^2 + s^2 + 2s + K}$

(B) $\frac{2s^4 + (K + 2)s^3 + Ks^2}{s^3 + s^2 + 2s + K}$

(C) $\frac{s^3 + s^2 + 2s + K}{s^3 + s^4 + 2s^3 + (K + 2)s^2 + (K + 2)s + K}$

(D) $\frac{s^4 + s^2 + 2s + K}{2s^4 + (K + 2)s^3 + Ks^2}$

15. The poles location for this system is shown in fig. P6.2.15. The value of $K$ is

(A) 4  
(B) $-4$
(C) 2  
(D) $-2$
27. The forward path transfer of uf b system is
\[ G(s) = \frac{1}{4s^2(s^2 + 1)} \]

The system is
(A) stable (B) unstable (C) marginally stable (D) More information is required.

28. The forward-path transfer function of a uf b system is
\[ G(s) = \frac{G(s)}{2s^4 + 5s^3 + s^2 + 2s} \]

The system is
(A) stable (B) unstable (C) marginally stable (D) More information is required.

29. The open loop transfer function of a system is as
\[ G(s)H(s) = \frac{K(s + 0.1)}{s(s - 0.2)(s^2 + s + 0.6)} \]

The range of \( K \) for stable system will be
(A) \( K > 0.355 \) (B) \( 0.149 < K < 0.355 \) (C) \( 0.236 < K < 0.44 \) (D) \( K > 0.44 \)

30. The open-loop transfer function of a uf b control system is given by
\[ G(s) = \frac{K}{s(T_1 + 1)(sT_2 + 1)} \]
For the system to be stable the range of \( K \) is
(A) \( 0 < K < \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \) (B) \( K > \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \)
(C) \( 0 < K < T_1T_2 \) (D) \( K > T_1T_2 \)

31. The closed loop transfer function of a system is
\[ T(s) = \frac{(s + 8)(s + 6)}{s^3 - s^4 + 4s^3 - 4s^2 + 3s - 2} \]
The number of poles in RHP and in LHP are
(A) 4, 1 (B) 1, 4 (C) 3, 2 (D) 2, 3

32. The closed loop transfer function of a system is
\[ T(s) = \frac{s^2 + 3s^2 + 7s + 24}{s^5 - 2s^4 + 3s^3 - 6s^2 + 2s - 4} \]
The number of poles in LHP, in RHP, and on jω axis are
(A) 2, 1, 2 (B) 0, 1, 4 (C) 1, 0, 4 (D) 1, 2, 2

33. For the system shown in fig. P6.2.34, the number of poles on RHP, LHP, and imaginary axis are
(Fig. P6.2.34)

(A) 2, 3, 0 (B) 3, 2, 0 (C) 2, 1, 2 (D) 1, 2, 2

34. A Routh table is shown in fig. P6.2.36. The location of pole on RHP, LHP and imaginary axis are
(Fig. P6.2.35)

(A) 1, 2, 4 (B) 1, 6, 0 (C) 1, 0, 6 (D) None of the above

35. For the open loop system of fig. P6.2.35 location of poles on RHP, LHP, and imaginary axis are
(Fig. P6.2.35)

(A) 3, 3, 0 (B) 1, 3, 2 (C) 1, 1, 4 (D) 3, 1, 2
11. (C) \[ T(s) = \frac{G(s)}{1 + G(s)} \]
\[ = \frac{K(s - 2)(s + 4)(s + 5)}{Ks^3 + (7K + 1)s^2 + 2Ks + (3 - 40K)} \]
Routh table is as shown in fig. S.6.211

<table>
<thead>
<tr>
<th>( s^3 )</th>
<th>( K )</th>
<th>( 2K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>( 7K + 1 )</td>
<td>( 3 - 40K )</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( \frac{54K^2 - K}{2K} )</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>3 - 40K</td>
<td></td>
</tr>
</tbody>
</table>

Fig. S.6.211

\( K > 0, \)
\[ 7K + 1 > 0 \quad \Rightarrow \quad K > -\frac{1}{7} \]
\[ \frac{54K^2 - K}{7K + 1} > 0 \quad \Rightarrow \quad K > \frac{1}{54} \]
\[ \Rightarrow \quad \frac{1}{54} < K < \frac{3}{40} \]

12. (A) \[ T(s) = \frac{1}{s^3 + 6s^2 + 5s + K} \]
Routh table is as shown in fig. S.6.212

<table>
<thead>
<tr>
<th>( s^3 )</th>
<th>1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>6</td>
<td>( K )</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( 30 - K )</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>( K )</td>
<td></td>
</tr>
</tbody>
</table>

Fig. S.6.212

13. (D) \[ T(s) = \frac{K(s + 10)(s + 20)}{s^3 + (K + 2)s^2 + 30Ks + 200K} \]
Routh table is as shown in fig. S.6.213

<table>
<thead>
<tr>
<th>( s^3 )</th>
<th>1</th>
<th>30K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>( K + 2 )</td>
<td>200K</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( 30K^2 - 140K )</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>200K</td>
<td></td>
</tr>
</tbody>
</table>

Fig. S.6.213

\( 200K > 0 \quad \Rightarrow \quad K > 0, \quad 30K^2 - 140K > 0 \)
\[ \Rightarrow \quad K > \frac{14}{3} \]
\( 5 \) satisfy this condition.

14. (B) First combine the parallel loop \( \frac{K}{s^2} \) and \( \frac{2}{s} \) giving \( \frac{K}{s^2} + \frac{2}{s} \). Then apply feedback formula with \( \frac{K}{s^3} + \frac{2}{s} \) and \( \frac{1}{s + 1} \) and then multiply with \( s^2 \).

\[ T(s) = \frac{\frac{K}{s^3} + \frac{2}{s}}{1 + \frac{1}{s + 1}} \]
\[ = \frac{2s^4 + (K + 2)s^3 + Ks^2}{s^3 + s^2 + 2s + K} \]

15. (C) Denominator \( s^3 + s^2 + 2s + K \)
Routh table is as shown in fig. S.6.215

<table>
<thead>
<tr>
<th>( s^3 )</th>
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<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>1</td>
<td>( K )</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( 2 - K )</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>( K )</td>
<td></td>
</tr>
</tbody>
</table>

Fig. S.6.215

Row of zeros when \( K = 2, \)
\( s^2 + 2 = 0, \quad \Rightarrow \quad s = -1, \quad j\sqrt{2}, \quad -j\sqrt{2} \)

16. (D) Applying the feedback formula on the inner loop and multiplying by \( K \) yield
\[ G_c(s) = \frac{K}{s(s^2 + 5s + 7)} \]
\[ T(s) = \frac{K}{s^3 + 5s^2 + 7s + K} \]

17. (B) Routh table is as shown in fig. S.6.217

<table>
<thead>
<tr>
<th>( s^3 )</th>
<th>1</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^2 )</td>
<td>5</td>
<td>( K )</td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( \frac{35 - K}{K} )</td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>( K )</td>
<td></td>
</tr>
</tbody>
</table>

Fig. S.6.217

\( K > 0, \quad \frac{35 - K}{5} > 0 \quad \Rightarrow \quad K < 35 \)

18. (C) At \( K = 35 \) system will oscillate.
Auxiliary equation \( 5s^2 + 35 = 0, \quad \Rightarrow \quad s = \pm j\sqrt{7} \)

19. (B) For inner loop
\[ G_c(s) = \frac{K}{(s - a)(s + 3a)(s + 4a)} = \frac{K}{P(s)}, \]
\[ T_c(s) = \frac{K}{P(s) + K} \]
For outer loop, \( G_c(s) = T_c(s) = \frac{K}{P(s) + K} \),
\[ T_i(s) = \frac{K}{P(s) + 2K}, \]

Therefore if inner loop is stable for \( X < K < Y \), then outer loop will be stable for \( X < 2K < Y \)
\[ \Rightarrow \frac{X}{2} < K < \frac{Y}{2}. \]

20. (D) \( T(s) = \frac{K(s + 2)}{s^4 + 3s^3 - 3s^2 + (K + 3)s + (2K - 4)} \)

Routh table is as shown in fig. S.6.2.20

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & -3 & 2K - 4 \\
\text{s}^3 & 3 & K + 3 \\
\text{s}^2 & \frac{-\left(K + 12\right)}{3} & 2K - 4 \\
\text{s}^1 & \frac{K(K + 33)}{K - 12} \\
\text{s}^0 & 2K - 4 \\
\end{array}
\]

\( \frac{-(K + 12)}{3} > 0 \Rightarrow K < -12, \ 2K - 4 > 0 \)
\[ \Rightarrow K > 2 \ \text{and} \ K > -33, \] These condition cannot be met simultaneously. System is unstable for any value of \( K \).

21. (D) Routh table is as shown in fig. S.6.2.21

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & 1 & 1 \\
\text{s}^3 & K & 1 \\
\text{s}^2 & \frac{K - 1}{K} & 1 \\
\text{s}^1 & \frac{K - 1 - K^2}{K - 1} \\
\text{s}^0 & 1 \\
\end{array}
\]

\( K > 0, \ K - 1 > 0 \Rightarrow K > 1, \ \frac{K - 1 - K^2}{K - 1} > 0, \) But for \( K > 1 \) third term is always -ive. Thus the three condition cannot be fulfilled simultaneously.

22. (D) Routh table is as shown in fig. S.6.2.22

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & 4 + K & 25 \\
\text{s}^3 & 2 & 9 \\
\text{s}^2 & \frac{2K - 1}{2} & 25 \\
\text{s}^1 & \frac{18K - 109}{2K - 1} \\
\text{s}^0 & 25 \\
\end{array}
\]

\( \frac{2K - 1}{2} > 0 \Rightarrow K > \frac{1}{2} \)
\( \frac{18K - 109}{2K - 1} > 0 \Rightarrow K > \frac{109}{18} \)
\[ \Rightarrow K > \frac{109}{18} \]

23. (B) Characteristic equation
\[ s^4 + 9s^3 + 20s^2 + Ks + K = 0 \]

Routh table is as shown in fig. S.6.2.23

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & 20 & K \\
\text{s}^3 & 9 & K \\
\text{s}^2 & \frac{180 - K}{3} & K \\
\text{s}^1 & \frac{K(K - 39)}{K - 190} \\
\text{s}^0 & K \\
\end{array}
\]

For stability \( 0 < K < 99 \)

24. (C) \( T(s) = \frac{K(s + 2)}{s^4 + 3s^3 - 3s^2 + (K + 3)s + (2K - 4)} \)

Routh table is as shown in fig. S.6.2.24

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & -3 & 2K - 4 \\
\text{s}^3 & 3 & K + 3 \\
\text{s}^2 & \frac{-K + 12}{3} & 2K - 4 \\
\text{s}^1 & \frac{K(K + 33)}{K - 12} \\
\text{s}^0 & 2K - 4 \\
\end{array}
\]

\( K > -33, \ 1 \text{ sign change} \)
For \( -33 < K < -12, \ 1 \text{ sign change} \)
For \( -12 < K < 0, \ 1 \text{ sign change} \)
For \( 0 < K < 2, \ 3 \text{ sign change} \)
For \( K > 2, \ 2 \text{ sign change} \)
Therefore \( K > 2 \) yield two RHP pole.

25. (B) Routh table is as shown in fig. S.6.2.25

\[
\begin{array}{c|cccc}
\text{s}^4 & 1 & 8 & 15 \\
\text{s}^3 & 4 & 20 \\
\text{s}^2 & 3 & 15 \\
\text{s}^1 & 6 & \text{ROZ} \\
\text{s}^0 & 15 \\
\end{array}
\]

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\( P(s) = 3s^2 + 15, \frac{d P(s)}{ds} = 6s, \) No sign change from \( s^2 \) to \( s^0 \) on \( j\omega \)-axis 2 roots, RHP 0, LHP 2.

**26. (B)** Closed-loop transfer function is
\[
T(s) = \frac{G(s)}{1 + G(s)} = \frac{240}{s^4 + 10s^3 + 35s^2 + 50s + 264}
\]
Routh table is as shown in fig. S.6.2.26

| \( s^4 \) | 1 | 35 | 264 |
| \( s^3 \) | 10 | 50 |
| \( s^2 \) | 30 | 264 |
| \( s^1 \) | -386 | ROZ |
| \( s^0 \) | 264 |

Fig. S.6.2.26

Two sign change. RHP-2 poles. System is not stable.

**27. (C)** Closed loop transfer function
\[
T(s) = \frac{G(s)}{1 + G(s)} = \frac{1}{4s^4 + 8s^2 + 1}
\]
Routh table is as shown in fig. S.6.2.27

| \( s^4 \) | 4 | 4 | 1 |
| \( s^3 \) | 16 | 8 | ROZ |
| \( s^2 \) | 2 | 1 |
| \( s^1 \) | 46 | ROZ |
| \( s^0 \) | 1 |

Fig. S.6.2.27

\( P(s) = 4s^4 + 4s + 1, \frac{d P(s)}{ds} = 16s^3 + 3s \)

There is no sign change. So all pole are on \( j\omega \)-axis. So system is marginally stable.

**28. (B)** Closed loop transfer function
\[
T(s) = \frac{G(s)}{1 + G(s)} = \frac{1}{2s^4 + 5s^3 + s^2 + 2s + 1}
\]
Routh table is as shown in fig. S.6.2.28

| \( s^4 \) | 2 | 1 | 1 |
| \( s^3 \) | 5 | 2 |
| \( s^2 \) | \( \frac{1}{2} \) | 1 |
| \( s^1 \) | -23 |
| \( s^0 \) | 1 |

Fig. S.6.2.28

2 RHP poles so unstable.

**29. (B)** The characteristic equation is \( 1 + G(s)H(s) = 0 \)
\[
\Rightarrow \quad s(s - 0.2)(s^2 + s + 0.6) + K(s + 0.1) = 0
\]
\[
s^4 + 0.8s^3 + 0.4s^2 + (K - 0.12)s + 0.1K = 0
\]
Routh table is as shown in fig. S.6.2.29

| \( s^4 \) | 2 | 0.4 | 1 |
| \( s^3 \) | 0.8 | \( K - 0.12 \) |
| \( s^2 \) | \( 0.55 - 125K \) | 0.1K |
| \( s^1 \) | \( \frac{-1.25K^2 + 0.63K - 0.066}{0.55 - 1.25K} \) |
| \( s^0 \) | 0.1K |

Fig. S.6.2.29

\( K > 0, \ 0.55 - 125K > 0 \quad \Rightarrow \quad K < 0.44 \)
\[ -125K^2 + 0.63K - 0.066 > 0 \]
\[ (K - 0.149)(K - 0.355) < 0, \ 0.149 < K < 0.355 \]

**30. (A)** Characteristic equation
\[
s(sT_1 + 1)(sT_2 + 1) + K = 0
\]
\[
T_1T_2s^3 + (T_1 + T_2)s^2 + s + K = 0
\]
Routh table is as shown in fig. S.6.2.30

| \( s^3 \) | \( T_1T_2 \) | 1 |
| \( s^2 \) | \( T_1 + T_2 \) | \( K \) |
| \( s^1 \) | \( \frac{(T_1 + T_2)K}{T_1 + T_2} \) |
| \( s^0 \) | \( K \) |

Fig. S.6.2.30

\( K > 0, \ (T_1 + T_2) - T_1T_2K > 0 \quad \Rightarrow \quad 0 < K < \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \)

**31. (B)** Routh table is as shown in fig. S.6.2.31

| \( s^5 \) | 1 | 5 | 1 |
| \( s^4 \) | 3 | 4 | 3 |
| \( s^3 \) | 3.67 | 0 |
| \( s^2 \) | 4 | 3 |
| \( s^1 \) | -2.75 |
| \( s^0 \) | 3 |

In RHP -2 poles. In LHP -3 poles.
32. (C) Routh table is as shown in fig. S.6.2.32

\[
\begin{array}{c|cccc}
\varepsilon & s^5 & 1 & 4 & 3 \\
+ & + & s^4 & -1 & -4 & -2 \\
- & - & s^3 & \varepsilon & 1 \\
+ & - & s^2 & \frac{1-4\varepsilon}{\varepsilon} & -2 \\
+ & + & s^1 & \frac{2(\varepsilon+1)}{1-4\varepsilon} & \varepsilon \\
- & - & s^0 & -2 \\
\end{array}
\]

Fig. S.6.2.32

3 RHP, 2 LHP poles.

33. (B) Routh table is as shown in fig. S.6.2.33

\[
\begin{array}{c|cccc}
\varepsilon & s^5 & 1 & 3 & 2 \\
+ & + & s^4 & -2 & -6 & -4 \\
- & - & s^3 & -2 & -3 & ROZ \\
- & - & s^2 & -3 & -4 \\
- & - & s^1 & \frac{1}{3} & -4 \\
- & - & s^0 & -4 \\
\end{array}
\]

Fig. S.6.2.33

\[P(s) = -2s^4 - 6s^2, \quad \frac{dP(s)}{ds} = -8s^3 - 12s, \quad -2, \quad -3\]

No sign change exist from the \(s^4\) row down to the \(s^0\) row. Thus, the even polynomial does not have RHP poles. Therefore because of symmetry all four poles must be on \(j\omega\) axis.

- \(j\omega\) axis: 4 pole
- RHP: 1 pole (1 sign change)
- LHP: 0 pole

34. (D) Closed loop transfer function

\[T(s) = \frac{G(s)}{1 + G(s) + H(s)} = \frac{507s}{s^3 + 3s^4 + 10s^3 + 30s^2 + 169s + 507}\]

Routh table is as shown in fig. S.6.2.34

\[
\begin{array}{c|cccc}
\varepsilon & s^5 & 1 & 10 & 69 \\
+ & + & s^4 & 3 & 30 & 57 \\
+ & - & s^3 & 12 & 60 & ROZ \\
+ & + & s^2 & 15 & 507 \\
+ & + & s^1 & -345.6 \\
- & - & s^0 & 507 \\
\end{array}
\]

Fig. S.6.2.33

\[P(s) = 3s^4 + 30s^2 + 507, \quad \frac{dP(s)}{ds} = 12s^3 + 60\]

From \(s^4\) row down to \(s^0\) there is one sign change. So LHP–1 + 1 = 2 pole. RHP–1 pole, \(j\omega\) axis –2 pole.

35. (A) Notice that in \(s^5\) row there would be zero. In this row coefficient of \(\frac{dP(s)}{ds}\), where \(P(s) = s^6 + 2s^4 - s^2 - 2\) have been entered. From \(s^6\) to row down to the \(s^0\) row, there is one sign change. So there is one pole on RHP. Corresponding to this pole there is a pole on LHP. Corresponding to this pole there is a pole on LHP. Rest 4 out of 6 poles are on imaginary axis. Rest 1 pole is on LHP.

36. (A) Routh table is as shown in fig. S.6.2.36

\[
\begin{array}{c|cccc}
\varepsilon & s^6 & 1 & -6 & -6 \\
+ & + & s^5 & 1 & 0 \\
+ & - & s^4 & -6 & 0 \\
- & - & s^3 & -24 & 0 & ROZ \\
- & - & s^2 & \varepsilon \\
- & - & s^1 & \frac{144}{7} \\
- & - & s^0 & -6 \\
\end{array}
\]

Fig. S.6.2.36

\[P(s) = -6s^4 - 6, \quad \frac{dP(s)}{ds} = -24s^3, \]

There is two sign change from the \(s^4\) row down to the \(s^0\) row. So two roots are on RHS. Because of symmetry rest two roots must be in LHP. From \(s^6\) to \(s^4\) there is 1 sign change so 1 on RHP and 1 on LHP.

Total LHP 3 root, RHP 3 root.
3. A system is shown in fig. P6.3.8. The rise time and settling time for this system is

\[ T(s) = \frac{1}{s} \quad R(s) \quad \frac{10}{s + 10} \quad C(s) \]

Fig. P6.3.8

(A) 0.22s, 0.4s  \quad (B) 0.4s, 0.22s
(C) 0.12s, 0.4s  \quad (D) 0.4s, 0.12s

9. For a second order system settling time is \( T_s = 7 \) s and peak time is \( T_p = 3 \) s. The location of poles are

(A) \(-0.97 \pm j0.69\)  \quad (B) \(-0.69 \pm j0.97\)
(C) \(-1.047 \pm j0.571\)  \quad (D) \(-0.571 \pm j1.047\)

10. For a second order system overshoot =10% and peak time \( T_p = 5 \) s. The location of poles are

(A) \(-0.46 \pm j0.63\)  \quad (B) \(-0.63 \pm j0.46\)
(C) \(-0.74 \pm j0.92\)  \quad (D) \(-0.92 \pm j0.74\)

11. For a second-order system overshoot =12 % and settling time =0.6 s. The location of poles are

(A) \(-9.88 \pm j6.67\)  \quad (B) \(-6.67 \pm j9.88\)
(C) \(-4.38 \pm j6.46\)  \quad (D) \(-6.46 \pm j4.38\)

Statement for Q.12–13:

A system has a damping ratio of 1.25, a natural frequency of 200 rad/s and DC gain of 1.

12. The response of the system to a unit step input is

(A) \(1 + \frac{5}{3}e^{-50t} - \frac{2}{3}e^{-150t}\)  \quad (B) \(1 - \frac{4}{3}e^{-100t} + \frac{1}{3}e^{-400t}\)
(C) \(1 + \frac{1}{3}e^{-100t} - \frac{4}{3}e^{-400t}\)  \quad (D) \(1 + \frac{2}{3}e^{-50t} - \frac{5}{3}e^{-150t}\)

13. The system is

(A) overdamped  \quad (B) under damped
(C) critically damped  \quad (D) None of the above

14. Consider the following system

a. \( T(s) = \frac{5}{(s + 3)(s + 6)}\)

b. \( T(s) = \frac{10(s + 7)}{(s + 10)(s + 20)}\)

c. \( T(s) = \frac{20}{s^2 + 6s + 144}\)

d. \( T(s) = \frac{s + 2}{s^2 + 9}\)

e. \( T(s) = \frac{(s + 5)}{(s + 10)^2}\)

Consider the following response

1. Overdamped  \quad 2. Under damped

The correct match is

1 2 3 4
(A) a c d e
(B) b a d e
(C) c a e d
(D) c b e d

15. The forward-path transfer of a \( \text{ufb} \) control system is

\[ G(s) = \frac{1000}{(1 + 10s)(1 + 10s)} \]

The step, ramp, and parabolic error constants are

(A) 0, 1000, 0  \quad (B) 1000, 0, 0
(C) 0, 0, 0  \quad (D) 0, 0, 1000

16. The open-loop transfer function of a \( \text{ufb} \) control system is

\[ G(s) = \frac{K(1 + 2s)(1 + 4s)}{s^2(s^2 + 2s + 8)} \]

The position, velocity and acceleration error constants are respectively

(A) 0, 0, 4K  \quad (B) \infty, \frac{K}{8}, 0
(C) 0, 4K, \infty  \quad (D) \infty, \infty, \frac{K}{8}

17. The open-loop transfer function of a unit feedback system is

\[ G(s) = \frac{50}{(1 + 0.1s)(1 + 2s)} \]

The position, velocity and acceleration error constants are respectively

(A) 0, 0, 250  \quad (B) 50, 0, 0
(C) 0, 250, \infty  \quad (D) \infty, 50, 0
Statement for Q.18–19:

The forward-path transfer function of a unity feedback system is

\[ G(s) = \frac{K}{s^3(s + a)} \]

The system has 10% overshoot and velocity error constant \( K_v = 100 \).

18. The value of \( K \) is
   (A) \( 237 \times 10^3 \)
   (B) 144
   (C) 14 \times 10^3
   (D) 237

19. The value of \( a \) is
   (A) \( 237 \times 10^3 \)
   (B) 237
   (C) \( 14 \times 10^3 \)
   (D) 144

20. For the system shown in fig. P6.3.20 the steady state error component due to unit step disturbance is 0.000012 and steady state error component due to unit ramp input is 0.003. The values of \( K_1 \) and \( K_2 \) are respectively

21. The transfer function for a single loop nonunity feedback control system is

\[ G(s) = \frac{1}{s^3 + s + 2}, \quad H(s) = \frac{1}{s + 1} \]

The steady state error due to unit step input is
   (A) \( \frac{6}{7} \)
   (B) \( \frac{6}{5} \)
   (C) \( \frac{2}{3} \)
   (D) 0

22. For the system of fig. P6.3.22 the total steady state error due to a unit step input and a unit step disturbance is

23. The forward path transfer function of a \( ufb \) system is

\[ G(s) = \frac{K}{s(s + 4)(s + 8)(s + 10)} \]

If a unit ramp is applied, the minimum possible steady-state error is
   (A) 0.16
   (B) 6.25
   (C) 0.14
   (D) 7.25

24. The forward-path transfer function of a \( ufb \) system is

\[ G(s) = \frac{1000(s^2 + 4s + 20)(s^2 + 20s + 15)}{s^3(s + 2)(s + 10)} \]

The system has \( r(t) = t^2 \) applied to its input. The steady state error is
   (A) \( 4 \times 10^{-4} \)
   (B) 0
   (C) \( \infty \)
   (D) \( 2 \times 10^{-5} \)

25. The transfer function of a \( ufb \) system is

\[ G(s) = \frac{10^4(s + 3)(s + 20)}{s(s + 25)(s + a)(s + 30)} \]

The value of \( a \) to yield velocity error constant \( K_v = 10^4 \) is
   (A) 4
   (B) 0
   (C) 8
   (D) 16

26. A system has position error constant \( K_p = 3 \). The steady state error for input of 8tu(\( t \)) is
   (A) 2.67
   (B) 2
   (C) \( \infty \)
   (D) 0
27. The forward path transfer function of a unity feedback system is

\[ G(s) = \frac{1000}{(s + 20)(s^2 + 4s + 10)} \]

For input of 60u(t) steady state error is

(A) 0  (B) 300  (C) ∞  (D) 10

28. For unity feedback system shown in fig. P6.3.28 the transfer function is

\[ G(s) = \frac{20(s + 3)(s + 4)(s + 8)}{s^2(s + 2)(s + 15)} \]

If input is 30t^2, then steady state error is

(A) 0.9375  (B) 0  (C) ∞  (D) 64

29. The forward-path transfer function of a unity feedback control system is

\[ G(s) = \frac{450(s + 8)(s + 12)(s + 15)}{s(s + 38)(s^2 + 2s + 28)} \]

The steady state errors for the test input 37tu(t) is

(A) 0  (B) 0.061  (C) ∞  (D) 609

30. In the system shown in fig. P6.3.30, r(t) = 1 + t^2, t > 0. The steady state error \( \epsilon(t) \) is equal to

\[ \epsilon(t) = \frac{10t + 1}{s^2 + 2t} \]
1. (D) Characteristic equation is \( s^2 + 9s + 18 \).
\[ \omega_n^2 = 18, \quad 2\xi\omega_n = 9 \]
Therefore \( \xi = 106, \quad \omega_n = 4.24 \text{ rad/s} \)

2. (A) \( T(s) = \frac{1}{6(s + 0.8)^2 + (0.6)^2} \)
\[ \omega_n \sqrt{1-\xi^2} = 0.6, \quad \xi\omega_n = 0.8 \]
Hence \( \omega_n = 1, \quad \xi = 0.8 \)

3. (A) Characteristic equation is
\[ \Delta s = (s - (-3 + j4))(s - (-3 - j4)) = (s + 3)^2 + 4^2. \]
\[ s^2 + 6s + 25, \quad \omega_n^2 = 25 \Rightarrow \omega_n = 5 \text{ rad/s} \]
\[ 2\xi\omega_n = 6, \quad \xi = \frac{6}{2 \times 5} = 0.6 \]

4. (A) \( T(s) = \frac{16}{(4s^2 + 8s + 16)} = \frac{4}{(s^2 + 2s + 4)} \)
\[ \omega_n^2 = 4 \Rightarrow \omega_n = 2, \quad 2\xi\omega_n = 2, \quad \xi = 0.5 \]

5. (D) \( M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} = \frac{5}{100} = 0.05 \),
\[ \frac{\xi\pi}{\sqrt{1-\xi^2}} = 3 \Rightarrow \xi = 0.69, \]
\[ T(s) = \frac{1}{1 + G(s)} = \frac{K}{s^2 + 2s + K} \]
\[ 2\xi\omega_n = 2, \quad \omega_n = \frac{1}{0.69} = 1.45 \]
Peak time,
\[ T_p = \frac{\pi}{\omega_n} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} = \frac{\pi}{1.45\sqrt{(1-0.69^2)}} = 3 \text{ sec} \]
But the peak time \( T_p \) given is 1 sec. Hence these two specification cannot be met.

6. (C) \( T(s) = \frac{K_1}{s^2 + (s + \omega_n^2) + K_1} \),
\[ \omega_n^2 = K_1, \quad 2\xi\omega_n = 1 + K_2 \]
\[ \omega_n = 0.1, \quad \xi = 0.6, \quad \omega_n = \omega_n \sqrt{1-0.6^2} = 10 \]
\[ \omega_n = 12.5 \Rightarrow K_1 = 15625, \]
\[ 2\omega_n 3 = K_2 + 1 \]
\[ 2 \times 12.5 \times 0.6 = K_2 + 1 \Rightarrow K_2 = 14 \]

7. (A) \( M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}, \text{ At } \xi = 0, \quad M_p = 1 = 100\% \)

8. (A) \( C(s) = \frac{10}{s(s + 10)} = \frac{1}{s - \frac{1}{s + 10}} \)
\[ \Rightarrow \quad a(t) = 1 - e^{-10t} \]
\[ a = 10, \quad \text{Rise time } T_r = \frac{22}{a} = \frac{2.2}{10} = 0.22 \text{ s} \]
Settling time \( T_s = \frac{4}{a} = 0.4 \text{ s} \)

9. (D) \( \xi\omega_n = \frac{4}{T_s} = 0.571, \quad \omega_n \sqrt{1-\xi^2} = \frac{\pi}{T_p} = 1047 \)
Poles \( = -0.571 \pm j1.047 \)

10. (A) \( 0.1 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0.59 \)
\[ \omega_n = \frac{\pi}{T_p} \sqrt{1-\xi^2} = 0.797, \]
Poles \( = -\xi\omega_n \pm j\omega_n \sqrt{1-\xi^2} = -0.46 \pm j0.63 \)

11. (B) \( 0.12 = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0.56, \quad \omega_n = \frac{4}{\xi\omega_n} = 11.92 \)
Therefore Poles \( = -\xi\omega_n \pm j\omega_n \sqrt{1-\xi^2} = -6.67 \pm j9.88 \)
Note :
\[ T_s = \frac{32}{\xi\omega_n}, \text{ For } 0 < \xi < 0.69 \]
\[ T_s = \frac{45}{\xi\omega_n}, \text{ For } \xi > 0.69 \]

12. (B) \( T(s) = \frac{\omega_n^2}{s + 2\xi\omega_n s + \omega_n^2} = \frac{40000}{s^2 + 500s + 40000} \)
\[ = \frac{40000}{(s + 100)(s + 400)} \]
\[ R(s) = \frac{40000}{s(s + 100)(s + 400)} = \frac{1}{s} \frac{4}{3(s + 100)} + \frac{1}{3(s + 400)} \]
\[ r(t) = 1 - \frac{4}{3} e^{-100t} + \frac{1}{3} e^{-400t} \]

13. (A) System has two different poles on negative real axis. So response is over damped.

14. (A) 1. Overdamped response (a, b)
Poles : Two real and different on negative real axis.
2. Underdamped response (c)
Poles : Two complex in left half plane
3. Undamped response (d)
Poles : Two imaginary.
4. Critically damped (e)
Poles : Two real and same on negative real axis.
15. (B) \( K_p = \lim_{s \to 0} G(s) = 1000 \)
\[ K_v = \lim_{s \to 0} sG(s) = 0 \]
\[ K_a = \lim_{s \to 0} s^2G(s) = 0 \]

16. (D) \( H(s) = 1, \quad K_p = \lim_{s \to 0} G(s) \cdot H(s) = \infty \)
\[ K_v = \lim_{s \to 0} sG(s) \cdot H(s) = \infty \]
\[ K_a = \lim_{s \to 0} s^2G(s) \cdot H(s) = \frac{K}{8} \]

17. (B) \( H(s) = 1, \quad K_p = \lim_{s \to 0} G(s) \cdot H(s) = 50 \)
\[ K_v = \lim_{s \to 0} sG(s) \cdot H(s) = 0 \]
\[ K_a = \lim_{s \to 0} s^2G(s) \cdot H(s) = 0. \]

18. (C) System type = 1, so \( n = 1 \)
\[ K_v = \lim_{s \to 0} sG(s) = \frac{K}{a} = 100 \]

For 10% overshoot,
\[ 0.1 = e^{-\frac{\zeta}{s}} = \frac{2}{s} \Rightarrow \zeta = 0.6 \]
\[ T(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{s^2 + as + K} \]
\[ 2\zeta\omega_n = a, \quad \omega_n^2 = K \Rightarrow 2 \times 0.6\sqrt{K} = a \]
\[ K \times 0.6\sqrt{K} = 100 \Rightarrow K = 14400 \]

19. (D) \( \frac{K}{a} = 100, \quad K = 14400, \]
\[ \frac{14400}{a} = 100 \Rightarrow a = 144 \]

20. (C) If \( R(s) = 0 \)
\[ T_p(s) = \frac{K_2}{s(s + 4)} \cdot \frac{1}{1 + \frac{K_1K_2(s + 2)}{s(s + 4)(s + 3)}} \]
\[ = \frac{K_2}{s(s + 3)(s + 4) + K_1K_2(s + 2)} \]

Error in output due to disturbance
\[ E(s) = T_p(s)D(s), \]
If \( D(s) = \frac{1}{s} \),
\[ e_{sd} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{1}{s} \cdot T_p(s) = \lim_{s \to 0} T_p(s) = \frac{3}{2K_1} \]
\[ \frac{3}{2K_1} = 0.000012 \Rightarrow K_1 = 125 \times 10^3 \]

Error due to ramp input
\[ \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} = \frac{\lim_{s \to 0} sR(s)}{1 + 1000(s^2 + 20s + 15)(s^2 + 20s + 15)} \]
\[ = \frac{6}{s^3 + 1000(s^2 + 20s + 15)(s + 2)(s + 10)} \]
\[ = \frac{6}{0 + \frac{1000 \times 20 \times 15}{2 \times 10}} = 4 \times 10^{-4} \]

21. (C) \( E(s) = R(s) - C(s)H(s) \)
\[ R(s) = \frac{R(s)G(s)H(s)}{1 + G(s)H(s)} = \frac{R(s)}{1 + G(s)H(s)} \]
\[ e_{se} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s}{1 + \frac{1}{1 - \frac{1}{s^2 + s + 2} \frac{1}{s + 1}}} = \frac{2}{3} \]

22. (A) \( e_{se} = \lim_{s \to 0} s\frac{R(s) - sD(s)G_2(s)}{1 + G(s)G_2(s)} \]
where \( G_1(s) = \frac{1}{s + 5} \) and \( G_2(s) = \frac{100}{s + 2} \)
\[ R(s) = D(s) = \frac{1}{s} \]
\[ e_{se} = \frac{1 - 100}{1 + \frac{100}{2}} = -\frac{49}{11} \]

23. (A) Using Routh-Hurwitz Criterion, system is stable for \( 0 < K < 2000 \)
maximum \( K_v = \lim_{s \to 0} sG(s) = \frac{2000}{4 \times 8 \times 10} = 6.25 \)
minimum possible error \( \frac{1}{K_v} = \frac{1}{625} = 0.16 \)

24. (A) \( R(s) = \frac{6}{s^2}, \quad E(s) = \frac{R(s)}{1 + G(s)} \)
\[ e_{se} = \lim_{s \to 0} sE(s) \]
\[ = \lim_{s \to 0} \frac{6s}{1 + \frac{1000(s^2 + 4s + 20)(s^2 + 20s + 15)}{s^3 + 1000(s^2 + 20s + 15)s + 2000 \times 20 \times 15}} \]
\[ = \lim_{s \to 0} \frac{6}{s^3 + 1000(s^2 + 20s + 15)(s + 2)(s + 10)} \]
\[ = \frac{6}{0 + \frac{1000 \times 20 \times 15}{2 \times 10}} = 4 \times 10^{-4} \]
25. (A) \( K_v = \lim_{s\to 0} sG(s) \)
\[ 10^4 = \frac{10^4 \times 3 \times 10 \times 20}{25 \times a \times 30} \implies a = 4 \]

26. (C) System is zero type \( K_v = 0\), \( e_\infty = \frac{1}{K_v} = \infty \)

27. (D) \( K_p = \lim_{s\to 0} G(s) = 5 \)
For input \( 60u(t)\), \( e_\infty = \frac{60}{1 + K_p} = 10 \)

28. (A) \( K_a = \lim_{s\to 0} s^2G(s) = 64 \)
\[ e_\infty = \frac{30 \times 2}{64} = 0.9375 \]

29. (B) \( K_v = \lim_{s\to 0} sG(s) = 609.02 \)
\[ e_\infty = \frac{37}{K_v} = 0.0607 \]

30. (C) The system is type 2. Thus to step and ramp input error will be zero.
\[ E(s) = R(s) - C(s) = R(s) - \frac{G(s)R(s)}{1 + G(s)} = \frac{R(s)}{1 + G(s)} \]
\[ R(s) = \frac{1 + \frac{2}{s^2}}{s^2} = \frac{s + 2}{s^2} \]

31. (C) System is type 2. Therefore error due to \( 1 + t \) would be zero and due to \( \frac{t^2}{2} \) would be \( \frac{1}{K_a} \).
\[ K_a = \lim_{s\to 0} s^2G(s) = 10, \quad e_\infty(t) = \frac{1}{10} = 0.1 \]
Note that you may calculate error from the formula
\[ e_\infty(t) = \lim_{s\to 0} sE(s) = \frac{sR(s)}{1 + G(s)} \]

32. (D) \( K_p = \lim_{s\to 0} G(s) = K \)
\[ e_\infty(t) = \frac{1}{1 + K_p} = \frac{1}{1 + K} = 0.1 \implies K = 9. \]

33. (B) \( e_\infty = \lim_{t\to \infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)H(s)} \)
When \( K_t = 0 \) and \( K_a = 5 \)

\[ G(s) = \frac{5}{s(0.5s + 1)} \quad H(s) = 1 \quad R(s) = \frac{1}{s^2} \]
\[ e_\infty = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{5}{s(0.5s + 1)}} = \frac{1}{5} = 0.2 \]

34. (C) The equivalent open-loop transfer function
\[ G_s = \frac{K_a}{1 + \frac{sK_a}{s(0.5s + 1)}} = \frac{K_a}{s(0.5s + 1) + K_a} \]
\[ T(s) = \frac{G(s)}{1 + G(s)} = \frac{K_a}{0.5s^2 + s(1 + K_a) + K_a} \]
\[ = \frac{2K_a}{s^2 + 2s(1 + K_a) + 2K_a} \]
\[ \omega^2_n = 2K_a \implies \omega_n = \sqrt{2}K_a \]
\[ 2\zeta \omega_n = 2(1 + K_a) \]
\[ \zeta = 1 + \frac{K_t}{\sqrt{2}K_a} = 0.7 \quad \text{... (i)} \]
\[ e_\infty = \lim_{s\to 0} \frac{sR(s)}{1 + G_s(s)} = \frac{1 + K_t}{K_a} \]
\[ e_\infty = \frac{1 + K_t}{K_a} = 0.2 \quad \text{... (ii)} \]
Solving (i) and (ii)
\[ K_a = 24.5, \quad K_t = 39 \]

35. (C) The \( s \) has power of 4 and denominator has order of 7. So Type 4 and Order 7.

36. (D) For \( 8u(t)\), \( e_\infty = \frac{8}{1 + K_p} = 2. \)
For \( 8tu(t)\), \( e_\infty = \infty \), since the system is type 0.

37. (A) For equivalent unit feedback system the forward transfer function is
\[ G_s = \frac{G(s)}{1 + G(s)H(s) - G(s)} = \frac{1+ \frac{10(s + 10)}{s(1 + 2)}}{1 + \frac{10(s + 10)(s + 20)}{s(1 + 2)}} \]
\[ = \frac{10(s + 10)}{11s^2 + 132s + 300} \]
The system is of Type 0. Hence step input will produce a constant error constant.
**CHAPTER 6.5**

**FREQUENCY-DOMAIN ANALYSIS**

**Statement for Q.1–2:**

An under damped second order system having a transfer function of the form

\[ T(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

has a frequency response plot shown in fig. P6.5.1–2.

1. The system gain \( K \) is
   (A) 1  
   (B) 2  
   (C) \( \sqrt{2} \)  
   (D) \( \frac{1}{\sqrt{2}} \)

2. The damping factor \( \zeta \) is approximately
   (A) 0.6  
   (B) 0.2  
   (C) 1.8  
   (D) 2.4

3. For the transfer function
   \[ G(s)H(s) = \frac{1}{s(s + 1)(s + 0.5)} \]
   the phase cross-over frequency is
   (A) 0.5 rad/sec  
   (B) 0.707 rad/sec  
   (C) 1.732 rad/sec  
   (D) 2 rad/sec

4. The gain-phase plots of open-loop transfer function of four different system are shown in fig. P6.5.4. The correct sequence of the increasing order of stability of these four system will be

   (A) D, C, B, A  
   (B) A, B, C, D  
   (C) B, C, A, D  
   (D) A, D, B, C

5. The open-loop frequency response of a unity feedback system is shown in following table

   | \( \omega \) | \(|G(j\omega)|\) | \( \angle G(j\omega) \) |
   |---|---|---|
   | 2 | 8.5 | \(-119^\circ\) |
   | 3 | 6.4 | \(-128^\circ\) |
   | 4 | 4.8 | \(-142^\circ\) |
   | 5 | 2.56 | \(-156^\circ\) |
   | 6 | 1.4 | \(-164^\circ\) |
   | 8 | 1.00 | \(-172^\circ\) |
   | 10 | 0.63 | \(-180^\circ\) |
The gain margin and phase margin of the system are

(A) 2 dB, 8°
(B) 2 dB, −172°
(C) 4 dB, 8°
(D) 4 dB, −172°

Statement for Q.6–7:
Consider the gain-phase plot shown in fig. P6.5.6–7.

6. The gain margin and phase margin are
(A) −2 dB, 40°
(B) 2 dB, 40°
(C) 2 dB, 140°
(D) −2 dB, 140°

7. The gain crossover and phase crossover frequency are respectively
(A) 10 rad/sec, 100 rad/sec
(B) 100 rad/sec, 10 rad/sec
(C) 10 rad/sec, 2 rad/sec
(D) 100 rad/sec, 2 rad/sec

8. The phase margin of a system with the open loop transfer function
\[ G(s) = \frac{k}{s(s+1)(s+2)} \] is
(A) 68.3°
(B) 90°
(C) 0°
(D) ∞

9. Consider a u/fb system having an open-loop transfer function
\[ G(s) = \frac{2}{s+1} \]
For \( K = 1 \), the gain margin is 28 dB. When gain margin is 20 dB, \( K \) will be equal to
(A) 2
(B) 4
(C) 5
(D) 2.5

10. The gain margin of the u/fb system
\[ G(s) = \frac{2}{(s+1)(s+2)} \] is
(A) 1.76 dB
(B) 3.5 dB
(C) −3.5 dB
(D) −1.76 dB

11. The open-loop transfer function of a system is
\[ G(s)H(s) = \frac{K}{s(1+2s)(1+3s)} \]
The phase crossover frequency is
(A) 6 rad/sec
(B) 2.46 rad/sec
(C) 0.41 rad/sec
(D) 3.23 rad/sec

12. The open-loop transfer function of a u/fb system is
\[ G(s) = \frac{1+s}{s(1+0.5s)} \]
The corner frequencies are
(A) 0 and 2
(B) 0 and 1
(C) 0 and −1
(D) 1 and 2

13. In the Bode-plot of a unity feedback control system, the value of magnitude of \( G(j\omega) \) at the phase crossover frequency is \( \frac{1}{2} \). The gain margin is
(A) 2
(B) \( \frac{1}{2} \)
(C) \( \frac{1}{3} \)
(D) 3

14. In the Bode-plot of a u/fb control system, the value of phase of \( G(j\omega) \) at the gain crossover frequency is −120°. The phase margin of the system is
(A) −120°
(B) 60°
(C) −60°
(D) 120°

15. The transfer function of a system is given by
\[ G(s) = \frac{K}{s(sT+1)} \] ; \( K < \frac{1}{T} \)
The Bode plot of this function is
(A) (B)
16. The asymptotic approximation of the log-magnitude versus frequency plot of a certain system is shown in fig. P6.5.16. Its transfer function is

\[
\frac{50(s + 5)}{s(s + 2)(s + 25)} \quad \text{(A)} \\
\frac{20(s + 5)}{s(s + 2)(s + 25)} \quad \text{(B)} \\
\frac{10s^2(s + 5)}{(s + 2)(s + 25)} \quad \text{(C)} \\
\frac{20(s + 5)}{s(s + 2)(s + 25)} \quad \text{(D)}
\]

17. For the Bode plot shown in fig. P6.5.17 the transfer function is

\[
\frac{100s}{(s + 4)(s + 10)^2} \quad \text{(A)} \\
\frac{100(s + 4)}{s(s + 10)^2} \quad \text{(B)} \\
\frac{100}{(s + 4)(s + 10)} \quad \text{(C)} \\
\frac{100}{s^2(s + 4)(s + 10)} \quad \text{(D)}
\]

18. Bode plot of a stable system is shown in the fig. P6.3.18. The open-loop transfer function of the \( ufb \) system is

\[
\frac{100}{s + 10} \quad \text{(A)} \\
\frac{10}{s + 10} \quad \text{(B)} \\
\frac{1}{s + 10} \quad \text{(C)} \\
\text{None of the above} \quad \text{(D)}
\]

19. Consider the asymptotic Bode plot of a minimum phase linear system given in fig. P6.5.19. The transfer function is

\[
\frac{8s(s + 2)}{(s + 5)(s + 10)} \quad \text{(A)} \\
\frac{4(s + 5)}{(s + 2)(s + 10)} \quad \text{(B)} \\
\frac{4(s + 2)}{s(s + 5)(s + 10)} \quad \text{(C)} \\
\frac{8(s + 5)}{(s + 2)(s + 10)} \quad \text{(D)}
\]

20. The Bode plot shown in fig. P6.5.20 represent

\[
\frac{100s^2}{(1 + 0.1s)^3} \quad \text{(A)} \\
\frac{1000s^2}{(1 + 0.1s)^5} \quad \text{(B)} \\
\frac{100s^2}{(1 + 0.1s)^5} \quad \text{(C)} \\
\frac{1000s^2}{(1 + 0.1s)^5} \quad \text{(D)}
\]

**Statement for Q.21–22:**

The Bode plot of the transfer function \( K/(1 + sT) \) is given in the fig. P6.5.21–22.
Statement for Q.29–30:

Consider the Bode plot of a ufb system shown in fig. P6.5.29–30.

29. The steady state error corresponding to a ramp input is
   (A) 0.25
   (B) 0.2
   (C) 0
   (D) \( \infty \)

30. The damping ratio is
   (A) 0.063
   (B) 0.179
   (C) 0.483
   (D) 0.639

31. The Nyquist plot of a open-loop transfer function \( G(j\omega)H(j\omega) \) of a system encloses the \((-1, j0)\) point. The gain margin of the system is
   (A) less than zero
   (B) greater than zero
   (C) zero
   (D) infinity

32. Consider a ufb system
   \[ G(s) = \frac{K}{s(1 + sT_1)(1 + sT_2)(1 + sT_3)} \]
   The angle of asymptote, which the Nyquist plot approaches as \( \omega \to 0 \), is
   (A) \(-90^\circ\)
   (B) \(90^\circ\)
   (C) \(180^\circ\)
   (D) \(-45^\circ\)

33. If the gain margin of a certain feedback system is given as 20 dB, the Nyquist plot will cross the negative real axis at the point
   (A) \( s = -0.05 \)
   (B) \( s = -0.2 \)
   (C) \( s = -0.1 \)
   (D) \( s = -0.01 \)

34. The transfer function of an open-loop system is
   \[ G(s)H(s) = \frac{s + 2}{(s + 1)(s - 1)} \]
   The Nyquist plot will be of the form

35. Consider a ufb system whose open-loop transfer function is
   \[ G(s) = \frac{K}{s(s^2 + 2s + 2)} \]
   The Nyquist plot for this system is

36. The open loop transfer function of a system is
   \[ G(s)H(s) = \frac{K(1+s)^2}{s^3} \]
   The Nyquist plot for this system is
37. For the certain unity feedback system

\[ G(s) = \frac{K}{s(s + 1)(2s + 1)(3s + 1)} \]

The Nyquist plot is

38. The Nyquist plot of a system is shown in fig. P6.5.38. The open-loop transfer function is

\[ G(s)H(s) = \frac{4s + 1}{s^2(s + 1)(2s + 1)} \]

39. The Nyquist plot for this system is

40. Regarding the system consider the statements
1. Open-loop system is stable
2. Closed-loop system is unstable
3. One closed-loop poles is lying on the RHP
The correct statements are
(A) 1 and 2 (B) 1 and 3
(C) only 2 (D) All

41. The Nyquist plot shown in the fig. P6.5.41 is for
(A) type–0 system (B) type–1 system
(C) type–2 system (D) type–3 system

Statement for Q.42–43:

The open-loop transfer function of a feedback system is
\[ G(s)H(s) = \frac{K(1 + s)}{(1 - s)} \]

42. The Nyquist plot of this system is

(A) (B) (C) (D)

43. The system is stable for \( K \)
(A) \( K > 1 \) (B) \( K < 1 \)
(C) any value of \( K \) (D) unstable

Statement for Q.44–46:

A unity feedback system has open-loop transfer function
\[ G(s) = \frac{1}{s(2s + 1)(s + 1)} \]

44. The Nyquist plot for the system is

(A) (B)
SOLUTIONS

1. (A) \( T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \)
   
   \[ T(j\omega) = \frac{K\omega_n^2}{-\omega^2 + 2j\xi\omega_n \omega + \omega_n^2} \]
   
   \[ |T(j\omega)|^2 = \frac{K^2\omega_n^4}{(\omega_n^2 - \omega^2)^2 + 4\xi^2\omega_n^2 \omega^2} \]
   
   From the fig. P6.5.1–2, \( |T(j\omega)| = 1 \)
   
   \[ |T(j\omega)|^2 = \frac{K^2\omega_n^4}{4\xi^2\omega_n^2} = \frac{K^2}{4\xi^2} \Rightarrow |T(j\omega)| = \frac{K}{2\xi} = 2.5 \]
   
   \[ \xi = \frac{K}{5} = 0.2 \]

2. (B) The peak value of \( T(j\omega) \) occurs when the denominator of function \( |T(j\omega)|^2 \) is minimum i.e. when \( \omega_n^2 - \omega^2 = 0 \Rightarrow \omega = \omega_n \)
   
   \[ |T(j\omega)|^2 = \frac{K^2\omega_n^4}{4\xi^2\omega_n^2} = \frac{K^2}{4\xi^2} \Rightarrow |T(j\omega_n)| = \frac{K}{2\xi} = 2.5 \]
   
   \[ \xi = \frac{K}{5} = 0.2 \]

3. (B) \( G(j\omega)H(j\omega) = \frac{1}{j\omega(\omega + 1)(0.5 + j\omega)} \)
   \[ \phi = -90^\circ - \tan^{-1} 2\omega - \tan^{-1} \omega \]
   At phase cross over point \( \phi = -180^\circ \)

   \[ -\tan^{-1} 2\omega - \tan^{-1} \omega - 90^\circ = -180^\circ \]
   \[ \tan^{-1} 2\omega + \tan^{-1} \omega = 90^\circ \]

   \[ \frac{2\omega + \omega}{1 - (2\omega)(\omega)} = \tan 90^\circ = \infty \]

   \[ 1 - (2\omega)(\omega) = 0 \Rightarrow \omega = \frac{1}{\sqrt{2}} = 0.707 \text{ rad/sec} \]

4. (B) For a stable system gain at \( 180^\circ \) phase must be negative in dB. More magnitude more stability.

5. (C) At \( 180^\circ \) gain is 0.63. Hence gain margin is
   \[ -20 \log \frac{1}{0.63} = 4 \text{ dB} \]
   At unity gain phase is \(-172^\circ\),

   Phase margin \( 180^\circ - 172^\circ = 8^\circ \)

6. (A) At \( \angle G(j\omega) = 180^\circ \) gain is \(-2 \text{ dB} \). Hence gain margin is \( 2 \text{ dB} \). At \( 0 \text{ dB} \) gain phase is \(-140^\circ \). Hence phase margin is \( 180^\circ - 140^\circ = 40^\circ \).

7. (A) At \( \omega = 100 \text{ rad/sec} \) phase is \( 180^\circ \). Phase cross-over frequency \( \omega_n = 100 \text{ rad/sec} \).

At \( \omega = 10 \text{ rad/sec} \) gain is 0 dB. Gain cross over frequency \( \omega = 10 \text{ rad/sec} \).

8. (D) \( |GH(j\omega)| < 1 \), for any value of \( \omega \) Thus phase margin is \( \infty \).

9. (D) For 28 dB gain Nyquist plot intersect the real axis at \( a, \)
   \[ 20 \log \frac{1}{a} = 28 \Rightarrow a = 0.04 \]

For 20 dB gain Nyquist plot should intersect at \( b, \)
   \[ 20 \log \frac{1}{b} = 20 \Rightarrow b = 0.1 \]

This is achieved if the system gain is increased by factor \( \frac{0.1}{0.04} = 25 \). Thus \( K = 25 \).

10. (B) Here \( K = 2, \quad T_1 = 1, \quad T_2 = \frac{1}{2} \)

   Gain Margin \( = \left[ \frac{KT_2}{T_1 + T_2} \right]^{-1} = \left[ \frac{(2)(0.5)}{1 + 0.5} \right]^{-1} = 15 = 35 \text{ dB} \)

11. (C) For phase crossover frequency
   \( \angle GH(j\omega) = -180^\circ \)

   \[ GH(j\omega) = \frac{K}{j\omega(1 + 2j\omega)(1 + 3j\omega)} \]

   \[ -90^\circ - \tan^{-1} 2\omega_n - \tan^{-1} 3\omega_n = -180^\circ \]
   \[ \tan^{-1} 2\omega_n + \tan^{-1} 3\omega_n = 90^\circ \]

   \[ \frac{2\omega_n + 3\omega_n}{1 - (2\omega_n)(3\omega_n)} = \tan 90^\circ \]

   \[ 1 - 6\omega_n^2 = 0 \Rightarrow \omega_n = 0.41 \text{ rad/s} \]

12. (D) \( G(s) = \frac{s + 1}{s(1 + 0.5s)} = \frac{s + 1}{s(\frac{s}{2} + 1)} \)

   The Bode plot of this function has break at \( \omega = 1 \) and \( \omega = 2 \). These are the corner frequencies.

13. (A) G.M. \( = \left| \frac{1}{GH(j\omega)} \right| = \frac{1}{\sqrt{2}} = 2 \)

14. (B) P.M. \( = 180^\circ + \angle GH(j\omega) = 180^\circ - 120^\circ = 60^\circ \)

15. (D) Due to pole at origin initial plot has a slope of \(-20 \text{ dB/decade} \). At \( s = j\omega = \frac{1}{T} \) Slope increases to \(-40 \text{ dB/decade} \). At \( \omega = \frac{1}{T}, \)
   \[ |G(j\omega)| = KT < 1 \text{ ,Gain in dB < 0} \]
27. (C) Initially slope is \(-20\) dB/decade. Hence there is a pole at origin and system type is 1. For type–1 system position error coefficient is \(\infty\).
\[
20 \log K = 6 \quad \Rightarrow \quad K = 2,
\]

28. (B) The system is type \(-0\),
\[
20 \log K_p = 40, \quad K_p = 100, \quad e_{\text{amp}}(\infty) = \frac{1}{1 + K_p} = \frac{1}{101}.
\]

29. (A) The Bode plot is as shown in fig. S6.5.29

\[
\begin{align*}
32 \text{ dB} & \quad -20 \text{ dB/dec} \\
18 \text{ dB} & \quad \text{-}40 \text{ dB/dec} \\
0 \text{ dB} & \quad 0.1 \\
1 & \quad 0.5 \\
1.4 & \quad 2
\end{align*}
\]
Fig. S6.5.29

\[
K_v = 4, \quad e_{\text{amp}}(\infty) = \frac{1}{K_v} = \frac{1}{4} = 0.25
\]

30. (B) From fig. S6.5.29 \(\xi = \frac{\omega_0}{2\omega_b} = \frac{0.5}{2(1.4)} = 0.179\)

31. (A) If Nyquist plot encloses the point \((-1, j0)\), the system is unstable and gain margin is negative.

32. (A) \(GH(j\omega) = \frac{K}{j\omega(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_3)}\)

\[
\lim_{\omega \to 0} GH(j\omega) = \lim_{\omega \to 0} \frac{K}{j\omega} = \lim_{\omega \to 0} \frac{K}{\omega} < 90^\circ
\]

Hence, the asymptote of the Nyquist plot tends to an angle of \(-90^\circ\) as \(\omega \to 0\).

33. (C) \(20 \log \left| \frac{1}{GH(j\omega)} \right| = 20\)

\[
\frac{1}{\left| GH(j\omega) \right|} = 10 \quad \Rightarrow \quad \left| GH(j\omega) \right| = 0.1
\]

Since system is stable, it will cross at \(s = -0.1\).

34. (B) \(GH(s) = \frac{s + 2}{(s^2 - 1)}\)

\[
GH(j\omega) = \frac{j\omega + 2}{(-1 - \omega^2)}
\]

At \(\omega = 0\), \(GH(j\omega) = 2 \angle -180^\circ\)
At \(\omega = \infty\), \(GH(j\omega) = 0 \angle -270^\circ\)

Hence (B) is correct option.

35. (C) \(GH(j\omega) = \frac{K}{j\omega(-\omega^2 + 2j\omega + 2)}\)

\[
\angle GH(j\omega) = -\tan^{-1} \frac{2\omega}{2 - \omega^2} - 90^\circ
\]

\[
\left| GH(j\omega) \right| = \frac{K}{\omega\sqrt{(2 - \omega)^2 + 4\omega^2}}
\]

At \(\omega = 0\), \(GH(j\omega) = \infty \angle -90^\circ\)
At \(\omega = \infty\), \(GH(j\omega) = 0 \angle -270^\circ\)
At \(\omega = 1\), \(GH(j\omega) = \frac{K}{\sqrt{5}} \angle -153.43^\circ\)
At \(\omega = 2\), \(GH(j\omega) = \frac{K}{2\sqrt{18}} \angle -206.6^\circ\)

Due to \(s\) there will be an infinite semicircle. Hence (C) is correct option.

36. (B) \(GH(j\omega) = \frac{K(1 + j\omega^2)}{(j\omega)^3}\)

\[
\left| GH(j\omega) \right| = \frac{K}{\omega^3}
\]

\[
\angle GH(j\omega) = -270^\circ + 2\tan^{-1} \omega
\]

For \(\omega = 0\), \(GH(j\omega) = \infty \angle -270^\circ\)
For \(\omega = 1\), \(\angle GH(j\omega) = -180^\circ\)
For \(\omega = \infty\), \(GH(j\omega) = 0 \angle -90^\circ\)

As \(\omega\) increases from 0 to \(\infty\), phase goes \(-270^\circ\) to \(-90^\circ\).
Due to \(s^3\) term there will be 3 infinite semicircles.

37. (A) \(\left| GH(j\omega) \right| = \frac{K}{\sqrt{1 + \omega^2 \sqrt{1 + 4\omega^2 \sqrt{1 + 9\omega^2}}}}\)

\[
\angle GH(j\omega) = -90^\circ - \tan^{-1} \omega - \tan^{-1} 2\omega - \tan^{-1} 3\omega
\]

For \(\omega = 0\), \(GH(j\omega) = \infty \angle -90^\circ\)
For \(\omega = \infty\), \(GH(j\omega) = 0 \angle -360^\circ\)

Hence (A) is correct option.

38. (C) The open-loop poles in RHP are \(P = 0\). Nyquist path enclosed 2 times the point \((-1 + j0)\). Taking clockwise encirclements as negative \(N = -2\).
\(N = P - Z, \quad -2 = 0 - Z, \quad Z = 2\) which implies that two poles of closed-loop system are on RHP.

39. (B) \(G(s)H(s) = \frac{-1}{2s(1 - 20s)}\)

\[
\left| GH(j\omega) \right| = \frac{1}{2\omega \sqrt{1 + 400\omega^2}}
\]

\[
\angle GH(j\omega) = 180^\circ - 90^\circ - \frac{-20\omega}{1}
\]

At \(\omega = 0\) \(GH(j\omega) = \infty \angle 90^\circ\)
At \(\omega = \infty\) \(GH(j\omega) = 0 \angle 180^\circ\)
At \(\omega = 0.1\) \(GH(j\omega) = 224 \angle 153.43^\circ\)
At \(\omega = 0.01\) \(GH(j\omega) = 49 \angle 91.15^\circ\)
40. (C) One open-loop pole is lying on the RHP. Thus open-loop system is unstable and \( P = 1 \). There is one clockwise encirclement. Hence \( N = -1 \).
Hence there are 2 closed-loop poles on the RHP and system is unstable.

41. (B) There is one infinite semicircle. Which represent single pole at origin. So system type is 1.

42. (D) \( \angle GH(j\omega) = \tan^{-1} \frac{\omega}{1} = -\frac{\omega}{1} \)
At \( \omega = 0 \) \( GH(j\omega) = K \angle 0^\circ \),
At \( \omega = 1 \) \( GH(j\omega) = K \angle 90^\circ \),
At \( \omega = 2 \) \( GH(j\omega) = K \angle 127^\circ \),
At \( \omega = \infty \) \( GH(j\omega) = K \angle 180^\circ \),

43. (A) RHP poles of open-loop system \( P = 1 \), \( Z = P - N \).
For closed loop system to be stable,
\( Z = 0 \), \( 0 = 1 - N \) \( \Rightarrow \) \( N = 1 \)
There must be one anticlockwise rotation of point \((-1 + j0)\). It is possible when \( K > 1 \).

44. (C) \( G(s) = \frac{1}{s(2s + 1)(s + 1)} \), \( H(s) = 1 \)
\( GH(s) = \frac{1}{s(2s + 1)(s + 1)} \)
\( GH(j\omega) = \frac{1}{j\omega(2j\omega + 1)(j\omega + 1)} \)
\( \lim_{\omega \to 0} GH(j\omega) = \lim_{\omega \to 0} \frac{1}{j\omega} = \infty \angle -90^\circ \)
\( \lim_{\omega \to \infty} GH(j\omega) = \lim_{\omega \to \infty} \frac{1}{2(j\omega)^3} = 0 \angle -270^\circ \)
The intersection with the real axis can be calculated as \( \text{Im}(GH(j\omega)) = 0 \), The condition gives \( \omega(2\omega^2 - 1) = 0 \)
i.e. \( \omega = 0, \frac{1}{\sqrt{2}}, \quad GH\left( j\frac{1}{\sqrt{2}} \right) = \frac{-2}{3} \)
With the above information the plot in option (C) is correct.

45. (C) The Nyquist plot crosses the negative real axis at \( \omega = 1 \frac{\sqrt{2}}{2} \) rad/sec. Hence phase crossover frequency is \( \omega_c = 1 \frac{\sqrt{2}}{2} = 0.707 \) rad/sec.

The frequency at which magnitude unity is
\( \omega_1 \frac{2}{1 + \omega_1^2}(1 + 4\omega_1^2) = 1 \)
\( \omega_1^2 = 0.326, \quad \omega_1 = 0.57 \text{ rad/sec} \)

46. (D) G.M. = \( 20 \log \frac{1}{|GH(j\omega)|}, \quad |GH(j\omega)| = \frac{2}{3} \)
Gain Margin = 20 log \( \frac{3}{2} = 352 \text{ dB} \).
\( \angle GH(j\omega) = -90^\circ - \tan^{-1} \frac{\omega}{1 + 2\omega}, \)
At unit gain \( \omega_c = 0.57 \text{ rad/sec} \),
Phase at this frequency is \( \angle GH(j\omega_c) = -90^\circ - \tan^{-1} 0.57 = -168.42^\circ \)
Phase margin = \(-168.42^\circ + 180^\circ = 11.6^\circ \)
Note that system is stable. So gain margin and phase margin are positive value. Hence only possible option is (D).

***************

---
1. The term ‘reset control’ refers to
   (A) Integral control  (B) Derivative control
   (C) Proportional control  (D) None of the above

2. If stability error for step input and speed of response be the criteria for design, the suitable controller will be
   (A) P controller  (B) PI controller
   (C) PD controller  (D) PID controller

3. The transfer function \(\frac{1 + 0.5s}{1 + s}\) represent a
   (A) Lag network  
   (B) Lead network
   (C) Lag–lead network
   (D) Proportional controller

4. A lag compensation network
   (a) increases the gain of the original network without affecting stability.
   (b) reduces the steady state error.
   (c) reduces the speed of response
   (d) permits the increase of gain of phase margin is acceptable.

   In the above statements, which are correct
   (A) a and b  
   (B) b and c
   (C) b, c, and d  
   (D) all

5. Derivative control
   (A) has the same effect as output rate control
   (B) reduces damping
   (C) is predictive in nature
   (D) increases the order of the system

6. Consider the List I and List II

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Derivative control</td>
<td>1. Improved overshoot response</td>
</tr>
<tr>
<td>Q. Integral control</td>
<td>2. Less steady state errors</td>
</tr>
<tr>
<td>R. Rate feed back control</td>
<td>3. Less stable</td>
</tr>
<tr>
<td>S. Proportional control</td>
<td>4. More damping</td>
</tr>
</tbody>
</table>

   The correct match is
   (A) 1 2 3 4  
   (B) 4 3 1 2  
   (C) 3 2 1 4  
   (D) 4 1 2 3

7. Consider the List–I (Transfer function) and List–II (Controller)

<table>
<thead>
<tr>
<th>List I</th>
<th>List II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.  (\frac{K_1 + K_2}{K_p})</td>
<td>1. P–controller</td>
</tr>
<tr>
<td>Q.  (\frac{K_1}{K_p})</td>
<td>2. PI–controller</td>
</tr>
<tr>
<td>R.  (\frac{K_1}{K_p})</td>
<td>3. PD–controller</td>
</tr>
<tr>
<td>S.  (\frac{K_1}{K_p})</td>
<td>4. PID–controller</td>
</tr>
</tbody>
</table>

   The correct match is
   (A) 3 4 2 1  
   (B) 4 3 1 2  
   (C) 3 2 1 4  
   (D) 4 1 2 3
8. The transfer function of a compensating network is of form \((1 + \alpha Ts)/(1 + Ts)\). If this is a phase–Lag network, the value of \(\alpha\) should be
   (A) greater than 1
   (B) between 0 and 1
   (C) exactly equal to 1
   (D) exactly equal to 0

9. The poll–zero configuration of a phase–lead compensator is given by
   (A) 
   (B) 
   (C) 
   (D) 

10. While designing controller, the advantage of pole–zero cancellation is
    (A) The system order is increased
    (B) The system order is reduced
    (C) The cost of controller becomes low
    (D) System’s error reduced to optimum levels

11. A proportional controller leads to
    (A) infinite error for step input for type 1 system
    (B) finite error for step input for type 1 system
    (C) zero steady state error for step input for type 1 system
    (D) zero steady state error for step input for type 0 system

12. The transfer function of a phase compensator is given by \((1 + aTs)/(1 + Ts)\) where \(a > 1\) and \(T > 0\). The maximum phase shift provided by a such compensator is
    (A) \(\tan^{-1}\left(\frac{a + 1}{a - 1}\right)\)
    (B) \(\sin^{-1}\left(\frac{a - 1}{a + 1}\right)\)
    (C) \(\tan^{-1}\left(\frac{a - 1}{a + 1}\right)\)
    (D) \(\cos^{-1}\left(\frac{a - 1}{a + 1}\right)\)

13. For an electrically heated temperature controlled liquid heater, the best controller is
    (A) Single–position controller
    (B) Two–position controller
    (C) Floating controller
    (D) Proportional–position controller

14. In case of phase–lag compensation used is system, gain crossover frequency, band width and undamped frequency are respectively
    (A) decreased, decreased, decreased
    (B) increased, increased, increased
    (C) increased, increased, decreased
    (D) increased, decreased, decreased

15. A process with open–loop model
    \[G(s) = \frac{Ke^{-\tau_s}}{\tau_s s + 1}\]
    is controlled by a PID controller. For this purpose
    (A) the derivative mode improves transient performance
    (B) the derivative mode improves steady state performance
    (C) the integral mode improves transient performance
    (D) the integral mode improves steady state performance.

    The correct statements are
    (A) (a) and (c)  (B) (b) and (c)
    (C) (a) and (d)  (D) (b) and (d)

16. A lead compensating network
    (a) improves response time
    (b) stabilizes the system with low phase margin
    (c) enables moderate increase in gain without affecting stability.
    (d) increases resonant frequency

    In the above statements, correct are
    (A) (a) and (b)  (B) (a) and (c)
    (C) (a), (c) and (d)  (D) All

17. A Lag network for compensation normally consists of
    (A) \(R, L\) and \(C\) elements
    (B) \(R\) and \(L\) elements
    (C) \(R\) and \(C\) elements
    (D) \(R\) only
18. The pole–zero plot given in fig. P6.6.18 is that of a

(A) PID controller
(B) PD controller
(C) Integrator
(D) Lag–lead compensating network

19. The correct sequence of steps needed to improve system stability is
(A) reduce gain, use negative feedback, insert derivative action
(B) reduce gain, insert derivative action, use negative feedback
(C) insert derivative action, use negative feedback, reduce gain
(D) use negative feedback, reduce gain, insert derivative action.

20. In a derivative error compensation
(A) damping decreases and setting time decreases
(B) damping increases and setting time increases
(C) damping decreases and setting time increases
(D) damping increases and setting time decreases

21. An ON–OFF controller is a
(A) P controller
(B) PID controller
(C) integral controller
(D) non linear controller

**********
Chapter 6.7

The State-Variable Analysis

1. Consider the SFG shown in Fig. P6.7.1

For this system dynamic equation is

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \]

(A) \[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \]

(B) \[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \]

(C) None of the above

Statement for Q.2–4:

Represent the given system in state-space equation \( \dot{x} = A \cdot x + B \cdot u \). Choose the correction option for matrix \( A \).

2. Fig. P6.7.2

3. Fig. P6.7.3

4. Fig. P6.7.4
5. The state equation of a LTI system is represented by
\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

The state transition matrix \( \Phi(t) \) is
(A) \( e^{-3t} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)  (B) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)
(C) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)  (D) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

6. The state equation of a LTI system is
\[ \dot{x} = \begin{bmatrix} 3 \\ -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

The state transition matrix \( \Phi(t) \) is
(A) \( e^{-3t} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)  (B) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)
(C) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)  (D) \( e^{-y(t)} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \)

7. The state equation of a LTI system is
\[ \dot{x} = \begin{bmatrix} 2 \\ -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \]

The state transition matrix is
(A) \( \begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix} \)  (B) \( \begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix} \)
(C) \( \begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix} \)  (D) \( \begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix} \)

Statement for Q.11–13:

A system is described by the dynamic equations
\[ x(t) = A \cdot x(t) + B \cdot u(t), \quad y(t) = C \cdot x(t) \]
where
\[ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \ 0 \ 0] \]

11. The Eigen values of \( A \) are
(A) 0.325, -1.662 ± j0.562
(B) 2.325, 0.338 ± j0.562
(C) -2.325, -0.338 ± j0.562
(D) -0.325, 1.662 ± j0.562

12. The transfer-function relation between \( X(s) \) and \( U(s) \) is
(A) \( \frac{1}{s^3 + 3s^2 + 2s + 1} \)  (B) \( \frac{1}{s^3 + 3s^2 + 2s + 1} \)
(C) \( \frac{1}{s^3 + 3s^2 + 2s + 1} \)  (D) None of the above

13. The output transfer function \( Y(s)/U(s) \) is
(A) \( s(s^3 + 3s^2 + 2s + 1)^{-1} \)  (B) \( s(s^3 + 3s^2 + 2s - 1)^{-1} \)
(C) \( (s^3 + 3s^2 + 2s + 1)^{-1} \)  (D) None of the above
14. A system is described by the dynamic equation 
\[
\dot{x}(t) = A \cdot x(t) + B \cdot u(t), \ y(t) = C \cdot x(t)
\]
where
\[
A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 \end{bmatrix}
\]

The output transfer function \(Y(s)/U(s)\) is
(A) \(\frac{(s + 1)}{(s + 2)^2}\)  
(B) \(\frac{s + 1}{s + 2}\)  
(C) \(\frac{(s + 2)}{(s + 1)}\)  
(D) None of the above

15. The state-space representation of a system is given by
\[
\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t), \ y(t) = [1 \ 1] x(t)
\]

The transfer function of this system is
(A) \(\frac{(s^2 + 3s + 2)}{s} \)  
(B) \(\frac{(s^2 + 2)}{s}\)  
(C) \(\frac{s(s^2 + 3s + 2)}{s}\)  
(D) \(\frac{(s + 1)}{s}\)

16. The state-space representation for a system is
\[
\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} u, \ y = [1 \ 0 \ 0] x
\]

The transfer function \(Y(s)/U(s)\) is
(A) \(\frac{10(2s^2 + 3s + 1)}{s^3 + 3s^2 + 2s + 1}\)  
(B) \(\frac{10(2s^2 + 3s + 1)}{s^3 + 2s^2 + 3s + 1}\)  
(C) \(\frac{10(2s^2 + 3s + 2)}{s^3 + 3s^2 + 2s + 1}\)  
(D) \(\frac{10(2s^2 + 3s + 2)}{s^3 + 2s^2 + 3s + 1}\)

Statement for Q.17–18:

Determine the state-space representation for the transfer function given in question. Choose the state variable as follows
\[x_1 = c, \ x_2 = \frac{dc}{dt} = \ddot{c}, \ x_3 = \frac{d^2c}{dt^2} = \dddot{c}, \ x_4 = \frac{d^3c}{dt^3} = \dddot{\dddot{c}}\]

17. \[
C(s) = \frac{24}{s^3 + 9s^2 + 26s + 24}
\]

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

18. A state-space representation of a system is given by
\[
\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \ y = [1 \ -1] x, \text{ and } x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

The time response of this system will be
(A) \(\sin \sqrt{2}t\)  
(B) \(\frac{3}{\sqrt{2}} \sin \sqrt{2}t\)  
(C) \(-\frac{1}{\sqrt{2}} \sin \sqrt{2}t\)  
(D) \(\sqrt{3} \sin \sqrt{2}t\)

19. For the transfer function
\[
Y(s) = \frac{s + 3}{(s + 1)(s + 2)}
\]

The state model is given by \(\dot{x} = A \cdot x + B \cdot u\), \(y = C \cdot x\). The \(A\), \(B\), \(C\) are
29. \[ \dot{x} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} r, \ y = [-1 \ 2 \ 1] x \]

\( e_{\text{step}}(\infty) \quad e_{\text{ramp}}(\infty) \)
(A) 1.0976      \quad 0
(B) 1.0976      \quad \infty
(C) 0          \quad 1.0976
(D) \infty       \quad 4.86

30. \[ \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r, \ y = [1 \ 0 \ 0] x \]

\( e_{\text{step}}(\infty) \quad e_{\text{ramp}}(\infty) \)
(A) 0          \quad 0.714
(B) \infty       \quad 0.714
(C) 0          \quad 4.86
(D) \infty       \quad 4.86

**Statement for Q.31–33:**

Consider the system shown in fig. P6.7.31-33.

![Fig. P6.7.31-33](image)

31. The controllability matrix is

(A) \[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
(B) \[ \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \]
(C) \[ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]
(D) \[ \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix} \]

32. The observability matrix is

(A) \[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
(B) \[ \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \]
(C) \[ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]
(D) \[ \begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix} \]

33. The system is

(A) Controllable and observable
(B) Controllable only
(C) Observable only
(D) None of the above

**Statement for Q.34–36:**

Consider the system shown in fig. P6.7.34-36.

![Fig. P6.7.34-36](image)

34. The controllability matrix for this system is

(A) \[ \begin{bmatrix} -10 & 0 & -20 \\ 10 & -20 & 40 \end{bmatrix} \]
(B) \[ \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} \]
(C) \[ \begin{bmatrix} -10 & 0 & 20 \\ 10 & -20 & -40 \end{bmatrix} \]
(D) \[ \begin{bmatrix} 1 & 6 & -1 \\ 1 & -4 & -4 \end{bmatrix} \]

35. The observability matrix is

(A) \[ \begin{bmatrix} -10 & 0 & -20 \\ 10 & -10 & 40 \end{bmatrix} \]
(B) \[ \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} \]
(C) \[ \begin{bmatrix} -10 & 0 & 20 \\ 10 & -10 & -40 \end{bmatrix} \]
(D) \[ \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} \]

36. The system is

(A) Controllable and observable
(B) Controllable only
(C) Observable only
(D) None of the above

**Statement for Q.37–38:**

A state flow graph is shown in fig. P6.7.37-38.

![Fig. P6.7.37-38](image)

37. The state and output equation for this system is

(A) \[ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & 21 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u, \ y = \begin{bmatrix} 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]
40. The state-space representation for this network is

\[
\begin{bmatrix}
0 \\
-5 - \frac{21}{4}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix} u,
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
4 \\
5
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

38. The system is
(A) Observable and controllable
(B) Controllable only
(C) Observable only
(D) None of the above

39. Consider the network shown in fig. P6.7.39. The state-space representation for this network is

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
= \begin{bmatrix}
0 & -1 & 21/4 \\
-5 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
1
\end{bmatrix} u,
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
4 \\
5
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

41. The state variable may be
(A) \( i_{r1}, i_{r2} \)
(B) \( i_L, i_C \)
(C) \( v_C, i_L \)
(D) None of the above

42. If state variable are chosen as in previous question, then the matrix \( A \) is
(A) \( \begin{bmatrix}
1 & -1 \\
-1 & 3
\end{bmatrix} \)
(B) \( \begin{bmatrix}
1 & -1 \\
-1 & 3
\end{bmatrix} \)
(C) \( \begin{bmatrix}
-1 & 3 \\
1 & -1
\end{bmatrix} \)
(D) \( \begin{bmatrix}
-1 & 3 \\
1 & -1
\end{bmatrix} \)

43. The matrix \( B \) is
(A) \( \begin{bmatrix}
3 \\
-1
\end{bmatrix} \)
(B) \( \begin{bmatrix}
-1 \\
3
\end{bmatrix} \)
(C) \( \begin{bmatrix}
-3 \\
1
\end{bmatrix} \)
(D) \( \begin{bmatrix}
1 \\
-3
\end{bmatrix} \)

Statement for Q.44–47:

Consider the network shown in fig. P6.7.44-47

\[
\begin{bmatrix}
\dot{v}_1 \\
\dot{v}_2
\end{bmatrix}
= \begin{bmatrix}
1 & -1 & 0 \\
-3 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0
\end{bmatrix} v_r,
\begin{bmatrix}
\dot{v}_1 \\
\dot{v}_2
\end{bmatrix}
= \begin{bmatrix}
1 \\
0
\end{bmatrix} \begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
= \begin{bmatrix}
1 & -3 & 0 \\
-1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0
\end{bmatrix} u_r
\]

\[
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
= \begin{bmatrix}
1 & -3 & 0 \\
-1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0
\end{bmatrix} u_r
\]
UNIT 6

44. The state variable may be
(A) $i_2$, $i_4$
(B) $i_2$, $i_4$, $u_o$
(C) $i_1$, $i_3$
(D) $i_1$, $i_3$, $i_3$

45. In state space representation matrix $A$ is

\[
\begin{bmatrix}
2 & 1 & 1 \\
3 & 3 & 3 \\
1 & 2 & 2 \\
3 & 3 & 3
\end{bmatrix}
\]

(A) \[
\begin{bmatrix}
2 & 1 & 1 \\
3 & 3 & 3 \\
1 & 2 & 2 \\
3 & 3 & 3
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
1 & 1 & 2 \\
3 & 3 & 3 \\
1 & 2 & 1 \\
3 & 3 & 3
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
2 & 1 & 1 \\
3 & 3 & 3 \\
1 & 2 & 2 \\
3 & 3 & 3
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
2 & 1 & 1 \\
3 & 3 & 3 \\
1 & 2 & 1 \\
3 & 3 & 3
\end{bmatrix}
\]

46. The matrix $B$ is

\[
\begin{bmatrix}
2 \\
3 \\
1 \\
3
\end{bmatrix}
\]

(A) \[
\begin{bmatrix}
2 \\
3 \\
1 \\
3
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
1 \\
3 \\
1 \\
3
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
2 \\
3 \\
1 \\
3
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
1 \\
3 \\
1 \\
3
\end{bmatrix}
\]

47. If output is $v_o$, then matrix $C$ is

(A) $[-1 0 0]$
(B) $[1 0 0]$
(C) $[0 0 -1]$
(D) $[0 0 1]$

**************

SOLUTIONS

1. (B) From the SFG
$\dot{x}_3 = -3x_1 - 2x_2 - x_3 + u$
$x_2 = \frac{x_3}{s} \Rightarrow \dot{x}_2 = x_3$
$x_1 = \frac{x_2}{s} \Rightarrow \dot{x}_1 = x_2$

2. (A) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-2 & 1 & 0 \\
0 & -2 & 0 \\
0 & 0 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u
\]

3. (C) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

$y = 5x_1 + 5x_2 + 5x_3$

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-2 & 0 & 1 \\
0 & -2 & 0 \\
0 & 0 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]

4. (C) $\dot{x}_1 = -4x_1 + x_2$, $\dot{x}_2 = x_3 + 2u$, $\dot{x}_3 = -3x_3 + u$

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} =
\begin{bmatrix}
-4 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
2 \\
1
\end{bmatrix} u
\]

5. (B) $\Delta s = |sI - A| = s^2 + s + 2 = 0 \Rightarrow s = -0.5 \pm j1.323$

6. (A) $(sI - A) = \begin{bmatrix} s + 3 & 0 \\ 0 & s + 3 \end{bmatrix}$, $|sI - A| = \frac{1}{(s + 3)^2}$

$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s + 3} & 0 \\ 0 & \frac{1}{s + 3} \end{bmatrix}$

$\Phi(t) = L^{-1}[(sI - A)] = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$

7. (A) $(sI - A) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $|sI - A| = s^2 + 4$

$(sI - A)^{-1} = \frac{1}{s^2 + 4} \begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix} = \begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix}$

$\Phi(t) = L^{-1}[(sI - A)] = \begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$

8. (A) $(sI - A) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $\Delta s = |sI - A| = s^2 + 4$

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10. (C) (A) is not a state-transition matrix, since \( \Phi(0) \neq I \)
(B) is not a state-transition matrix since \( \Phi(0) \neq I \)
(C) is a state-transition matrix since \( \Phi(0) = I \) and 
\[ [\Phi(t)]^{-1} = \Phi(-t) \]

11. (C) \( (sI-A) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix} \)

\[ |sI-A| = s^3 + 3s^2 + 2s + 1, \]
\[ \Rightarrow s = -2.325, -0.338 \pm j0.562 \]

12. (B) \[ X(s) = \frac{1}{(s^3 + 3s^2 + 2s + 1)} B \]

\[ = \frac{1}{s^3 + 3s^2 + 2s + 1} \begin{bmatrix} s & 3 & 2 \\ s & 3 & 2 \\ s & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]

\[ = \frac{1}{s^3 + 3s^2 + 2s + 1} \begin{bmatrix} 1 \\ s \end{bmatrix} \frac{1}{s^3 + 3s^2 + 2s + 1} \]

13. (C) \[ Y(s) = \frac{CX(s)}{U(s)} \]

\[ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & s & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \]

\[ = \frac{1}{s^3 + 3s^2 + 2s + 1} \]

14. (D) \[ Y(s) = \frac{C(sI-A)^{-1}B}{U(s)} \]

\[ B = \begin{bmatrix} 1 & 1 \\ 0 & s + 1 \end{bmatrix} \]

\[ = \frac{1}{s^2 + 2} \]

15. (D) \[ T(s) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \]

\[ (sI-A)^{-1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \]

16. (C) \[ \dot{x} = A \dot{x} + Bu, \ y = C \cdot x + Du \]

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \ B = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, \ C = [1 \ 0 \ 0], \ D = 0 \]

\[ T(s) = \frac{Y(s)}{U(s)} = C (sI-A)^{-1}B + D \]

\[ (sI-A)^{-1} = \frac{1}{s^3 + 3s^2 + 2s + 1} \begin{bmatrix} s^3 + 3s^2 + s + 3 & 1 \\ -1 & s(s + 3) & s \\ -s & -2s - 1 & s^2 \end{bmatrix} \]

Substituting the all values,
\[ T(s) = \frac{10(2s + 3s^2 + 2s + 1)}{s^3 + 3s^2 + 2s + 1} \]

17. (A) \[ C(s) = \frac{b_0}{R(s)} = \frac{s^3 + a_3s^2 + a_1s + a_0}{(s^3 + 9s^2 + 26s + 24)} \]

\[ (s^3 + a_3s^2 + a_1s + a_0)C(s) = b_0R(s) \]

Taking the inverse Laplace transform assuming zero initial conditions
\[ \ddot{c} + a_2\ddot{c} + a_1\dot{c} + a_0c = b_0r \]
\[ x_1 = \dot{c}, \ x_2 = \dot{x}_1, \ x_3 = \dot{x}_2, \ x_4 = \dot{x}_3 \]
\[ x_5 = \ddot{c}, \ x_6 = \ddot{x}_1, \ x_7 = \ddot{x}_2, \ x_8 = \ddot{x}_3 \]
\[ x_9 = \dddot{c}, \ x_{10} = \dddot{x}_1, \ x_{11} = \dddot{x}_2, \ x_{12} = \dddot{x}_3 \]
\[ y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} \]

18. (B) Fourth order hence four state variable
20. (C) Find the transfer function of option

For (A) \( \frac{Y(s)}{U(s)} = \frac{1}{s - 2} \)

For (B) \( \frac{Y(s)}{U(s)} = \frac{1}{s - 2} \)

For (C), \( \frac{Y(s)}{U(s)} = \frac{1}{s + 1(s + 2)} \left[ \frac{s + 2}{s + 1} \right] = \frac{s + 3}{s + 1(s + 3)} \)

So (C) is correct option.

21. (C) \( A = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix} \)

\(|sI-A| = s^2 + 7s + 7 \Rightarrow s = -5.79, -1.21\)

22. (B) \( (sI-A) = \begin{bmatrix} s & -2 \\ 0 & s - 6 \end{bmatrix} \)

\(|sI-A| = s^3 - 8s^2 - 11s + 8 \Rightarrow s = 9.11, 0.53, -1.64\)

23. (D) \( X(s) = (sI-A)^{-1}(x(0) + B \cdot u) \)

\[= \frac{1}{s - 2} \begin{bmatrix} s + 1 & 0 \\ -1 & s + 2 \end{bmatrix} \]

\[= \begin{bmatrix} \frac{s + 1}{s(s + 1)} & 1 \\ \frac{1}{s(s + 1)} & 0 \end{bmatrix} \]

\[Y(s) = \frac{1}{s(s + 1)(s + 2)} \left[ \begin{bmatrix} x(0) + B \cdot u \end{bmatrix} \right] \]

\[= \frac{1}{s^2(s + 1)(s + 2)} \]

24. (B) \( X(s) = (sI-A)^{-1}(x(0) + B \cdot u) \)

\[= \frac{1}{s(s + 1)(s + 2)} \left[ \begin{bmatrix} x(0) + B \cdot u \end{bmatrix} \right] \]

\[= \begin{bmatrix} \frac{s + 1}{s(s + 1)} & 1 \\ \frac{1}{s(s + 1)} & 0 \end{bmatrix} \]

\[Y(s) = \frac{1}{s(s + 1)(s + 2)} \left[ \begin{bmatrix} x(0) + B \cdot u \end{bmatrix} \right] \]

\[= \frac{1}{s^2(s + 1)(s + 2)} \]
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30. (B) \( A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1.286 & 0.143 & -0.714 \end{bmatrix} \)

\( e_{	ext{step}}(\infty) = 1 + \frac{1}{1 + \infty} = 1 + 0.0976 = 1.0976 \)

\( e_{	ext{ramp}}(\infty) = \lim_{t \to \infty} \left( (1 + CA^{-1}B)t + CA(B)^{-1}B \right) = \infty \)

31. (B) \( \dot{x}_1 = x_2 + u, \quad \dot{x}_2 = -5x_2 - 6x_1 - 2u \)

\( x = \begin{bmatrix} 0 \\ -6 \\ -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u, \quad A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \)

\( C_M = [B \ AB] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad O_M = \begin{bmatrix} 1 \end{bmatrix} \)

32. (A) \( y = x_1, \quad y = [1 \ 0]x, \quad C = \begin{bmatrix} 1 \ 0 \end{bmatrix}, \quad \text{CA} = \begin{bmatrix} 1 \ 0 \end{bmatrix}, \quad O_M = \begin{bmatrix} 1 \end{bmatrix} \)

33. (C) \( \text{det} \ C_M = 0. \) Hence system is not controllable. \( \text{det} \ O_M = 1. \) Hence system is observable.

34. (B) \( \dot{x}_1 = -x_1 + x_2, \quad \dot{x}_2 = -x_2 + u, \quad \dot{x}_3 = -2x_3 + u \)

\( x = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u, \quad A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \)

\( AB = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \quad A^2B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad A^3B = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \)

\( C_M = [B \ AB] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -2 & 4 \end{bmatrix} \)

35. (A) \( y = 10x_1 - 10x_2 + 10x_3, \quad y = [10 \ -10 \ 10]x \)

36. (A) \( \text{det} \ C_M = \text{det} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} = -1, \quad \text{det} O_M = \text{det} \begin{bmatrix} 1 & -10 & 10 \\ 10 & -10 & 10 \\ 10 & 10 & 10 \end{bmatrix} = -3000 \)

Since the determinant is not zero, the 3 \times 3 matrix is nonsingular and system is controllable.

The rank of \( O_M \) is 3. Hence system is observable.

37. (B) \( \dot{x}_2 = -x_2 - \frac{2}{4}x_2 + u, \quad \dot{x}_1 = x_2, \quad y = 5x_1 + 4x_2 \)

\( \dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 5 & 4 \end{bmatrix} \)

38. (B) \( O_M = \begin{bmatrix} 1 \end{bmatrix} \)

39. (B) \( \frac{dv}{dt} = i_c, \quad \frac{di_l}{dt} = \frac{v_l}{4} = 0.25v_L, \quad v_C \text{ and } i_L \text{ are state variable.} \)

\( i_L = i_C + i_R, \quad i_C = i_L - i_R = i_L - \frac{v_C}{2}, \quad v_L = v_s - v_C \)

Hence equations are \( \frac{dv_L}{dt} = \frac{-v_C}{4} = -0.5v_C + i_l \)

\( \frac{di_l}{dt} = 0.25(v_s - v_C), \quad 0.25v_C + 0.25v_s \)

\( \begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -0.25 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 1 \end{bmatrix} v_s, \quad \begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_C \\ i_C \\ i_R \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \)

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40. (B) \[ \frac{dv_1}{dt} = i_2, \quad \frac{dv_3}{dt} = v_L \]

Hence \(v_1\) and \(v_3\) are state variable.

\[
i_2 = i_1 - i_3 = (v_1 - v_i) - i_3, \quad i_2 = -i_4 + v_1
\]

\[
v_2 = v_L = v_1 - v_2 = v_1 - i_3 + v_L
\]

\[
\frac{dv_1}{dt} = -v_1 - i_2 + v_2, \quad \frac{dv_3}{dt} = -3v_i - i_2
\]

\[
\begin{bmatrix}
  v_1 \\
  v_3
\end{bmatrix}
= \begin{bmatrix}
  -1 & -1 \\
  -3 & -1
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  v_3
\end{bmatrix}
+ \begin{bmatrix}
  1 \\
  0
\end{bmatrix}, \quad \begin{bmatrix}
  i_1 \\
  i_3
\end{bmatrix} = \begin{bmatrix}
  4 \\
  1
\end{bmatrix}
\]

41. (C) Energy storage elements are capacitor and inductor. \(v_C\) and \(i_L\) are available in differential form and linearly independent. Hence \(v_C\) and \(i_L\) are suitable for state-variable.

42. (B) \[ \frac{1}{2} \frac{dv_C}{dt} = i_C \quad \Rightarrow \quad \frac{dv_C}{dt} = 2i_C \]

\[ \frac{1}{2} \frac{di_L}{dt} = v_L \quad \Rightarrow \quad \frac{di_L}{dt} = 2v_L \]

![Fig. S6.7.42](image)

\[
v_L = v_C + v_{R_2} = v_C + i_{R_2}, \quad i_C + 4v_L = i_{R_2}
\]

\[
v_L = v_C + i_C + 4v_L, \quad -3v_L = v_C + i_C
\]

\[
i_C = i_R - i_L, \quad i_C = i_R - \frac{v_L}{1}
\]

Solving equation (i) and (ii)

\[
-3(i_R - i_L - i_C) = v_C + i_C, \quad 2i_C = v_C - 3i_L + 3i_s
\]

\[
-3v_L = v_C + i_s - v_L + i_L, \quad 2v_L = -v_C + i_L - i_s
\]

\[
\frac{dv_C}{dt} = v_C - 3i_L + 3i_s, \quad \frac{di_L}{dt} = -v_C + i_L - i_s
\]

\[
\begin{bmatrix}
  v_C \\
  i_L
\end{bmatrix}
= \begin{bmatrix}
  1 & -3 \\
  -1 & 1
\end{bmatrix}
\begin{bmatrix}
  v_C \\
  i_L
\end{bmatrix}
+ \begin{bmatrix}
  3 \\
  -1
\end{bmatrix}, \quad \begin{bmatrix}
  v_C \\
  i_L
\end{bmatrix} = \begin{bmatrix}
  3 \\
  -1
\end{bmatrix}
\]

43. (A) \[ \mathbf{B} = \begin{bmatrix}
  3 \\
  -1
\end{bmatrix} \]

44. (B) There are three energy storage elements, hence 3 variable, \(i_2\), \(i_4\) and \(v_o\) are available in differentiated form hence these are state variable.
10. If machine is not properly adjusted, the product resistance change to the case where \( a_x = 1050 \Omega \). Now the rejected fraction is
(A) 50.46%  
(B) 10.57%  
(C) 2.18%  
(D) 6.43%

11. Cannon shell impact position, as measured along the line of fire from the target point is described by a gaussian random variable \( X \). It is found that 15.15% of shell falls 11.2 m or farther from the target in a direction toward the cannon, while 5.05% fall farther from the 95.6 m beyond the target. The value of \( a_x \) and \( \sigma_x \) for \( X \) is (Given that \( F(0.103) = 0.8485 \) and \( F(1.64) = 0.9495 \))
(A) \( T + 40 \text{ m and } 50 \text{ m} \)  
(B) \( T + 40 \text{ m and } 30 \text{ m} \)  
(C) \( T + 10 \text{ m and } 50 \text{ m} \)  
(D) \( T + 30 \text{ m and } 40 \text{ m} \)

12. A gaussian random voltage \( X \) for which \( a_x = 0 \) and \( \sigma_x = 4.2 \text{ V} \) appears across a 100 \( \Omega \) resistor with a power rating of 0.25 W. The probability, that the voltage will cause an instantaneous power that exceeds the resistor's rating, is
(A) \( 2Q\left(\frac{5}{42}\right) \)  
(B) \( Q\left(\frac{5}{42}\right) \)  
(C) \( 1 + Q\left(\frac{5}{42}\right) \)  
(D) \( 1 - Q\left(\frac{5}{42}\right) \)

**Statement for Question 13 - 14:**

Assume that the time of arrival of bird at Bharatpur sanctuary on a migratory route, as measured in days from the first year (January 1 is the first day), is approximated as a gaussian random variable \( X \) with \( a_x = 200 \) and \( \sigma_x = 20 \) days. Given that : \( F(0.5) = 0.6915 \), \( F(1.0) = 0.8413 \), \( F(1.5) = 0.8531 \), \( F(1.55) = 0.9394 \) and \( F(2.0) = 0.9773 \).

13. What is the probability that birds arrive after 160 days but on or before the 210th day ?
(A) 0.6687  
(B) 0.8413  
(C) 0.8531  
(D) 0.9773

14. What is the probability that bird will arrive after 231st day ?
(A) 0.0432  
(B) 0.1123  
(C) 0.0606  
(D) 0.0732

**Statement for Question 15-16:**

The life time of a system expressed in weeks is a Rayleigh random variable \( X \) for which
\[
f_X(x) = \frac{x}{200} e^{-\frac{x^2}{400}} \quad 0 \leq x \\
0 \quad x < 0
\]

15. The probability that the system will not last a full week is
(A) 0.01%  
(B) 0.25%  
(C) 0.40%  
(D) 0.60%

16. The probability that the system lifetime will exceed in year is
(A) 0.01%  
(B) 0.05%  
(C) 0.12%  
(D) 0.22%

17. The cauchy random variable has the following probability density function
\[
f_X(x) = \frac{b}{\pi (b^2 + (x-a)^2)}
\]

For real numbers \( 0 < b \) and \( -\infty < a < \infty \). The distribution function of \( X \) is
(A) \( \frac{1}{\pi} \tan^{-1}\left(\frac{x-a}{b}\right) \)  
(B) \( \frac{1}{\pi} \cot^{-1}\left(\frac{x-a}{b}\right) \)  
(C) \( \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x-a}{b}\right) \)  
(D) \( \frac{1}{2} + \frac{1}{\pi} \cot^{-1}\left(\frac{x-a}{b}\right) \)

**Statement for Question 18 - 19**

The number of cars arriving at ICICI bank drive-in window during 10-min period is Poisson random variable \( X \) with \( b = 2 \).

18. The probability that more than 3 cars will arrive during any 10 min period is
(A) 0.249  
(B) 0.143  
(C) 0.346  
(D) 0.543

19. The probability that no car will arrive is
(A) 0.516  
(B) 0.459  
(C) 0.246  
(D) 0.135
20. The power reflected from an aircraft of complicated shape that is received by a radar can be described by an exponential random variable $W$. The density of $W$ is

$$f_w(w) = \begin{cases} \frac{1}{W_0} e^{-w/W_0} & w > 0 \\ 0 & w < 0 \end{cases}$$

where $W_0$ is the average amount of received power. The probability that the received power is larger than the power received on the average is

(A) $e^{-2}$ \quad (B) $e^{-1}$ \quad (C) $1 - e^{-1}$ \quad (D) $1 - e^{-2}$

**Statement for Question 21-23:**

Delhi averages three murder per week and their occurrences follow a poisson distribution.

21. The probability that there will be five or more murder in a given week is

(A) 0.1847 \quad (B) 0.2461 \quad (C) 0.3927 \quad (D) 0.4167

22. On the average, how many weeks a year can Delhi expect to have no murders ?

(A) 1.4 \quad (B) 1.9 \quad (C) 2.6 \quad (D) 3.4

23. How many weeds per year (average) can the Delhi expect the number of murders per week to equal or exceed the average number per week ?

(A) 15 \quad (B) 20 \quad (C) 25 \quad (D) 30

24. A discrete random variable $X$ has possible values $x_i = i^2$, $i = 1, 2, 3, 4$ which occur with probabilities 0.4, 0.25, 0.15, 0.1. The mean value $\bar{X} = E[X]$ of $X$ is

(A) 6.85 \quad (B) 4.35 \quad (C) 3.96 \quad (D) 1.42

25. The random variable $X$ is defined by the density

$$f_X(x) = \frac{1}{2} x e^{-x/2}$$

The expected value of $g(X) = X^2$ is

(A) 48 \quad (B) 192 \quad (C) 36 \quad (D) 72

26. The mean of random variable $X$ is

(A) $1/4$ \quad (B) $1/6$ \quad (C) $1/3$ \quad (D) $1/5$

27. The variance of random variable $X$ is

(A) $1/10$ \quad (B) $3/80$ \quad (C) $5/16$ \quad (D) $3/16$

28. A Random variable $X$ is uniformly distributed on the interval $(-5, 15)$. Another random variable $Y = e^{-X}$ is formed. The value of $E[Y]$ is

(A) 2 \quad (B) 0.667 \quad (C) 1.387 \quad (D) 8.267

29. A random variable $X$ has $X_1, X_2, X_3, X_4$ which occur with probabilities 0.4, 0.25, 0.15, 0.1. The mean value $\bar{X} = E[X]$ of $X$ is

(A) $1/4$ \quad (B) $1/6$ \quad (C) $1/3$ \quad (D) $1/5$

30. The probability of the event $\{X \leq 2.5, Y \leq 6\}$ is

(A) 0.45 \quad (B) 0.50 \quad (C) 0.55 \quad (D) 0.60

31. The probability of the event $\{X \leq 3\}$ is

(A) 0.45 \quad (B) 0.50 \quad (C) 0.55 \quad (D) 0.60

**Statement for Question 32-34:**

Random variable $X$ and $Y$ have the joint distribution

$$F_{X,Y}(x, y) = \begin{cases} 5 \left( \frac{x + e^{-(y+1)^2}}{x+1} \right) e^{-y^2} u(y), & 0 \leq x \leq 4 \\ 0, & x < 0 \text{ or } y < 0 \\ 1 + \frac{1}{4} e^{-5y^2} - \frac{5}{4} e^{-y^2}, & 4 \leq x \text{ and any } y \geq 0 \end{cases}$$
32. The marginal distribution function $F_X(x)$ is

$$
\begin{align*}
(A) & \quad \frac{5x}{4(x+1)}, \quad -4 < x \leq 4 \\
0, & \quad x > 0 \\
1, & \quad x \leq -4 \\
(B) & \quad \frac{5x}{4(x+1)}, \quad 0 \leq x < 4 \\
1, & \quad x \geq 4 \\
(C) & \quad \frac{5x}{4(x+1)}, \quad -4 < x \leq 0 \\
0, & \quad x > 0 \\
1, & \quad x \leq -4 \\
(D) & \quad \frac{5x}{4(x+1)}, \quad 0 \leq x < 4 \\
1, & \quad x \geq 4
\end{align*}
$$

33. The marginal distribution function $F_Y(y)$ is

$$
\begin{align*}
(A) & \quad \frac{5}{4} e^{-y^2}, \quad y > 0 \\
1 + \frac{4}{4} e^{-y^2}, & \quad y \leq 0 \\
0, & \quad y > 0 \\
(B) & \quad 1 + \frac{4}{4} e^{-y^2} - \frac{5}{4} e^{-y^2}, \quad y \leq 0 \\
\frac{-5}{4} e^{-y^2}, & \quad y < 0 \\
1 + \frac{1}{4} e^{-y^2} - \frac{5}{4} e^{-y^2}, & \quad y \geq 0 \\
0, & \quad y > 0 \\
1 + \frac{1}{4} e^{-y^2} - \frac{5}{4} e^{-y^2}, & \quad y \geq 0
\end{align*}
$$

34. The probability $P\{3 < X \leq 5, \ 1 < y \leq 2\}$ is

(A) 0.001 (B) 0.002 (C) 0.003 (D) 0.004

**Statement for Question 35-39:**

Two random variables $X$ and $Y$ have a joint density

$$
F_{X,Y}(x,y) = \frac{10}{4} [u(x) - u(x-4)] u(y) y e^{-(x-1)^2+y^2}
$$

35. The marginal density $f_X(x)$ is

$$
\begin{align*}
(A) & \quad 5 \frac{u(x) - u(x-4)}{(x+1)^2} \\
(B) & \quad 5 \frac{u(x) - u(x-4)}{(x+1)} \\
(C) & \quad \frac{5 u(x) - u(x-4)}{4 \ (x+1)^2} \\
(D) & \quad \frac{5 u(x) - u(x-4)}{4 \ (x+1)}
\end{align*}
$$

36. The marginal density $f_Y(y)$ is

$$
\begin{align*}
(A) & \quad \frac{\pi}{2} y^2 [e^{-y^2} - e^{-5y^2}] u(y) \\
&B) & \quad \frac{\pi}{2} y^2 [e^{-y^2} - e^{-5y^2}] u(y) \\
(C) & \quad \frac{\pi}{2} y^2 [e^{-y^2} - e^{-5y^2}] u(y) \\
(D) & \quad \frac{\pi}{2} y^2 [e^{-y^2} - e^{-5y^2}] u(y)
\end{align*}
$$

37. The marginal distribution function $F_Y(y)$ is

$$
\begin{align*}
(A) & \quad \frac{5}{4} \left(4(x-4)-4x\right) \\
(B) & \quad \frac{5}{2} \left(4(x-4)-4x\right) \\
(C) & \quad \frac{5}{4} \left(4(x-4)-4x\right) \\
(D) & \quad None of the above
\end{align*}
$$

38. The marginal distribution function $F_Y(y)$ is

$$
\begin{align*}
(A) & \quad 1 - e^{-y^2} u(y) \\
(B) & \quad \frac{1}{4} [1 - e^{-y^2} u(y)] \\
(C) & \quad \frac{1}{4} [1 - e^{-y^2} u(y)] \\
(D) & \quad None of the above
\end{align*}
$$

39. The joint distribution function is

$$
\begin{align*}
(A) & \quad \frac{5}{4} \left[\frac{x + e^{-(x+1)^2}}{x + 1} - e^{-x^2}\right], \quad 0 \leq x \leq 4 \text{ and } y > 0 \\
1 + \frac{1}{4} \left[e^{-y^2} - 5e^{-y^2}\right], & \quad x > 4 \text{ and } y > 0 \\
\frac{5}{8} \left[\frac{x + e^{-(x+1)^2}}{x + 1} - e^{-x^2}\right], & \quad 0 \leq x \leq 4 \text{ and } y > 0 \\
1 + \frac{1}{2} \left[e^{-y^2} - 5e^{-y^2}\right], & \quad x > 4 \text{ and } y > 0 \\
\frac{5}{8} \left[\frac{x + e^{-(x+1)^2}}{x + 1} - e^{-x^2}\right], & \quad 0 \leq x \leq 4 \text{ and } y > 0 \\
1 + \frac{1}{4} \left[e^{-y^2} - 5e^{-y^2}\right], & \quad x > 4 \text{ and } y > 0 \\
\frac{5}{4} \left[\frac{x + e^{-(x+1)^2}}{x + 1} - e^{-x^2}\right], & \quad 0 \leq x \leq 4 \text{ and } y > 0 \\
1 + \frac{1}{2} \left[e^{-y^2} - 5e^{-y^2}\right], & \quad x > 4 \text{ and } y > 0
\end{align*}
$$

40. The function

$$
F_{X,Y}(x,y) = \pi \left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{\frac{\pi}{2}}\right)\right] + \pi \left[\frac{\pi}{2} + \tan^{-1}\left(\frac{y}{\frac{\pi}{3}}\right)\right]
$$

is a valid joint distribution function for random variables $X$ and $Y$ if the constant $a$ is

(A) $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ (C) $\frac{4}{\pi}$ (D) $\frac{8}{\pi}$
Random variable $X$ and $Y$ have the joint distribution function

$$F_{X,Y}(x,y) = \begin{cases} 
0, & \text{if } x < 0 \text{ or } y < 0 \\
\frac{27}{26} x \left(1 - \frac{y^2}{27}\right), & \text{if } 0 \leq x < 1 \text{ and } 1 \leq y \\
\frac{27}{26} y \left(1 - \frac{x^2}{27}\right), & \text{if } 1 \leq x \text{ and } 0 \leq y < 1 \\
\frac{27}{26} xy \left(1 - \frac{x^2y^2}{27}\right), & \text{if } 0 \leq x < 1 \text{ and } 0 \leq y < 1 \\
1, & \text{if } 1 \leq x \text{ and } 1 \leq y 
\end{cases}$$

The probability of the event $[0 < X \leq 0.5, 0 < Y \leq 0.25]$ is

(A) 0.13  
(B) 0.24  
(C) 0.69  
(D) 1

**Statement for question 42-43:**

The joint probability density function of random variable $X$ and $Y$ is given by

$$p_{XY}(x,y) = xy e^{-\frac{x^2 + y^2}{2}} u(x)u(y)$$

The $p_x(x)$ is

(A) $2xe^{-x^2} u(x)$  
(B) $xe^{-\frac{x^2}{2}} u(x)$  
(C) $xe^{-x^2} u(x)$  
(D) $2xe^{-x^2} u(x)$

The $p_{Y/X}(y/x)$ is

(A) $\frac{1}{x} ye^{-y^2} u(y)$  
(B) $ye^{-y^2} u(y)$  
(C) $ye^{-\frac{y^2}{2}} u(y)$  
(D) $\frac{1}{x} ye^{-\frac{y^2}{2}} u(y)$

The probability density function of a random variable $X$ is given as $f_X(x)$. A random variable $Y$ is defined as $y = ax + b$ where $a < 0$. The PDF of random variable $Y$ is

(A) $bf_X\left(\frac{y-b}{a}\right)$  
(B) $af_X\left(\frac{y-b}{a}\right)$  
(C) $\frac{1}{a} f_X\left(\frac{y-b}{a}\right)$  
(D) $\frac{1}{b} f_X\left(\frac{y-b}{a}\right)$

The function

$$f_{X,Y}(x,y) = \begin{cases} 
be^{-(x+y)} & 0 < x < a \text{ and } 0 < y < \infty \\
0 & \text{elsewhere}
\end{cases}$$

is a valid joint density function if $b$ is

(A) $\frac{a^2}{1-e^{-a}}$  
(B) $\frac{a}{1-e^{-a}}$  
(C) $\frac{1}{1-e^{-a}}$  
(D) None of the above

**Statement for Question 46-47:**

Random variable $X$ and $Y$ have the joint density

$$f_{X,Y}(x,y) = \frac{1}{2} u(x)u(y)e^{-\frac{1}{2}y^2}$$

The probability of the event $[2 < X \leq 4, -1 < Y \leq 5]$ is

(A) 0.1936  
(B) 6.2964  
(C) 0  
(D) None of the above

The probability of the event $[0 < X < \infty, < y \leq -2]$ is

(A) 0.2349  
(B) 0.3168  
(C) 0.4946  
(D) None of the above

Let $X$ and $Y$ be two statistically independent random variables uniformly distributed in the ranges $(-1, 1)$ and $(-2, 1)$ respectively. Let $Z = X + Y$. Then the probability that $Z \leq -2$ is

(A) zero  
(B) $\frac{1}{6}$  
(C) $\frac{1}{3}$  
(D) $\frac{1}{12}$

The probability density function of two statistically independent random variable $X$ and $Y$ are

$$f_X(x) = 5u(x)e^{-5x}$$

$$f_Y(y) = 24(y)e^{-2y}$$

The density of the sum $W = X + Y$ is

(A) $\frac{10}{e^{2\alpha} - e^{5\alpha}} u(w)$  
(B) $\frac{5}{e^{2\alpha} - e^{5\alpha}} u(w)$  
(C) $\frac{10}{e^{2\alpha} - e^{5\alpha}} u(w)$  
(D) $\frac{10}{e^{2\alpha} - e^{5\alpha}} u(w)$

The density function of two random variable $X$ and $Y$ is

$$f_{X,Y}(x,y) = \begin{cases} 
\frac{1}{24} & 0 < x < 6 \text{ and } 0 < y < 4 \\
0 & \text{elsewhere}
\end{cases}$$

The expected value of the function $g(x, y) = (XY)$ is

(A) 64  
(B) 96  
(C) 32  
(D) 48
51. The density function of two random variable X and Y is

\[ f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+2\sigma^2}{2\sigma^2}\right) \]

with \( \sigma^2 \) a constant. The mean value of the function \( g(X, Y) = X^2 + Y^2 \) is

(A) \( \sigma^2 \)  \hspace{1cm} (B) \( \sigma \)

(C) \( 2\sigma^2 \)  \hspace{1cm} (D) \( 2\sigma \)

**Statement for Question 52-54:**

The statistically independent random variable X and Y have mean values \( \bar{X} = E[X] = 2 \) and \( \bar{Y} = E[Y] = Y \). They have second moments \( \bar{X}^2 = E[X^2] = 8 \) and \( \bar{Y}^2 = E[Y^2] = 25 \). Consider a random variable \( W = 3X - Y \).

52. The mean value \( E[W] \) is

(A) 2  \hspace{1cm} (B) 4  

(C) 8  \hspace{1cm} (D) 25

53. The second moment of \( W \) is

(A) 145  \hspace{1cm} (B) 49  

(C) 97  \hspace{1cm} (D) 0

54. The variance of the random variable is

(A) 4  \hspace{1cm} (B) 45  

(C) 49  \hspace{1cm} (D) 54

55. Two random variable X and Y have the density function

\[ f_{X,Y}(x,y) = \begin{cases} \frac{xy}{9}, & 0 < x < 2 \text{ and } 0 < y < 3 \\ 0, & \text{elsewhere} \end{cases} \]

The X and Y are

(A) Correlated but statistically independent  

(B) uncorrelated but statistically independent  

(C) Correlated but statistically dependent  

(D) Uncorrelated but statistically dependent
1. (A) \[ P(X > 1) = \int_{1}^{\infty} p_X(x)dx = \int_{1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}dx = 0.368 \]

2. (C) \[ P(-1 < X \leq 2) = \int_{-1}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}dx + \int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}dx = 0.429 \]

3. (A) Test 1: \( f_X(x) \geq 0 \) is true
   Test 2: area must be 1 i.e. \[ \int_{-\infty}^{\infty} \frac{1}{4} \left( \frac{x^2}{3} - \frac{1}{3} \right) \]Thus \( b = \frac{1}{3} \ln 13 \)

4. (C) \[ \rho(v) = \int_{-\infty}^{\infty} x^2p(x)dx = \int_{0}^{\infty} \frac{x^2}{8} dx = 8 \]

5. (B) For \( x = 0 \), \( F_X(0) = \frac{1}{2} e^{\frac{-x^2}{2}} = \left[ 1 - e^{-\frac{1}{2}} \right] u(x) \)
   \( P(A) = F_X(3) - F_X(1) = e^{-\frac{9}{2}} - e^{-\frac{1}{2}} = 0.3834 \)

6. (D) \( P(B) = F_X(2.5) - 1 - e^{-\frac{2.5}{2}} = 0.7135 \)

7. (D) \( C = A \cap B = \{1 < X < 2.5\} \)
   \( P(C) = F_X(2.5) - F_X(1) = e^{-\frac{9}{2}} - e^{-\frac{2.5}{2}} = 0.3200 \)

8. (A) \( P(\mid X \mid > 2) = P(2 < x) + P(x < -2) \)
   \( = 1 - P(x \leq 2) + P(x < -2) = 1 - P(2) + P(-2) \)
   We know that for gaussian function \( F(-x) = 1 - F(x) \)
   Thus \( P(\mid X \mid > 2) = 1 - F(2) + 1 - F(-2) \)
   \( = 2 - 2F(2) = 2 - 2(0.9772) = 0.0456 \)

9. (C) Rejected resistor corresponds to \( \{X < 900\} \) and \( \{X > 1100\} \). Fraction rejected corresponds to probability of rejection.
   \( P(\text{resistor rejected}) = P(X < 900) + P(X < 1100) \)

10. (B) \( P(\text{resistor rejected}) = F \left( \frac{900 - 1050}{40} \right) + 1 \)

11. (D) \( P(x > T + 95.6) = 0.0505 \)
   \( = 1 - F \left( \frac{T + 95.6 - a_x}{\sigma_x} \right) = 0.9495 \)
   This occurs when \( \frac{T + 95.6 - a_x}{\sigma_x} = 1.64 \) \( \ldots \) (i)

12. (A) 0.25 exceeds when \( \frac{|X|^2}{100} > 0.21 \) or \( |X| > 5 \)

13. (A) \( P(160 < X \leq 210) = F_X(210) - F_X(160) \)

14. (C) \( P(X > 231) = 1 - P(X \leq 231) = 1 - F_X(231) \)

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15. (B) We use the Rayleigh distribution with \( a = 0 \) and \( b = 400 \)

For probability \( P(X \leq 1) = F_X(1) = 1 - e^{-\frac{1}{400}} \)

\[ = 0.0025 \text{ or } 0.25\% \]

16. (C) \( P(X \geq 52) = 1 - F_X(52) \)

\[ = 1 - \left[ 1 - e^{-\frac{52^2}{400}} \right] = 1 - e^{-\frac{52^2}{400}} = 0.00116 \text{ or } 0.12\% \]

17. (C) \( F_X = \frac{\pi}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{(b/\pi)du}{b^2 + (u-a)^2} \)

Let \( v = u - a \) and \( dv = du \) to get

\[ F_X(x) = \frac{b}{\pi} \int_{-\infty}^{\infty} \frac{1}{b^2 + v^2} \left[ \frac{1}{\pi} \tan^{-1} \left( \frac{v}{b} \right) \right] \]

\[ = \frac{1}{1 + \frac{1}{\pi} \tan^{-1} \left( \frac{x-a}{b} \right)} \]

18. (B) Here \( f_X(x) = e^{-\sum_{k=0}^{\infty} \frac{3^k}{k!}} \delta(x - k) \)

\[ P(x > 0) = 1 - P(x \leq 3) \]

\[ = 1 - P(x = 0) - P(x = 1) - P(x = 2) - P(x = 3) \]

\[ = 1 - e^{-e^{-\frac{2^0}{0!} + \frac{2^1}{2!} + \frac{2^2}{3!} + \frac{2^3}{4!}}} = 1 - e^{-\frac{19}{3}} = 0.1429 \]

19. (D) \( P(x = 0) = e^{-e^{-\frac{2^0}{0!}}} = 0.135 \)

20. (B) \( P(W > W_o) = 1 - P(W \leq W_o) = 1 - F_{W}(W_o) \)

\[ = 1 - \left[ 1 - e^{-\frac{W}{W_o}} \right] = e^{-\frac{W}{W_o}} \]

21. (A) \( P \{5 \text{ or more}\} = 1 - P(0) - P(1) - P(2) - P(4) \)

\[ = 1 - e^{-e^{-\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}}} = 1 - e^{-\frac{131}{8}} e^{-3} = 0.1847 \]

22. (C) \( P(0) = e^{-3} = 0.0498 \)

average number of week, per year with no murder 52e^{-3} = 2588.99 week.

23. (D) \( P \{3 \text{ or more}\} = 1 - P(0) - P(1) - P(2) \)

\[ = 1 - e^{-e^{-\frac{1^3}{1!} + \frac{2^2}{2!}}} = 1 - e^{-\frac{17}{2}} e^{-3} = 0.5768 \]

Average number of weeks per year that number of murder exceeds the average

\[ = 52 \left( 1 - \frac{17}{2} e^{-3} \right) = 29.994 \text{ weeks} \]

24. (B) \( E[X] = \bar{X} = \sum_{i=1}^{5} x_i P(x_i) \)

\[ = 10(0.4) + 4(0.25) + 9(0.15) + 6(0.1) = 4.35 \]

25. (A) \( E[g(X)] = E[X^3] = \frac{\pi^2}{2} x^2 e^{-x^2} = \frac{\pi}{2} \left[ \frac{6}{(1/2)^2} \right] = 48 \)

26. (A) Mean of \( X = \frac{\pi}{2} x f_x(x) dx = \int_{0}^{1} x^2(1-x)^2 dx = \frac{1}{4} \)

27. (B) Variance of \( X \) is \( \sigma_x^2 = E[X^2] - \mu_x^2 \)

\[ = \int_{0}^{1} x^2(1-x)^2 dx = \frac{1}{10} \]

\[ \sigma_x^2 = \frac{1}{10} - \left( \frac{1}{4} \right)^2 = \frac{3}{80} \]

Hence B is correct option

28. (B) Here \( Y = g(X) = e^{-\frac{X}{2}} \)

So \( E[Y] = E[g(Y)] = \int_{-\infty}^{\infty} e^{-\frac{X}{2}} g(X) dx = \int_{-5}^{5} e^{-\frac{X}{2}} \frac{1}{15}(-5) dx \)

\[ = \frac{1}{20} \left[ -5 e^{-\frac{X}{2}} \right]_{-5}^{15} = \frac{1}{5} \left[ e^{1} - e^{-3} \right] = 0.667 \]

29. (C) \( E[Y] = E[2X - 3] = 2\bar{X} - 3 = 2(-3) - 3 = -9 \)

\[ E[Y^2] = E[(2X - 3)^2] = 4\bar{X}^2 - 12\bar{X} - 9 \]

\[ = 4(11) - 12(-3) - 9 = 89 \]

\[ \sigma_Y^2 = \bar{Y}^2 - \bar{Y} = 89 - 9^2 = 8 \]

30. (A) \( F_{XY}(x, y) = 0.1u(x-1)u(y-1) + 0.35u(x-2)u(y-2) \)

\[ + 0.05u(x-3)u(y-3) + 0.5u(x-y)u(y-4) \]

\[ P(X \leq 2.5, Y \leq 6.0) = f_{XY}(2.5, 6.0) = 0.1 + 0.35 = 0.45 \]

31. (B) \( P(X \leq 3.0) = F_X(3.0) = F_{XY}(3.0, \infty) \)

\[ = 0.1 + 0.35 + 0.05 = 0.5 \]

32. (B) \( F_X(x) = F_{X,Y}(x, \infty) \)

\[ \lim_{y \to \infty} \left[ \frac{5}{4} \left( \frac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2} \right) u(y) = \frac{5x}{4(x+1)} \right] \]

\[ \lim_{y \to \infty} \left( 1 + \frac{1}{4} e^{5y^2} - \frac{5}{4} e^{-y^2} \right) = 1 \]

33. (B) \( F_Y(y) = F_{X,Y}(\infty, y) \)
\[ \begin{align*}
&= \int_{-\infty}^{\infty} 5e^{-u}u(w - y)dy = 10e^{-w}dy, \quad w > 0 \\
&= \frac{10}{3} u(w)[e^{-2w} - e^{-5w}]
\end{align*} \]

50. (A) \( E[(XY)^2] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2y^2 f_{X,Y}(x,y) dx dy \)

\[ = 4 \int_{y=0}^{6} \int_{x=-\infty}^{\infty} \frac{x^2y^2}{24} dx dy = 64 \]

51. (C) \( E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{x^2+y^2}{2\sigma^2}} dx dy \)

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-\frac{x^2+y^2}{2\sigma^2}}}{\sqrt{2\pi \sigma^2}} dx dy = \frac{1}{\sqrt{2\pi \sigma^2}} \]

Both double integrals are of the same form. The second factors equal 1 because they are area of a Gaussian density. The first factor equals \( \sigma^2 \) because they are second moment of Gaussian density with zero mean and variance \( \sigma^2 \).

Thus \( E[g(x,y)] = E[(x^2 + y^2)]2\sigma^2 \)

52. (A) \( E[W] = E[3X - Y] = 3\bar{X} - \bar{Y} = 6 - 4 = 2 \)

53. (B) \( E[W^2] = E[(3X - Y)^2] = E[9X^2 - 6XY + Y^2] \)

\[ = 9\bar{X}^2 - 6\bar{XY} + \bar{Y}^2 = 9(3)^2 - 6(3)(2) + (2)^2 \]

\[ = 9(8) - 6(2)(4) + 25 = 25 \]

54. (B) \( \sigma_w^2 = E[(W - \bar{W})^2] = E[W^2 - 2W\bar{W} + \bar{W}^2] \)

\[ = \bar{W}^2 - \bar{W}^2 = 25 - 4 = 21 \]

55. (B) \( R_{XY} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{X,Y}(x,y) dx dy = \int_{0}^{3} \int_{0}^{\frac{x^2}{9}} \frac{x^2y^2}{9} dx dy = \frac{8}{3} \)

\[ E[X] = \int_{0}^{3} \int_{0}^{\frac{x^2}{9}} \frac{x^2y}{9} dx dy = \frac{4}{3} \]

\[ E[Y] = \int_{0}^{3} \int_{0}^{\frac{x^2}{9}} \frac{xy}{9} dx dy = 2 \]

Since \( R_{XY} = \frac{8}{3} = E[X]E[Y] = 2(\frac{4}{3}) = \frac{8}{3} \), we have \( X \) and \( Y \) uncorrelated.

From marginal densities \( f_x(x) = \frac{x}{9} \), \( 0 < x < 2 \)

\[ f_y(y) = \frac{y}{9} \], \( 0 < y < 3 \)

we have \( f_x(x) f_y(y) = \frac{xy}{9} \), \( 0 < x < 2 \) and \( 0 < y < 3 \)

Thus \( f_{X,Y}(x,y) = f_x(x) f_y(y) \) and \( X \) and \( Y \) are statistically independent.

56. (C) \( \sigma_x^2 = \bar{X}^2 - \bar{X}^2 = \frac{5}{2} - \left(\frac{1}{2}\right)^2 = \frac{9}{4} \)

\( \sigma_y^2 = \bar{Y}^2 - \bar{Y}^2 = \frac{11}{2} - (2)^2 = \frac{9}{2} \)

\( R_{XY} = \bar{X} \bar{Y} = C_{XY} + \bar{X} \bar{Y} = -\frac{1}{2\sqrt{3}} + \frac{1}{2}(2) = \frac{1}{2}(2 - \frac{1}{\sqrt{3}}) \)

\[ \rho = \frac{C_{XY}}{\sigma_x \sigma_y} = \frac{-1/2\sqrt{3}}{(\sqrt{914})/(\sqrt{112})} = \frac{-1}{2} \]

57. (B) \( \bar{W} = (X + Y)^2 + 2X + 3 \)

\[ = 3 + 2\bar{X} + \bar{Y}^2 + 2\bar{X} + 9Y^2 \]

\[ = 3 + 2\left(\frac{1}{2}\right) + 5 + 6\left(\frac{1}{2}\right) + \frac{3}{2} + 9\left(\frac{19}{2}\right) = 98 - \sqrt{3} \]
CHAPTER 7.2
RANDOM PROCESS

Statement for Question 1 - 4:
A random process $X(t)$ has periodic sample functions as shown in figure where $A$, $T$ and $4\tau_0 \leq T$ are constant but $\varepsilon$ is random variable uniformly distributed on the interval $(0, T)$.

1. The first order density function is
   
   \begin{cases} 
   \frac{T-2\tau_0}{T} \alpha(x) + \frac{2\tau_0}{AT} & 0 \leq x < A \\
   0 & \text{else where}
   \end{cases}

   \begin{cases} 
   \alpha(x) + \frac{2\tau_0}{AT} & 0 \leq x < A \\
   0 & \text{else where}
   \end{cases}

2. The value of $E[X(t)]$ is
   
   (A) $\frac{t_0A}{2T}$
   (B) $\frac{t_0A}{T}$
   (C) $\frac{t_0A}{4T}$
   (D) 0

3. The value of $E[X^2(t)]$ is
   
   (A) $\frac{t_0A^2}{T}$
   (B) $\frac{t_0A^2}{3T}$
   (C) $\frac{2t_0A^2}{3T}$
   (D) 0

4. The value of $\sigma_X^2$ is
   
   (A) $\frac{t_0A}{T} \left[ \frac{2}{3} - \frac{\tau_0}{T} \right]$
   (B) $\frac{t_0A}{T} - \frac{\tau_0}{T}$
   (C) $\frac{t_0A}{T} \left[ \frac{2}{3} + \frac{\tau_0}{T} \right]$
   (D) $\frac{t_0A^2}{T} \left[ \frac{2}{3} + \frac{\tau_0}{T} \right]$

5. An ergodic random power $x(t)$ has an auto-correlation function
   
   $$R_{xx}(\tau) = 18 + \frac{2}{6 + \tau^2} + 1 + 4 \cos(12\tau)$$

   The $X$ is
   
   (A) $\pm\sqrt{18}$
   (B) $\pm\sqrt{13}$
   (C) $\pm\sqrt{17}$
   (D) $\pm\sqrt{18} \pm \sqrt{17}$

6. For random process $\overline{X} = 6$ and
   
   $$R_{xx}(t, t + \tau) = 36 + 25e^{-1\tau}.$$
A random process is defined by \( X(t) + A \) where \( A \) is continuous random variable uniformly distributed on \((0,1)\). The auto correlation function and mean of the process is

(A) 1/2 & 1/3  
(B) 1/3 & 1/2  
(C) 1 & 1/2  
(D) 1/2 & 1

**Statement for Question 8 - 9 :**

A random process is defined by \( Y(t) = X(t) \cos(\omega_0 t + \theta) \) where \( X(t) \) is a wide sense stationary random process that amplitude modulates a carrier of constant angular frequency \( \omega_0 \) with a random phase \( \theta \) independent of \( X(t) \) and uniformly distributed on \((\pi / \pi)\).

8. The mean value \( E[Y(t)] \) is

(A) \( E[X(t)] \)  
(B) \(-E[X(t)]\)  
(C) 1  
(D) 0

9. The autocorrelation function of \( Y(t) \) is

(A) \( R_{XX}(\tau) \cos(\omega_0 \tau) \)  
(B) \( \frac{1}{2} R_{XX}(\tau) \cos(\omega_0 \tau) \)  
(C) \( 2R_{XX}(\tau) \cos(\omega_0 \tau) \)  
(D) None of the above

**Statement for Question 10 - 11 :**

Consider a low-pass random process with a white-noise power spectral density \( S_X(\omega) = \frac{4}{2} \) as shown in fig.P7.2.10-11.

![Fig.P7.2.10-11](image)

10. The autocorrelation function \( R_X(\tau) \) is

(A) \( 4B \) sinc \((2\pi B)\tau\)  
(B) \( \pi 4B \) sinc \((2\pi B)\tau\)  
(C) \( 4B \) sinc \((2\pi B)\tau\)  
(D) None of the above

11. The power \( P_X \) is

(A) \( 2NB \)  
(B) \( \pi NB \)  
(C) \( NB \)  
(D) \( \frac{NB}{2\pi} \)

12. If \( X(t) \) is a stationary process having a mean value \( E[X(t)] = 3 \) and autocorrelation function \( R_{XX}(\tau) = 9 + 2e^{-t^2} \).

The variance of random variable \( Y = \int_0^2 X(t)dt \) will be

(A) 1  
(B) 2.31  
(C) 4.54  
(D) 0

13. A random process is defined by \( X(t) = A \cos(\pi t) \) where \( A \) is a gaussian random variable with zero mean and variance \( \sigma^2 \). The density function of \( X(0) \)

(A) \( \frac{1}{\sqrt{2\pi} \sigma_x} e^{-\frac{x^2}{2\sigma^2}} \)  
(B) \( \sqrt{2\pi} \sigma_x e^{-\frac{x^2}{2\sigma^2}} \)  
(C) 0  
(D) 1

**Statement for Question 14-15 :**

The two-level semi-random binary process is defined by

\( X(t) = A \) or \(-A \)

where \((n-1/T < t < nT)\) and the levels \( A \) and \(-A \) occur with equal probability. \( T \) is a positive constant and \( n = 0, \pm 1, 2 \)

14. The mean value \( E[X(t)] \) is

(A) 1/2  
(B) 1/4  
(C) 1  
(D) 0

15. The auto correlation \( R_{XX}(t_1 = 0.5T, t_2 = 0.7T) \) will be

(A) 1  
(B) 0  
(C) \( A^2 \)  
(D) \( A^2/2 \)

16. A random process consists of three samples function \( X(t, s_1) = 2, X(t, s_2) = 2 \cos t_1 \) and \( X(t, s_3) = 3 \sin t \) each occurring with equal probability. The process is

(A) First order stationary  
(B) Second order stationary  
(C) Wide-sense stationary  
(D) Not stationary in any sense

**Statement for Question 17 - 19 :**

The auto correlation function of a stationary ergodic random process is shown in fig.P7.2.17-19

![Fig.P7.2.17-19](image)
17. The mean value $E[X(t)]$ is 
(A) 50  
(B) $\sqrt{50}$  
(C) 20  
(D) $\sqrt{20}$

18. The $E[X^2(t)]$ is 
(A) 10  
(B) $\sqrt{10}$  
(C) 50  
(D) $\sqrt{50}$

19. The variance $\sigma^2_X$ is 
(A) 20  
(B) 50  
(C) 70  
(D) 30

20. Two zero mean jointly wide sense stationary random process $X(t)$ and $Y(t)$ have no periodic components. It is known that $E[X^2(t)] = 5$ and $E[Y^2(t)] = 10$. The function, that can apply to the process is 
(A) $R_{XX} \sin(\omega t)$  
(B) $R_{YY} \sin(\omega t)$  
(C) $R_{XY} \sin(\omega t)$  
(D) None of the above

21. A stationary zero mean random process $X(t)$ is ergodic has average power of 24 W and has no periodic component. The valid auto correlation function is 
(A) $16 + 18 \cos(3t)$  
(B) $24 \delta(t - \tau)$  
(C) $e^{(-6t)}/(1 + 3t^2)$  
(D) $24 \delta(t - \tau)$

22. Air craft of Jet Airways at Ahmedabad airport arrive according to a poisson process at a rate of 12 per hour. All aircraft are handled by one air traffic controller. If the controller takes a 2 - minute coffee break, what is the probability that he will miss one or more arriving aircraft? 
(A) 0.33  
(B) 0.44  
(C) 0.55  
(D) 0.66

23. Delhi airport has two check-out lanes that develop waiting lines if more than two passengers arrives in any one minute interval. Assume that a poisson process describes the number of passengers that arrive for check-out. The probability of a waiting line if the average rate of passengers is 2 per minute, is 
(A) 0.16  
(B) 0.29  
(C) 0.32  
(D) 0.49

24. A complex random process $Z(t) = X(t) + jY(t)$ is defined by jointly stationary real process $X(t)$ and $Y(t)$. The $E[Z(t)^2]$ will be 
(A) $2R_{XY}(0) + R_{XX}(0) + R_{YY}(0)$  
(B) $R_{XX}(0) + R_{YY}(0)$  
(C) $R_{XX}(0) - R_{YY}(0)$  
(D) $R_{YY}(0) - R_{XX}(0)$

25. Consider random process $X(t) = A_N \cos(\omega_N t + \theta)$ where $A_N$ and $\omega_N$ are constant and $\theta$ is a random variable uniformly distributed on the interval $(0, \pi)$. The power in $X(t)$ is 
(A) $A_N^2$  
(B) $\frac{1}{3} A_N^2$  
(C) $\frac{1}{4} A_N^2$  
(D) None of the above

26. The non valid power spectral density function of a real random process is 
(A) $\frac{\delta^2}{\omega^2 + \delta^2}$  
(B) $\frac{\delta^2}{\omega^2 + 25}$  
(C) $\frac{\delta^2}{\omega^2 + 16}$  
(D) $\frac{\delta^2}{\omega^2 + 16}$

27. The valid power density spectrum is 
(A) $\frac{\delta^2}{\omega^2 + 1}$  
(B) $\frac{\delta^2}{\omega^2 + 1 - \delta}$  
(C) $e^{-\delta \omega^2}$  
(D) $\frac{\delta^2}{\omega^2 + 3\omega^2 + 3}$

28. A power spectrum is given as 
$p_{XX}(\omega) = \begin{cases} \frac{P}{1 + (\omega/W)^2} & |\omega| < KW \\ 0 & |\omega| > KW \end{cases}$

where $P$, $W$, and $K$ are real positive constants. The sums bandwidth of power spectrum is 
(A) $W \frac{\tan^{-1} k}{k}$  
(B) $W \frac{\sqrt{k}}{\tan^{-1} k}$  
(C) $W \frac{\sqrt{k}}{k} + 1$  
(D) $\infty$

29. Consider the power spectrum given by 
$p_{XX}(\omega) = \begin{cases} \frac{P}{1 + (\omega/W)^2} & |\omega| < W \\ 0 & |\omega| > W \end{cases}$

where $P$ and $W$ are real positive constants. The rms bandwidth of the power spectrum is 
(A) $\frac{W}{\sqrt{2}}$  
(B) $\frac{W^2}{3}$  
(C) $\frac{W}{\sqrt{3}}$  
(D) $\frac{W}{2}$
30. For a random process \( R_{XX}(\tau) = P \cos^4(\omega_b \tau) \) where \( P \) and \( \omega_b \) are constants. The power in process is
(A) \( P \)  
(B) \( 2P \) 
(C) \( 3P \)  
(D) \( 4P \)

31. A random process has the power density spectrum \( \rho_{XX}(\omega) = \frac{\omega^2}{(\omega^2 + \omega_0^2)^2} \). The average power in process is
(A) \( \frac{1}{4} \)  
(B) \( \frac{3}{8} \)  
(C) \( \frac{5}{8} \)  
(D) \( \frac{1}{2} \)

32. A deterministic signal \( A \cos(\omega_b t) \), where \( A \) and \( \omega_b \) are real constants is added to a noise process \( N(t) \) for which \( \rho_{NN}(\omega) = \frac{\omega_0^2}{(\omega^2 + \omega_0^2)^2} \) and \( W > 0 \) is a constant. The ratio of average signal power to average noise power is
(A) \( 1 \)  
(B) \( \frac{A}{W} \)  
(C) \( \frac{2A}{W} \)  
(D) \( \frac{A^2}{W} \)

33. The autocorrelation function of a random process \( X(t) \) is
\[
R_{XX}(t, t + \tau) = 12e^{y_1 \tau} \cos^2(24t)
\]
The \( R_{XX}(\tau) \) is
(A) \( 6e^{-4\tau} \)  
(B) \( 12e^{-4\tau} \)  
(C) \( 48e^{-4\tau} \)  
(D) None of the above

34. If \( X(t) \) and \( Y(t) \) are real random process, the valid power density spectrum \( f_{XX}(\omega) \) is
(A) \( \frac{6}{6 + 7\omega^2} \)  
(B) \( \frac{4e^{-3\tau}}{1 + \omega^2} \)  
(C) \( 3 + j\omega^2 \)  
(D) \( 18\omega^2 \)

35. The cross correlation of jointly wide sense stationary process \( X(t) \) and \( Y(t) \) is \( R_{XY}(\tau) = A\tau e^{-Wt} \), where \( A > 0 \) and \( W > 0 \) are constants. The \( \rho_{XY}(\omega) \) is
(A) \( \frac{A}{W^2 - \omega^2} \)  
(B) \( \frac{A}{W^2 + \omega^2} \)  
(C) \( \frac{A}{W + j\omega} \)  
(D) \( \frac{A}{W - j\omega} \)

36. A random process \( X(t) \) is applied to a linear time invariant system. A response \( Y(t) = X(t) - X(t - \tau) \) occurs when \( \tau \) is a real constant. The system's transfer function is
(A) \( 1 - e^{j\omega\tau} \)  
(B) \( 2je^{-j\omega\tau/2} \sin \frac{\omega\tau}{2} \)  
(C) \( 2je^{-j\omega\tau/2} \cos \frac{\omega\tau}{2} \)  
(D) \( 1 + e^{-j\omega\tau} \)

37. A random process \( X(t) \) has an autocorrelation function \( R_{XX}(\tau) = A^2 + Be^{-\tau} \) where \( A \) and \( B \) are constants. A system have an input response
\[
h(t) = \begin{cases} 
  e^{-Wt} & \text{if } 0 < t \\
  0 & \text{if } t < 0 
\end{cases}
\]
where \( W \) is a real positive constant, which \( X(t) \) is its input. The mean value of the response is
(A) \( \frac{A}{W} \)  
(B) \( \frac{A}{2W} \)  
(C) \( 2\frac{A}{W} \)  
(D) \( 0 \)

38. In previous question if impulse response of system is
\[
h(t) = \begin{cases} 
  e^{-Wt} \sin(\omega_b t) & \text{if } 0 < t \\
  0 & \text{if } t < 0 
\end{cases}
\]
where \( W \) and \( \omega_b \) are real positive constants, the mean value of response is
(A) \( \frac{2A\omega_b}{\omega_b^2 + W^2} \)  
(B) \( \frac{2A}{\omega_b^2 (\omega_b^2 + W^2)} \)  
(C) \( \frac{2A}{\omega_b^2 (\omega_b^2 + W^2)} \)  
(D) \( \frac{A}{\omega_b^2 (\omega_b^2 + W^2)} \)

39. A stationary random process \( X(t) \) is applied to the input of a system for which \( h(t) = 3u(t)te^{-at} \). If \( E[X(t)] = 2 \), the mean value of the system's response \( Y(t) \) is
(A) \( \frac{1}{3a} \)  
(B) \( \frac{1}{a} \)  
(C) \( \frac{1}{12a} \)  
(D) \( \frac{1}{a^2} \)

Statement for Question 40-41:

A random process \( X(t) \) is applied to a network with impulse response \( h(t) = u(t)te^{-at} \) where \( a > 0 \) is a constant. The cross correlation of \( X(t) \) with the output \( Y(t) \) is known to have the same form \( R_{XX}(\tau) = u(\tau)te^{-at} \).

40. The auto correlation of \( Y(t) \) is
(A) \( \frac{4 + a\tau}{4a^2} e^{-\alpha\tau} \)  
(B) \( \frac{1 + a\tau}{3a^2} e^{-\alpha\tau} \)  
(C) \( \frac{4 + a\tau}{8a^2} e^{-\alpha\tau} \)  
(D) \( \frac{1 + a\tau}{4a^2} e^{-\alpha\tau} \)

41. The average power in \( Y(t) \) is
(A) \( \frac{1}{4a^2} \)  
(B) \( \frac{1}{a^2} \)  
(C) \( \frac{1}{2a^2} \)  
(D) None of the above
Statement for Question 42 - 43:

A random noise \( X(t) \) having a power spectrum \( \rho_{XX}(\omega) = \frac{3}{49 + \omega^2} \) is applied to a differentiator that has a transfer function \( H(\omega) = j\omega \). The output is applied to a network for which \( h(t) = u(t)t^2e^{-2t} \).

42. The average power in \( X(t) \) is

(A) 5/21  
(B) 5/24  
(C) 5/42  
(D) 3/14

43. The power spectrum of \( Y(t) \) is

\[ P_{YY}(\omega) = \frac{12\omega^2}{(49 + \omega^2)^2} \]

(A) \( \frac{4\omega^2}{(49 + \omega^2)^2} \)  
(B) \( \frac{12\omega^2}{(49 + \omega^2)^2} \)  
(C) None of the above

44. White noise with power density \( k/2 \) is applied to a lowpass network for which \( |H(0)| = 2 \). It has a noise bandwidth of 2 MHz. If the average output noise power is 0.1 W in a 1-\( \Omega \) resistor, the value of \( k \) is

(A) 12.5 nW/Hz  
(B) 12.5 \( \mu \)W/Hz  
(C) 25 nW/Hz  
(D) 25 \( \mu \)W/Hz

45. An ideal filter with a mid-band power gain of 8 and bandwidth of 4 rad/s has noise \( X(t) \) at its input with power spectrum \( \rho_{XX}(\omega) = \frac{12}{3\pi} \omega e^{-3\omega^2/8} \). The noise power at the network’s output is \( F(2) = 0.9773 \)

(A) 60.8  
(B) 90.3  
(C) 20.2  
(D) 100.4

46. White noise with power density \( k/2 = 6 \) \( \mu \)W/Hz is applied to an ideal filter of gain 1 and bandwidth \( W \) rad/s. If the output’s average noise power is 15 watts, the bandwidth \( W \) is

(A) \( 2.5 \times 10^{-6} \)  
(B) \( 2.5\pi \times 10^{-6} \)  
(C) \( 5 \times 10^{-6} \)  
(D) \( \pi 5 \times 10^{-6} \)

47. A system have the transfer function \( |H(\omega)|^2 = \frac{1}{1 + (\omega/W)^2} \)

where \( W \) is a real positive constant. The noise bandwidth of the system is

(A) \( \frac{1}{8} \pi W \sqrt{2} \)  
(B) \( \frac{1}{8} \pi W \sqrt{2} \)  
(C) \( \frac{1}{8} \pi W \sqrt{2} \)  
(D) None of the above

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SOLUTION

1. (A) Let \( \varepsilon \) have value \( e \). Now

\[ P[X \leq x|\varepsilon = e] = F_x(X|\varepsilon = e) \]

and for any \( \varepsilon \) must be zero for \( x < 0 \) because \( x(t) \) is never negative. The event \( \{ X \leq 0 \} \) is satisfied whenever \( x(t) \) is zero. This happens during the fraction of time \( (T - 2t_0) / T \). Hence

\[ F_x(x|\varepsilon = e) = \frac{(T - 2t_0)}{T} u(x), \quad 0 \leq x < A \]

\[ = 1, A \leq x \]

\[ = 0, x < 0 \]

By differentiation

\[ f_x(x|\varepsilon = e) = \frac{(T - 2t_0)}{T} \delta(x) + \frac{2t_0}{T^2}, \quad 0 \leq x < A \]

\[ = 0 \text{ else where} \]

\[ f_x(x) = \int_{-\infty}^{\infty} f_x(x, e)de \]

\[ = \frac{(T - 2t_0)}{T^2} \delta(x) + \frac{2t_0}{AT}, \quad 0 \leq x < A \]

\[ = 0 \text{ elsewhere} \]

2. (B) \( E[X(t)] = \int_{-\infty}^{\infty} x f_x(x)dx \)

\[ = \int_{-\infty}^{\infty} x \left( \frac{T - 2t_0}{T} \delta(x) + \frac{2t_0}{AT} \right) dx = \frac{t_0 A^2}{T} \]

3. (C) \( E[X^2(t)] = \int_{-\infty}^{\infty} x^2 f_x(x)dx = \int_{-\infty}^{\infty} \frac{2t_0 x^2}{AT} + \frac{2t_0 A^2}{3T} = \frac{2t_0 A^2}{3T} \)

4. (D) \( \sigma_x^2 = E[X^2(t)] - [E[X(t)]]^2 \)

\[ = \frac{2t_0 A^2}{3T} - \frac{t_0 A^2}{T^2} = \frac{t_0 A^2}{T} \left[ \frac{2}{3} - \frac{t_0}{T} \right] \]

5. (A) We know that (i) if \( X(t) \) has a periodic component then \( R_{XX}(\tau) \) will have a periodic component with the same period. (ii) if \( E[X(t)] = \bar{X} \neq 0 \) and \( X(t) \) is ergodic with no periodic components then \( \lim_{\tau \to \infty} R_{XX}(\tau) = \bar{X}^2 \).

Thus we get \( \bar{X}^2 = 18 \) or \( \bar{X} = \pm \sqrt{18} \).
6. (C) $\bar{X}$ = Constant and $R_{XX}(\tau)$ is not a function of $t$, so $X(t)$ is a wide sense stationary. So 1 is false & 3 is true.

Thus 2 is false if $X(t)$ has a periodic component, then $R_{XX}(\tau)$ will have a periodic component with the same period. Thus 4 is false.

7. (B) $R_{XX}(t + \tau) = E[X(t)X(t + \tau)] = E[A^2] = \int_0^1 a^2 \, da = \frac{1}{3}$

where $E_x[\cdot]$ represent expectation with respect to $X$ only.

8. (D) $E[Y(t)] = E[X(t)\cos(\omega_0 t + \theta)]$

$= E_x[X(t)]\cos(\omega_0 t + \theta) \frac{1}{2\pi} d\theta = 0$

9. (B) $R_{YY}(t, t + \tau)$

$= E[X(t)\cos(\omega_0 t + \theta)X(t + \tau)\cos(\omega_0 t + \theta + \omega_0 \tau)]$

$= R_{XX}(\tau) \frac{1}{2} [\cos(\omega_0 \tau) + \cos(2\omega_0 t + 2\theta + \omega_0 \tau)]$

$= \frac{1}{2} R_{XX}(\tau) \cos(\omega_0 \tau)$

10. (C) $S_{X}(\omega) = \frac{4}{\pi} \text{rect} \left( \frac{\omega}{4B} \right)$

We know that $R_{X}(\tau) \longrightarrow S_{X}(\omega)$

$W \sin(Wt) \longrightarrow \text{rect} \left( \frac{\omega}{2W} \right)$

Here $W = 2\pi B$

Hence $R_{X}(\tau) = \frac{2\pi B}{\pi} \frac{4}{2} \sin(2\pi B t) = 4B \sin(2\pi B t)$

11. (C) $P_{X} = \bar{X}^2 = R_{X}(0) = NB \text{ since } \sin(0) = 1$

12. (C) $E[Y] = E_x \left[ \frac{1}{2} \int_0^1 X(t)dt \right] = \frac{1}{2} \int_0^1 E[X(t)] dt = 3\int_0^1 dt = 6$

$E[Y^2] = E_x \left[ \frac{2}{2} \left[ \int_0^1 X(t)dt \right]^2 \right] = \frac{2}{2} \int_0^1 E[X(t)X(u)du] dt$

$= \frac{2}{2} \int_0^1 R_{XX}(t-u)du = \frac{2}{2} \int_0^1 [9 + 2e^{-3t-\tau}] dt du$

$= 36 + 2\int_0^1 e^{-3t-\tau} dt du = 4(10 + e^{-2})$

$\sigma_y^2 = E[Y^2] - (E[Y])^2 = 4(1 + e^{-2}) = 4.541$

13. (A) For $t = 0, X(0) = A$, So $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_A} e^{-\frac{x^2}{2\sigma_A^2}}$

14. (D) $E[X(t)] = AP(A) + (-A)P(-A) = A \cdot \frac{A}{2} - \frac{A}{2} = 0$

15. (C) Here $R_{XX}(t_1, t_2) = A^2$

If both $t_1$ and $t_2$ are in the same interval $(n-1)T < t, t_2 < nT, n = 0, \pm, \pm 2, \ldots$ and $R_{XX}(t_1, t_2) = 0$ otherwise

Hence $R_{XX}(0.5T, 0.7T) = A^2$

16. (D) Let $x_1 = 2$, $x_2 = 2\cos(t)$ and $x_3 = 3\sin(t)$

Then $f_X(x) = \frac{1}{2} \delta(x - x_1) + \frac{1}{3} \delta(x - x_2) + \frac{1}{3} \delta(x - x_3)$

and $E[X(t)] = \int_\infty^{-\infty} xf_x(x)dx$

$= \frac{1}{2} \left[ \frac{1}{3} \delta(x - x_1) + \frac{1}{3} \delta(x - x_2) + \frac{1}{3} \delta(x - x_3) \right]$

$= \frac{1}{3} [2 + 2\cos(t) + 3\sin(t)]$

The mean value is time dependent so $X(t)$ is not stationary in any sense.

17. (D) We know that for ergodic with no periodic component

$\lim_{t \to \infty} R_{XX}(\tau) = \bar{X}^2$, Thus $\bar{X}^2 = 20$ or $\bar{X} = \sqrt{20}$

18. (C) $R_{XX}(0) = E[X^2(t)] = R_{XX}(0) = 50 = \bar{X}^2$

19. (D) $\sigma_x^2 = \bar{X}^2 - \bar{X}^2 = 50 - 20 = 30$

20. Here $\bar{X} = 0$, $\bar{Y} = 0$, $R_{XY}(0) = 5$, $\sigma_Y^2 = R_{YY}(0) = 10$

For (A) : Function does not have even symmetry

For (B) : Function does not satisfy $R_{YY}(0) = 10$

For (C) : Function does not satisfy $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)} = \sqrt{50}$

21. (D) For (A) : It has a periodic component.

For (B) : It is not even in $\tau$, total power is also incorrect.

For (C) : It depends on $t$ not even in $\tau$ and average power is $\infty$.

22. (A) $P$ (miss/or more aircraft) = $1 - P$(miss 0)

$= 1 - P(0 \text{ arrive}) = 1 - \frac{(\lambda t)^6 e^{-\lambda t}}{0!}$
\[ E[Y(t)] = A \int_{-\infty}^{\infty} h(\xi)d\xi = A \int_{0}^{\infty} e^{-\xi} dt = \frac{A}{W} \]

38. (A) \( \overline{X} = A \)

\[ E[Y(t)] = \overline{Y} = \overline{X} \int_{-\infty}^{\infty} h(t)dt = A \int_{0}^{\infty} e^{-\xi} \sin(\omega_0 t)dt = \frac{A\omega_0}{\omega_0^2 + W^2} \]

39. (C) \( \overline{Y} = \overline{X} \int_{-\infty}^{\infty} h(t)dt = 2 \int_{0}^{\infty} 3t^2e^{-s_1} dt = \frac{3}{128} \)

40. (D) \( R_{yy}(\tau) = \int_{-\infty}^{\infty} R_{xy}(\tau + \xi) h(\xi)d\xi \)

\[ = e^{-\tau} \int_{-\infty}^{\infty} u(\xi)u(\xi + \tau)(\xi^2 + \xi^2)e^{-s_1} d\xi \]

There are two cases of interest \( \tau \geq 0 \) and \( \tau < 0 \) Since \( R_{yy}(\tau) \) is an even function we solve only the ease \( \tau \geq 0 \)

\[ R_{yy}(\tau) = e^{-\tau} \int_{0}^{\infty} (\tau^2 + \xi^2)e^{-s_1} d\xi = \frac{1 + 4\tau^2}{4\tau^2} e^{-s_1} \]

41. (B) Power in \( y(t) = R_{yy}(0) = \frac{1}{4a^2} \)

42. (D) \( P_{xx} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_{xx}(\omega) d\omega = \frac{3}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{49 + \omega^2} = \frac{3}{14} \)

43. (B) \( h_2 = 49t^2e^{-7t} \)

\[ \leftrightarrow \quad \frac{2}{(7 + j\omega)^2} = H_2(\omega) \]

\( s_{yy}(\omega) = s_{xx}(\omega) = |H_1(\omega)H_2(\omega)|^2 = \frac{12\omega^2}{(49 + \omega^2)^2} \)

44. (A) \( P_{yy} = \frac{N_0 |H(0)|^2}{2\pi} \)

So \( N_0 = \frac{2\pi(0.1)}{|H(0)|^2 W_n} = \frac{2\pi(0.1)}{(2)^2 2\pi \times 2 \times 10^8} = 125 \times 10^{-8} \) W/Hz = 12.5 nW/Hz

45. (A) \( P_{yy} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_{xx}(\omega)|H(\omega)|^2 d\omega \)

\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{50}{\sqrt{8\pi}} \frac{e^{-\frac{\omega^2}{2\pi}}}{\sqrt{2\pi}} = 200 \frac{1}{\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-\frac{\omega^2}{2\pi}} \]

\[ = 200 \frac{1}{\pi} [F(2) - F(-2)] = \frac{200}{\pi} [2F(2) - 1] = 60.8 \]

46. (B) \( P_{yy} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_{xx}(\omega)|H(\omega)|^2 d\omega \)

\[ = \frac{6 \times 10^{-5}}{\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} = \frac{6 \times 10^{-5} W}{\pi} \]

So \( W = 2.5 \pi \times 10^6 \)

47. (B) Noise bandwidth \( W_n = \int_{0}^{\infty} |H(\omega)|^2 d\omega \)

\[ W_n = \int_{0}^{\infty} \frac{d\omega}{1 + (\omega/W)^4} = \frac{\pi W}{2\sqrt{2}} \]

***********
1. The power spectral density of a bandpass white noise \( n(t) \) is \( A/2 \) as shown in Fig. P.7.3.1. The value of \( n^2 \) is

   \[ \begin{array}{c}
   S_n(\omega) \\ 
   \frac{A}{4} \\ 
   \omega \\ 
   \end{array} \]

   Fig. P7.3.1

   (A) \( \cdot A \)  
   (B) \( 2 \cdot A \)  
   (C) \( 2 \pi \cdot A \)  
   (D) \( \frac{A}{\pi} \)

2. In a receiver the input signal is 100 \( \mu V \), while the internal noise at the input is 10 \( \mu V \). With amplification the output signal is 2 V, while the output noise is 0.4 V. The noise figure of receiver is

   (A) 2  
   (B) 0.5  
   (C) 0.2  
   (D) None of the above

3. A receiver is operated at a temperature of 300 K. The transistor used in the receiver have an average output resistance of 1 k\( \Omega \). The Johnson noise voltage for a receiver with a bandwidth of 200 kHz is

   (A) 1.8 \( \mu V \)  
   (B) 8.4 \( \mu V \)  
   (C) 4.3 \( \mu V \)  
   (D) 12.6 \( \mu V \)

4. A resistor \( R = 1 \) k\( \Omega \) is maintained at 17\(^\circ\)C. The rms noise voltage generated in a bandwidth of 10 kHz is

   (A) \( 16 \times 10^{-14} \) V  
   (B) 0.4 \( \mu V \)  
   (C) 4 \( \mu V \)  
   (D) \( 16 \times 10^{-18} \) V

5. A mixer stage has a noise figure of 20 dB. This mixer stage is preceded by an amplifier which has a noise figure of 9 dB and an available power gain of 15 dB. The overall noise figure referred to the input is

   (A) 11.07  
   (B) 18.23  
   (C) 56.48  
   (D) 97.38

6. A system has three stage cascaded amplifier each stage having a power gain of 10 dB and noise figure of 6 dB. The overall noise figure is

   (A) 1.38  
   (B) 6.8  
   (C) 4.33  
   (D) 10.43

7. A signal process \( m(t) \) is mixed with a channel noise \( n(t) \). The power spectral density are as follows

   \[ S_m(\omega) = \frac{6}{9 + \omega^2}, \quad S_n(\omega) = 6 \]

   The optimum Wiener-Hopf filter is

   (A) \( \frac{\omega^2 + 9}{\omega^2 + 10} \)  
   (B) \( \frac{1}{\omega^2 + 9} \)  
   (C) \( \frac{\omega^2 + 10}{\omega^2 + 9} \)  
   (D) None of the above

**Statement for Question 8-9**

A sonar echo system on a submarine transmits a random noise \( n(t) \) to determine the distance to another targeted submarine. Distance \( R \) is given by \( v \tau_R / 2 \) where \( v \) is the speed of the sound wave in water and \( \tau_R \) is the time it takes the reflected version of \( n(t) \) to return. Assume that \( n(t) \) is a sample function of an ergodic random process \( N(t) \) and \( T \) is very large.
3. The V will be
(A) $2R_{NN}(\tau_R - \tau_T)$
(B) $R_{NN}(\tau_R - 2\tau_T)$
(C) $R_{NN}(\tau_R - \tau_T)$
(D) $\frac{1}{2}R_{NN}(\tau_R - \tau_T)$

4. What value of the delay $\tau_T$ will cause $v$ to be maximum?
(A) $\tau_R$
(B) $2\tau_R$
(C) $3\tau_R$
(D) None of the above

10. Two resistor with resistance $R_1$ and $R_2$ are connected in parallel and have Physical temperatures $T_1$ and $T_2$ respectively. The effective noise temperature $T_{eq}$ of an equivalent resistor is
(A) $\frac{T_1R_1 + T_2R_2}{R_1 + R_2}$
(B) $\frac{T_1R_1 + T_2R_2}{R_1 + R_2}$
(C) $\frac{T_1T_2(R_1 + R_2)^2}{(T_1 + T_2)R_1R_2}$
(D) $\frac{(T_1 + T_2)R_1R_2}{T_1 + T_2(R_1 + R_2)^2}$

Statement for Question 11-12:

An amplifier has a standard spot noise figure $F_0 = 6.31 (8.0 \text{ dB})$. The amplifier, that is used to amplify the output of an antenna have antenna temperature of $T_a = 180 \text{ K}$

11. The effective input noise temperature of this amplifier is
(A) 2520 K
(B) 2120 K
(C) 2710 K
(D) 1540 K

12. The operating spot noise figure is
(A) 3.2 dB
(B) 6.4 dB
(C) 9.8 dB
(D) 11.9 dB

13. An amplifier has three stages for which $T_{s1} = 200 \text{ K}$ (first stage), $T_{s2} = 450 \text{ K}$, and $T_{s3} = 1000 \text{ K}$ (last stage). If the available power gain of the second stage is 5, what gain must the first stage have to guarantee an effective input noise temperature of 250 K?
(A) 10
(B) 13
(C) 16
(D) 19

Statement for Question 14-16:

An amplifier has an operating spot noise figure of 10 dB when driven by a source of effective noise temperature 225 K.

14. The standard spot noise figure of amplifier is
(A) 4 dB
(B) 5 dB
(C) 7 dB
(D) 9 dB

15. If a matched attenuator with a loss of 3.2 dB is placed between the source and the amplifier's input, what is the operating spot noise figure of the attenuator amplifier cascade if the attenuator's physical temperature is 290 K?
(A) 9 dB
(B) 10.4 dB
(C) 11.3 dB
(D) 13.3 dB

16. In previous question what is the standard spot noise figure of the cascade?
(A) 10.3 dB
(B) 12.2 dB
(C) 14.9 dB
(D) 17.6 dB

17. Omega Electronics sells a microwave receiver (A) having an operating spot noise figure of 10 dB when driven by a source with effective noise temperature 130 K. Digilink (B) sells a receiver with a standard spot noise figure of 6 dB. Microtronics (C) sells a receiver with standard spot noise figure of 8 dB when driven by a source with effective noise temperature 190 K. The best receiver to purchase is
(A) A
(B) B
(C) C
(D) all are equal

Statement for Question 18-20:

An amplifier has three stages for which $T_{s1} = 150 \text{ K}$ (first stage), $T_{s2} = 350 \text{ K}$, and $T_{s3} = 600 \text{ K}$ (output stage). Available power gain of the first stage is 10 and overall input effective noise temperature is 190 K.

18. The available power gain of the second stage is
(A) 12
(B) 14
(C) 16
(D) 18

19. The cascade's standard spot noise figure is
(A) 1.3 dB
(B) 2.2 dB
(C) 4.3 dB
(D) 5.3 dB

20. What is the cascade's operating spot noise figure when used with a source of noise temperature $T_s = 50 \text{ K}$?
(A) 1.34 dB
(B) 3.96 dB
(C) 6.81 dB
(D) None of the above.
21. Three networks are cascaded. Available power gains are \( G_1 = 8, G_2 = 6 \) and \( G_3 = 20 \). Respective input effective spot noise temperature are \( T_{e1} = 40 \) K, \( T_{e2} = 100 \) K and \( T_{e3} = 180 \) K.

(A) 58.33 K  
(B) 69.41 K  
(C) 83.90 K  
(D) 98.39 K

22. Three identical amplifier, each having a spot effective input noise temperature of 125 K and available power \( G \) are cascaded. The overall spot effective input noise temperature of the cascade is 155 K. The \( G \) is

(A) 3  
(B) 5  
(C) 7  
(D) 9

23. Three amplifier that may be connected in any order in a cascade are defined as follows:

<table>
<thead>
<tr>
<th>Amplifier</th>
<th>Effective Input Noise Temperature</th>
<th>Available Power Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>110 K</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>120 K</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>150 K</td>
<td>12</td>
</tr>
</tbody>
</table>

The sequence of connection that will give the lowest overall effective input noise temperature for the cascade is

(A) ABC  
(B) CBA  
(C) ACB  
(D) BAC

24. What is the maximum average effective input noise temperature that an amplifier can have if its average standard noise figure is to not exceed 1.7 ?

(A) 203 K  
(B) 215 K  
(C) 235 K  
(D) 255 K

25. An amplifier has an average standard noise figure of \( 2.0 \) dB and an average operating noise figure of \( 6.5 \) dB when used with a source of average effective source temperature \( T_e \). The \( T_e \) is

(A) 156.32 K  
(B) 100.81 K  
(C) 48.93 K  
(D) None of the above

Statement for Question

An antenna with average noise temperature 60 K connects to a receiver through various microwave elements that can be modeled as an impedance matched attenuator with an overall loss of 2.4 dB and a physical temperatures of 275 K. The overall system noise temperature of the receiver \( T_{sys} = 820 \) K.

26. The average effective input noise temperature of the receiver is

(A) 420.5 K  
(B) 320.5 K  
(C) 220.5 K  
(D) 10.5 K

27. The average operating noise figure of the attenuator-receiver cascade is

(A) 13.67 d  
(B) 11.4 dB  
(C) 1.4 dB  
(D) 1.367 dB

28. If receiver has an available power gain of 110 dB and a noise bandwidth of 10 MHz, the available output noise power of receiver is

(A) 6.5 mW  
(B) 8.9 mW  
(C) 10.3 mV  
(D) 11.4 mV

29. If antenna attenuator cascade is considered as a noise source, its average effective noise temperature is

(A) 63 K  
(B) 149 K  
(C) 263 K  
(D) 249 K

Statement for question 30-32:

An amplifier when used with a source of average noise temperature 60 K, has an average operating noise figure of 5.

30. The \( T_e \) is

(A) 70 K  
(B) 110 K  
(C) 149 K  
(D) 240 K

31. If the amplifier is sold to engineering public, the noise figure that would be quoted in a catalog is

(A) 0.46  
(B) 0.94  
(C) 1.83  
(D) 2.93

32. What average operating noise figure results when the amplifier is used with an antenna of temperature 30 K ?

(A) 9.54 dB  
(B) 10.96 dB  
(C) 11.23 dB  
(D) 12.96 dB
33. An engineer of RS communication purchase an amplifier with average operating noise figure of 1.8 when used with a 60 Ω broadband source having average source temperature of 80 K. When used with a different 60 Ω source the average operating noise figure is 1.25. The average noise temperature of the source is (A) 125 K (B) 156 K (C) 256 K (D) 292 K

34. The $\overline{T}$ for unit 1 and 2 unit are, respectively (A) 126.4 K and 256.9 K (B) 256.9 K and 126.4 K (C) 527.8 K and 864.2 K (D) 864.2 K and 527.8 K

35. The excess noise power of unit 1 and unit 2 are respectively (A) 15.4 nW and 27.1 nW (B) 23.8 nW and 21.1 nW (C) 23.8 nW and 27.1 nW (D) 15.4 nW and 21.1 nW

36. Consider following statement
   $S_1$: If the source noise temperature $\overline{T}$ is very small, unit-2 is the best to purchase
   $S_2$: If the source noise temperature $\overline{T}$ is very small unit - 1 is the best to purchase.
   correct statement is (A) $S_1$ (B) $S_2$ (C) both $S_1$ and $S_2$ (D) None

37. A source has an effective noise temperature of $T_0(\omega) = \frac{800}{100 + \omega}$ and feeds an amplifier that has an available power gain of $G_a(\omega) = \left(\frac{8}{10 + \omega}\right)^2$. The $\overline{T}$ for this source is (A) 10 K (B) 20 K (C) 30 K (D) 40 K

38. A system have an impulse response $h = \begin{cases} e^{-Wt} & e < t \\ 0 & t < 0 \end{cases}$
   where $W$ is a real positive constant. White noise with power density $5w/Hz$ is applied to this system. The mean-squared value of response is (A) $1/W$ (B) $2.5/W$ (C) $4.5/W$ (D) $6/W$

39. White noise, for which $R_{xx}(\tau) = 10^{-2}8(\tau)$ is applied to a network with impulse response $h(t) = 4e^{3t} - 3e^{-4t}$. The network's output noise power in a 1 Ω resistor is (A) 0.15 mW (B) 0.35 mW (C) 0.55 mW (D) 0.95 mW

40. White noise with power density $N_0/2 = 6(10^{-6})$ W/Hz is applied to an ideal fitter (gain= 1) with bandwidth $W$ (rad/sec). For output's average noise power to be 15 W, the $W$ must be (A) $2.5\pi(10^{-6})$ (B) $-2.5\pi(10^{6})$ (C) $4.5\pi(10^{-2})$ (D) $4.5\pi(10^{6})$

41. An ideal filter with a mid-band power gain of 8 and bandwidth of 4 rad/s has noise $X(t)$ at its input with power spectrum $F(2) = 0.9773$. The noise power at the network's output is (A) $164$ (B) $343$ (C) $211$ (D) $191$

42. A system has the power transfer function $H(\omega) = \frac{1}{1 + \left(\frac{\omega}{W}\right)^4}$
   where $W$ is a real positive constant. The noise bandwidth of the system is (A) $\frac{\pi W}{2\sqrt{2}}$ (B) $\frac{\pi W}{\sqrt{2}}$ (C) $\frac{\pi W}{2}$ (D) None of the above

43. White noise with power density $N_0/2$ is applied to a low pass network for which $|H(0)| = 2$. It has a noise bandwidth of 2 MHz. If the average output noise power is 8.1 W in a 1 Ω resistor, the $N_0$ is (A) $625 \times 10^8$ W/Hz (B) $625 \times 10^{-8}$ W/Hz (C) $125 \times 10^8$ W/Hz (D) $125 \times 10^{-8}$ W/Hz
Statement for Question 44-46:

An amplifier has a narrow bandwidth of 1 kHz and standard spot noise figure of 3.8 at its frequency of operation. The amplifier's available output noise power is 0.1 mW when its input is connected to a radio receiving antenna having an antenna temperature of 80 K.

44. The amplifier's input effective noise temperature $T_e$ is
   (A) 812 K  (B) 600 K  (C) 421 K  (D) 321 K

45. Its operating spot noise figure $F_{op}$ is
   (A) 5.16  (B) 7.98  (C) 11.15  (D) 16.23

46. Its available power gain $G_a$ is
   (A) $2 \times 10^{12}$  (B) $4 \times 10^{12}$  (C) $8 \times 10^{12}$  (D) $11 \times 10^{12}$

SOLUTION

1. (B) $n^2 = 2 \int \frac{f^{+B}}{f^{-B}} d\nu = 2 \cdot dB$

2. (A) $NF = \frac{S_f/N_f}{S_f/N_o} = \frac{100/10}{2/0.4} = 2$

3. (A) $v_n^2 = 4kTR = 4 \times 1.38 \times 10^{-23} \times 300 \times 200 \times 10^3 \times 10^3 = 3.3 \times 10^{-12}$
   $v_{rms} = 1.8 \mu V$

4. (B) $v_n^2 = 4kTR$, $T = (273 + 17) K = 290 K$, $R = 1000 \Omega$, $B = 10^4$ Hz, $k = 1.38 \times 10^{-23}$ J/K
   $v_n^2 = 4 \times 1.38 \times 10^{-23} \times 290 \times 10^3 \times 10^3 = 16 \times 10^{-14}$ $V^2$
   $v_{rms} = 0.4 \mu V$

5. (A) $F_1 = 9$ dB = 7.94
   $F_2 = 20$ dB = 100
   $A_1 = 15$ dB = 31.62
   $F = F_1 + \frac{F_2 - 1}{A} = 7.94 + \frac{100 - 1}{31.62} = 11.07$

6. (C) Gain of each stage $A_1 = A_2 = A_3 = 10$ dB
   Noise figure of each stage
   $F_1 = F_2 = F_3 = 6$ dB or $F_1 = F_2 = F_3 = 4$ dB
   $F = F_1 + \frac{F_2 - 1}{A_1} + \frac{F_3 - 1}{A_1A_2} = 4 + \frac{4 - 1}{10} + \frac{4 - 1}{100} = 4.33$

7. (B) $H_{op}(\omega) = \frac{S_m(\omega)}{S_m(\omega) + S_{m}(\omega)} = \frac{\frac{6}{\omega^2} \omega^2}{\frac{6}{\omega^2} + 6} = \frac{1}{10 + \omega^2}$

8. (C) $V = \frac{1}{2T} \int_T^n (t - \tau) n(t - \tau) d\tau$
   Since $T$ is very large
   $V = \lim_{T \to \infty} \frac{1}{2T} \int_T^n (t - \tau) n(t - \tau) d\tau = A[n(t - \tau) n(t - \tau)]$
   Since $N(t)$ is ergodic, $V \approx \varphi_{nn}(\tau - \tau)$

9. (A) Because $|\varphi_{nn}(\tau)| \leq \varphi_{nn}(0)$ for any auto correlation function, $V$ will be maximum if $\tau_R = \tau_T$

10. (B) Use the current form of equivalent circuit
    $i_n^2 = i_1^2 + i_2^2 = \frac{2kTd\omega}{\pi R_1} + \frac{2kTd\omega}{\pi R_2}$
    $i_n^2 = \frac{2kTd\omega}{\pi R}$
    Thus $T_s = \left( \frac{T_1 + T_2}{R_1} \right) R = \frac{T_1R_2 + T_2R_1}{R_1 + R_2}$
11. (D) $T_c = T_o(F_o - 1) = 290(6.31 - 1) = 1539.9 \text{ K}$

12. (C) $F_{op} = 1 + \frac{T_c}{T_o} = 1 + \frac{1540}{180} = 9.56$ or 9.8 dB

13. (B) $T_c = 290 = T_s + \frac{T_{c2}}{G_1} + \frac{T_{c3}}{G_2}$
   
   $250 = 200 + \frac{450}{G_1} + \frac{1000}{5G_2}$ or $G_1 = 13$

14. (D) $F_o = 1 + \frac{T_c}{T_o}(F_{op} - 1) = 1 + \frac{225}{290}(10 - 1)$
   
   $= 7.98$ or 9.0 dB

15. (D) Here $L = 2.089$ or 3.2 dB, $T_c = 290 \text{ K}$
   
   $T = T_s + \frac{T_{c2}}{G_1} = T_s(L - 1) + \frac{T_o(F_o - 1)}{1/L}$
   
   $= 290(2.089 - 1) + (2.089)(7.98 - 1) = 4544.4 \text{ K}$
   
   $F_{op} = 1 + \frac{4544.4}{225} = 212$ or 13.3 dB

16. (B) $F_o = 1 + \frac{4544.4}{290} = 16.67$ or 12.2 dB

17. (B) For A: $F_{op} = 10$ (or 10 dB) when $T_s = 130 \text{ K}$
   
   $T_{c4} = 130(10 - 1) = 11700 \text{ K}$
   
   For B: $F_o = 398$ (or 6 dB) when $T_c = 290 \text{ K}$
   
   $T_{c6} = 290(398 - 1) = 3642 \text{ K}$
   
   For C: $F_o = 6.3$ (or 8 dB) when $T_c = 190 \text{ K}$
   
   $T_{c6} = 190(6.3 - 1) = 1007 \text{ K}$, (B) is better as $T_{c6}$ is less.

18. (A) $T_s = T_{c4} + \frac{T_{c1}}{G_1} + \frac{T_{c3}}{G_1G_2}$
   
   $G_2 = \frac{T_{c4}}{G_1(T_s - T_{c1} - \frac{T_{c2}}{m})} = 600$
   
   $= 10(190 - 150 - \frac{100}{m}) = 12$

19. (B) $F_o = 1 + \frac{T_c}{T_o} = 1 + \frac{190}{290} = 1.655$ or 2.19 dB

20. (C) $F_{op} = 1 + \frac{T_c}{T_o} = 1 + \frac{190}{50} = 4.8$ or 6.81 dB

21. (A) $T_s = T_{c1} + \frac{T_{c2}}{G_1} + \frac{T_{c3}}{G_1G_2} = 40 + \frac{100}{8} + \frac{280}{8(6)} = 58.33 \text{ K}$

22. (B) $T_c = T_{c1} + \frac{T_{c2}}{G_1} + \frac{T_{c3}}{G_1G_2} = T_{c4} \left[ \frac{1}{G} + \frac{1}{G^2} \right]$
   
   or $(T_{c1} - T_c)G^2 + T_cG + T_{c1} = 0$

(125 - 155)G^2 + 125G + 125 = 0

6G^2 - 25G - 25 = 0 or $G = 5$

23. (A) Sequence $T_s$
   
   ABC110 + \frac{120}{4} + \frac{150}{4} = 146.25

ACB110 + \frac{150}{4} + \frac{120}{4(12)} = 150.00

BAC120 + \frac{110}{6} + \frac{150}{6(4)} = 144.583 ← Best

CBA150 + \frac{120}{12} + \frac{110}{(12)(6)} = 161.528

24. (A) $\overline{T_s} = T_o(\overline{F} - 1) \leq 290(1.7 - 1) = 203 \text{ K}$

25. (D) Here $\overline{F_o} = 1.585$ (or 2.0 dB) and $\overline{F_{op}} = 4.467$ (or 6.5 dB)
   
   $\overline{T_s} = T_o(\overline{F} - 1) = \frac{290(1.585 - 1)}{4.467 - 1} = 48.93 \text{ K}$

26. (B) Here $T_a = 60 \text{ K}$, $L = 1.738$ (or 2.4 dB), $T_c = 275 \text{ K}$
   
   and $\overline{T_s} = 820 \text{ K}$. We know that
   
   $\overline{T_R} = \frac{[\overline{T_{sys}} - T_a - T_s(L - 1)]}{L} = \frac{820 - 60 - 275(1.738 - 1)}{1738}$
   
   $= 320.5 \text{ K}$

27. (B) $\overline{F_{op}} = 1 + \frac{T_c}{T_o} = 1 + \frac{\overline{T_{sys}} - T_a}{T_s} = 1 + \frac{820 - 60}{60}$
   
   $= 13.67$ or 11.4 dB

28. (A) Here $G_p(\omega_0) = 10^{11}$ (or 110 dB)
   
   and $W_{pp} = 2\pi(10^7) \text{ Hz}$
   
   $N_{cl} = k\overline{T}_{sys}G_p(\omega)W_{sys} = \frac{138(10^{-23})(820)(10^{11})(10^7)}{2\pi L}$
   
   $= 651.110^{-5}$ or 6.51 mW

29. (C) $dN_{cl} = k[T_a + T_s(L - 1)] \frac{d\omega}{2\pi} = kT_s \frac{d\omega}{2\pi}$
   
   Thus $T_s = T_a + T_s(L - 1)$
   
   $= 60 + 275(1.738 - 1) = 263 \text{ K}$

30. (D) $\overline{T_s} = \overline{T_o}(\overline{F_{op}} - 1) = 60(5 - 1) = 240 \text{ K}$

31. (C) $\overline{F_o} = 1 + \frac{\overline{T_o}}{290} = 1 + \frac{240}{290} = 1.8276$

32. (A) $\overline{F_{op}} = 1 + \frac{\overline{T_c}}{290} = 1 + \frac{240}{30} = 9$ or 9.54 dB
Chapter 7.4

Amplitude Modulation

Statement for Question 1 - 3

An AM signal is represented by
\[ x(t) = (20 + 4 \sin 5000 \pi t) \cos(2\pi \times 10^5 t) \text{ V} \]

1. The modulation index is
   (A) 20
   (B) 4
   (C) 0.2
   (D) 10

2. The total signal power is
   (A) 208 W
   (B) 204 W
   (C) 408 W
   (D) 416 W

3. The total sideband power is
   (A) 4 W
   (B) 8 W
   (C) 16 W
   (D) 2 W

Statement for Question 4 - 5:

An AM signal has the form
\[ x(t) = [20 + 2 \cos 3000 \pi t + 10 \cos 6000 \pi t] \cos 2\pi f_c t \text{ where } f_c = 10^5 \text{ Hz}. \]

4. The modulation index is
   (A) \( \frac{201}{400} \)
   (B) \( -\frac{201}{400} \)
   (C) \( \frac{199}{400} \)
   (D) \( -\frac{199}{400} \)

5. The ratio of the sidebands power to the total power is
   (A) \( \frac{43}{225} \)
   (B) \( \frac{26}{225} \)
   (C) \( \frac{26}{225} \)
   (D) \( \frac{43}{225} \)

6. A 2 kW carrier is to be modulated to a 90% level. The total transmitted power would be
   (A) 3.62 kW
   (B) 2.81 kW
   (C) 1.4 kW
   (D) None of the above

7. An AM broadcast station operates at its maximum allowed total output of 50 kW with 80% modulation. The power in the intelligence part is
   (A) 12.12 kW
   (B) 31.12 kW
   (C) 6.42 kW
   (D) None of the above

8. The aerial current of an AM transmitter is 18 A when unmodulated but increases to 20 A when modulated. The modulation index is
   (A) 0.68
   (B) 0.73
   (C) 0.89
   (D) None of the above

9. A modulating signal is amplified by a 80% efficiency amplifier before being combined with a 20 kW carrier to generate an AM signal. The required DC input power to the amplifier, for the system to operate at 100% modulation, would be
   (A) 5 kW
   (B) 8.46 kW
   (C) 12.5 kW
   (D) 6.25 kW

10. A 2 MHz carrier is amplitude modulated by a 500 Hz modulating signal to a depth of 70%. If the unmodulated carrier power is 2 kW, the power of the modulated signal is
    (A) 2.23 kW
    (B) 2.36 kW
    (C) 1.18 kW
    (D) 1.26 kW

11. A carrier is simultaneously modulated by two sine waves with modulation indices of 0.4 and 0.3. The resultant modulation index will be
    (A) 1.0
    (B) 0.7
    (C) 0.5
    (D) 0.35
12. In a DSB-SC system with 100% modulation, the power saving is
(A) 50%  (B) 66%  (C) 75%  (D) 100%

13. A 10 kW carrier is sinusoidally modulated by two carriers corresponding to a modulation index of 30% and 40% respectively. The total radiated power is
(A) 11.25 kW  (B) 12.5 kW  (C) 15 kW  (D) 17 kW

14. In amplitude modulation, the modulation envelope has a peak value which is double the unmodulated carrier value. What is the value of the modulation index ?
(A) 25%  (B) 50%  (C) 75%  (D) 100%

15. If the modulation index of an AM wave is changed from 0 to 1, the transmitted power
(A) increases by 50%  (B) increases by 75%  (C) increases by 100%  (D) remains unaffected

16. A diode detector has a load of 1 kΩ shunted by a 10000 pF capacitor. The diode has a forward resistance of 1 Ω. The maximum permissible depth of modulation, so as to avoid diagonal clipping, with modulating signal frequency f_0 = 10 kHz will be
(A) 0.847  (B) 0.628  (C) 0.734  (D) None of the above

17. An AM signal is detected using an envelop detector. The carrier frequency and modulating signal frequency are 1 MHz and 2 kHz respectively. An appropriate value for the time constant of the envelope detector is.
(A) 500 µ sec  (B) 20 µ sec  (C) 0.2 µ sec  (D) 1 µ sec

18. An AM voltage signal s(t), with a carrier frequency of 1.15 GHz has a complex envelope g(t) = \text{A}_c[1 + m(t)], where \text{A}_c = 500 V, and the modulation is a 1 kHz sinusoidal test tone described by m(t) = 0.8 \sin(2\pi \times 10^3t) appearing across a 50 Ω resistive load. What is the actual power dissipated in the load ?
(A) 165 kW  (B) 82.5 kW  (C) 3.3 kW  (D) 6.6 kW

19. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 µ sec. Which of the following frequencies will NOT be present in the modulated signal?
(A) 990 KHz  (B) 1010 KHz  (C) 1020 KHz  (D) 1030 KHz

20. For an AM signal, the bandwidth is 10 kHz and the highest frequency component present is 705 kHz. The carrier frequency used for this AM signal is
(A) 695 kHz  (B) 700 kHz  (C) 705 kHz  (D) 710 kHz

21. A message signal \( m(t) = \text{sinc} t + \text{sinc}^2(t) \) modulates the carrier signal \( c(t) = A \cos 2\pi f_c t \). The bandwidth of the modulated signal is
(A) \( 2f_c \)  (B) \( \frac{3}{2}f_c \)  (C) 2  (D) \( \frac{\pi}{4} \)

22. The signal \( m(t) = \cos 2000\pi t + 2 \cos 4000\pi t \) is multiplied by the carrier \( c(t) = 100 \cos 2\pi f_c t \) where \( f_c = 1 \) MHz to produce the DSB signal. The expression for the upper side band (USB) signal is
(A) \( 100 \cos(2\pi f_c t + 1000t) + 200 \cos(2\pi f_c t + 200t) \)
(B) \( 100 \cos(2\pi f_c t - 1000t) + 200 \cos(2\pi f_c t - 2000t) \)
(C) \( 50 \cos(2\pi f_c t + 1000t) + 100 \cos(2\pi f_c t + 2000t) \)
(D) \( 50 \cos(2\pi f_c t - 1000t) + 100 \cos(2\pi f_c t - 100t) \)

Statement for Question 23-26 :

The Fourier transform \( M(f) \) of a signal \( m(t) \) is shown in figure. It is to be transmitted from a source to destination. It is known that the signal is normalized, meaning that \( -1 \leq m(t) \leq 1 \)

![M(f) graph](Fig.P7.4.23-26)

23. If USSB is employed, the bandwidth of the modulated signal is
(A) 5 kHz  (B) 20 kHz, 10 kHz  (C) 20 kHz  (D) None of the above

24. If DSB is employed, the bandwidth of the modulated signal is
(A) 5 kHz  (B) 10 kHz  (C) 20 kHz  (D) None of the above
25. If an AM modulation scheme with $\alpha = 0.8$ is used, the bandwidth of the modulated signal is.

(A) 5 kHz  (B) 10 kHz  
(C) 20 kHz  (D) None of the above

26. If an FM signal with $k_f = 60$ kHz is used, then the bandwidth of the modulated signal is

(A) 5 kHz  (B) 10 kHz  
(C) 20 kHz  (D) None of the above

27. A DSB modulated signal $x(t) = Am(t) \cos 2\pi f_c t$ is mixed (multiplied) with a local carrier $x_c(t) = \cos (2\pi f_c t + \theta)$ and the output is passed through a LPF with a bandwidth equal to the bandwidth of the message $m(t)$. If the power of the signal at the output of the low pass filter is $p_{out}$ and the power of the modulated signal is $p_{in}$, the $\frac{p_{out}}{p_{in}}$ is

(A) $0.5 \cos \theta$  (B) $\cos^2 \theta$  
(C) $0.5 \cos^2 \theta$  (D) $\frac{1}{2} \cos^2 \theta$

28. A DSB-SC signal is to be generated with a carrier frequency $f_c = 1$ MHz using a non-linear device with the input-output characteristic $v_o = a_0 v_i + a_1 v_i^3$ where $a_0$ and $a_1$ are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter. Let $v_i = A_m \cos (2\pi f_c t) + m(t)$ where $m(t)$ is the message signal. Then the value of $f_c^2$(in MHz) is

(A) 1.0  (B) 0.333  
(C) 0.5  (D) 3.0

29. A non-linear device with a transfer characteristic given by $i = (10 + 2v_i + 0.2v_i^2)$ mA is supplied with a carrier of 1 V amplitude and a sinusoidal signal of 0.5 V amplitude in series. If at the output the frequency component of AM signal is considered, the depth of modulation is

(A) 18 %  (B) 10 %  
(C) 20 %  (D) 33.33 %

Statement for Question 30-31

Consider the system shown in fig.7.4.30-31. The modulating signal $m(t)$ has zero mean and its maximum (absolute) value is $A_{m\text{max}}$. It has bandwidth $W_m$. The nonlinear device has a input-output characteristic $y(t) = ax(t) + bx^2(t)$.

30. The filter should be a

(A) LPP with bandwidth $W$  
(B) LPF with bandwidth $2W$  
(C) a BPF with center frequency $f_0$ and $BW = W$ such that $f_0 - W_m > f_0 - \frac{W}{2} > 2W_m$  
(D) a BPF with center frequency $f_0$ and $BW = W$ such that $f_0 - W_m > f_0 - \frac{W}{2} > W_m$

31. The modulation index is

(A) $\frac{2b}{a} A_m$  (B) $\frac{2a}{b} A_m$  
(C) $\frac{a}{b} A_m$  (D) $\frac{b}{a} A_m$

32. A message signal is periodic with period $T$, as shown in figure. This message signal is applied to an AM modulator with modulation index $\alpha = 0.4$. The modulation efficiency would be

(A) 51 %  (B) 11.8 %  
(C) 5.1 %  (D) None of the above

Statement for Question 33-36

The figure 6.54-57 shows the positive portion of the envelope of the output of an AM modulator. The message signal is a waveform having zero DC value.
34. The modulation efficiency is
   (A) 8.3 %  (B) 14.28 %  
   (C) 7.69 %  (D) None of the above

35. The carrier power is
   (A) 60 W  (B) 450 W  
   (C) 30 W  (D) 900 W

36. The power in sidebands is
   (A) 85 W  (B) 42.5 W  
   (C) 56 W  (D) 37.5 W

37. In a broadcast transmitter, the RF output is represented as
   \[ e(t) = 50[1 + 0.89 \cos 5000t + 0.30 \sin 9000t] \cos(6 \times 10^6 t) \text{V} \]
   What are the sidebands of the signals in radians?
   (A) 5 \times 10^3 and 9 \times 10^3  
   (B) 5.991 \times 10^6, 5.995 \times 10^6, 6.005 \times 10^6 and 6.009 \times 10^6  
   (C) 4 \times 10^3, 1.4 \times 10^4  
   (D) 1 \times 10^6, 1.1 \times 10^7, 3 \times 10^6, and 15 \times 10^7

38. An AM modulator has output
   \[ x(t) = 40 \cos 400 \pi t + 4 \cos 360 \pi t + 4 \cos 440 \pi t \]
   The modulation efficiency is
   (A) 0.01  (B) 0.02  
   (C) 0.03  (D) 0.04

39. An AM modulator has output
   \[ x(t) = A \cos 400 \pi t + B \cos 380 \pi t + B \cos 420 \pi t \]
   The carrier power is 100 W and the efficiency is 40%. The value of A and B are
   (A) 14.14, 8.16  (B) 50, 10  
   (C) 22.36, 13.46  (D) None of the above

**Statement for Question 42-43**

Consider the system shown in figure 6.69-70. The average value of \( m(t) \) is zero and maximum value of \( |m(t)| \) is \( M \). The square-law device is defined by
   \[ y(t) = 4x(t) + 10x(t) \]

40. The value of \( M \), required to produce modulation index of 0.8, is
   (A) 0.32  (B) 0.26  
   (C) 0.52  (D) 0.16

41. Let \( W \) be the bandwidth of message signal \( m(t) \). AM signal would be recovered if
   (A) \( f_c \geq W \)  (B) \( f_c > 2W \)  
   (C) \( f_c \geq 3W \)  (D) \( f_c > 4W \)

42. A super heterodyne receiver is designed to receive transmitted signals between 5 and 10 MHz. High-side tuning is to be used. The tuning range of the local oscillator for IF frequency 500 kHz would be
   (A) 4.5 MHz - 9.5 MHz  
   (B) 5.5 MHz - 10.5 MHz  
   (C) 4.5 MHz - 10.5 MHz  
   (D) None of the above

43. A super heterodyne receiver uses an IF frequency of 455 kHz. The receiver is tuned to a transmitter having a carrier frequency of 2400 kHz. High-side tuning is to be used. The image frequency will be
   (A) 2855 kHz  (B) 3310 kHz  
   (C) 1845 kHz  (D) 1490 kHz
46. In the circuit shown in fig.P7.4.46, the transformers are center tapped and the diodes are connected as shown in a bridge. Between the terminals 1 and 2 an a.c. voltage source of frequency 400 Hz is connected. Another a.c. voltage of 1.0 MHz is connected between 3 and 4. The output between 5 and 6 contains components at

![Fig.P7.4.46](image)

(A) 400 Hz, 1.0 MHz, 1000.4 kHz, 999.6 kHz
(B) 400 Hz, 1000.4 kHz, 999.6 kHz
(C) 1 MHz, 1000.4 kHz, 999.6 kHz
(D) 1000.4 kHz, 999.6 kHz

47. A superheterodyne receiver is to operate in the frequency range 550 kHz-1650 kHz, with the intermediate frequency of 450 kHz. Let \( \frac{R}{C} \) denote the required capacitance ratio of the local oscillator and \( I \) denote the image frequency (in kHz) of the incoming signal. If the receiver is tuned to 700 kHz, then

(A) \( \frac{R}{C} = 4.41, I = 1600 \)  
(B) \( \frac{R}{C} = 2.10, I = 1150 \)  
(C) \( R = 3, I = 1600 \)  
(D) \( R = 9.0, I = 1150 \)

48. Consider a system shown in Figure. Let \( X(f) \) and \( Y(f) \) denote the Fourier transforms of \( x(t) \) and \( y(t) \) respectively. The ideal HPF has the cutoff frequency 10 kHz. The positive frequencies where \( Y(f) \) has spectral peaks are

![Fig.P7.4.48](image)

(A) 1 kHz and 24 kHz  
(B) 2 kHz and 24 kHz  
(C) 1 kHz and 14 kHz  
(D) 2 kHz and 14 kHz

49. In fig.P7.4.49

\[ m(t) = \frac{2 \sin 2\pi t}{t}, \quad s(t) = \cos 200\pi t \]  
\[ n(t) = \frac{\sin 199\pi t}{t} \]

The output \( y(t) \) will be

(A) \( \frac{\sin 2\pi t}{t} \)  
(B) \( \frac{\sin 2\pi t}{t} + \frac{\sin \pi t}{t} \cos 3\pi t \)  
(C) \( \frac{\sin 2\pi t}{t} + \frac{\sin 0.5\pi t}{t} \cos 15\pi t \)  
(D) \( \frac{\sin 2\pi t}{t} + \frac{\sin \pi t}{t} \cos 0.75\pi t \)

50. 12 signals each band-limited to 5 kHz are to be transmitted over a single channel by frequency division multiplexing. If AM -SSNB modulation guard band of 1 kHz is used, then the bandwidth of the multiplexed signal will be

(A) 51 kHz  
(B) 61 kHz  
(C) 71 kHz  
(D) 81 kHz

51. Let \( x(t) \) be a signal band-limited to 1 kHz. Amplitude modulation is performed to produce signal \( g(t) = x(t) \sin 2000\pi t \). A proposed demodulation technique is illustrated in figure 6.83. The ideal low pass filter has cutoff frequency 1 kHz and pass band gain of 2. The \( y(t) \) would be

(A) \( 2y(t) \)  
(B) \( y(t) \)  
(C) \( \frac{1}{2} y(t) \)  
(D) 0

52. Suppose we wish to transmit the signal \( x(t) = \sin 200\pi t + 2 \sin 400\pi t \) using a modulation that create the signal \( g(t) = x(t) \sin 400\pi t \). If the product \( g(t) \sin 400\pi t \) is passed through an ideal LPF with cutoff frequency 400π and pass band gain of 2, the signal obtained at the output of the LPF is

(A) \( \sin 200\pi t \)  
(B) \( \frac{1}{2} \sin 200\pi t \)  
(C) \( 2 \sin 200\pi t \)  
(D) 0

53. In an AM signal the received signal power is \( 10^{-10} \) W with a maximum modulating signal of 5 kHz. The noise spectral density at the receiver input is \( 10^{-18} \) W/Hz. If the noise power is restricted to the message signal
bandwidth only, the signals-to-noise ratio at the input to the receiver is
(A) 43 dB  (B) 66 dB  (C) 56 dB  (D) 33 dB

**Statement for Question 54-55**

Consider the following Amplitude Modulated (AM) signal, where $f_m < B$

$$x_{AM}(t) = 10(1 + 0.5 \sin 2\pi f_m t) \cos 2\pi f_c t.$$  

54. The average side-band power for the AM signal given above is
(A) 25  (B) 12.5  (C) 6.25  (D) 3.125

55. The AM signal gets added to a noise with Power Spectra Density $S_n(f)$ given in the figure below. The ratio of average sideband power to mean noise power would be
(A) $\frac{25}{8N_0B}$  (B) $\frac{25}{4N_0B}$  (C) $\frac{25}{2N_0B}$  (D) $\frac{25}{N_0B}$

**Statement for Question 56-57**

A certain communication channel is characterized by 80 dB attenuation and noise power-spectral density of $10^{-10}$ W/Hz. The transmitter power is 40 kW and the message signal has a bandwidth of 10 kHz.

56. In the case of conventional AM modulation, the predetection SNR is
(A) $10^8$  (B) $2 \times 10^8$  (C) $10^3$  (D) $2 \times 10^2$

57. In case of SSB, the predetection SNR is
(A) $2 \times 10^2$  (B) $4 \times 10^2$  (C) $2 \times 10^3$  (D) $4 \times 10^3$

SOLUTION

1. (C) $u(t) = (20 + 4 \sin 500\pi t) \cos(2\pi \times 10^6 t) \ V$

$$= 20(1 + 0.2 \sin 500\pi t) \cos(2\pi \times 10^6 t) \ V, \quad \alpha = 0.2$$

2. (B) $P_i = \frac{20^2}{2} = 200 \ W, \quad P_t = P_i \left(1 + \frac{(0.2)^2}{2}\right) = 204 \ W$

3. (A) $P_{ab} = P_t - P_c = 204 - 20 = 4 \ W$

4. (B) $x(t)$

$$= (20 + 2 \cos(2\pi 1500 t) + 10 \cos(2\pi 3000 t)) \cos(2\pi f_c t)$$

$$= 20 \left( \frac{1}{10} \cos(2\pi 1500 t) + \frac{1}{2} \cos(2\pi 3000 t) \cos(2\pi f_c t) \right)$$

This is the form of a conventional AM signal with message signal $m(t) = \frac{1}{10} \cos(2\pi 1500 t) + \frac{1}{2} \cos(2\pi 3000 t)$

$$= \cos^2(2\pi 1500 t) + \frac{1}{10} \cos(2\pi 1500 t) - \frac{1}{2}$$

The minimum of $g(z) = z^2 + \frac{1}{10} z - \frac{1}{2}$ is achieved for $z = -\frac{1}{20}$ and it is $\min(g(z)) = -\frac{201}{400}$. Since $z = -\frac{1}{20}$ is in the range of $\cos(2\pi 1500 t)$, we conclude that the minimum value of $m(t)$ is $-\frac{201}{400}$. Hence, the modulation index is $\alpha = -\frac{201}{400}$

5. (B) $x(t) = 20 \cos(2\pi f_c t) + \cos(2\pi (f_c - 1500) t)$

$$= 5 \cos(2\pi (f_c - 1500) t) + 5 \cos(2\pi (f_c + 3000) t)$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{1}{2} + \frac{25}{2} + \frac{25}{2} = 26$$

The total power is $P_{\text{total}} = P_{\text{carrier}} + P_{\text{sidebands}} = 200 + 26 = 226$

The ratio of the sidebands power to the total power is

$$\frac{P_{\text{sidebands}}}{P_{\text{total}}} = \frac{26}{226}$$

6. (B) $P_i = P_c \left(1 + \frac{\alpha^2}{2}\right) = 2000 \left(1 + \frac{0.9^2}{2}\right) = 2810 \ W$

7. (A) $P_i = P_c \left(1 + \frac{\alpha^2}{2}\right)$  or  $50 \times 10^3 = P_i \left(1 + \frac{0.8^2}{2}\right)$

$$P_c = 37.88 \ kW, \quad P_t = (P_i - P_c) = (50 - 37.88) = 12.12 \ kW$$

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8. \( I_t = I_a \left( 1 + \frac{\alpha^2}{2} \right)^{\frac{1}{2}} \) or \( 20 = 18 \left( 1 + \frac{\alpha^2}{2} \right)^{\frac{1}{2}} \) or \( \alpha = 0.68 \)

9. (C) \( P_t = 20000 \left( 1 + \frac{1}{2} \right) \) \( P_t = 30 \) kW, 
   \( P_i = 30 - 20 = 10 \) kW  
   The DC input power \( \frac{10}{0.8} = 12.5 \) kW.

10. (A) \( P_c = 2 \) kW, \( \alpha = 70\% = 0.7 \) 
    \( P_t = P_c \left( 1 + \frac{\alpha^2}{2} \right) = 2 \left( 1 + \frac{0.7^2}{2} \right) = 2.23 \) kW

11. (C) \( \alpha^2 = \alpha_1^2 + \alpha_2^2 = 0.3^2 + 0.4^2 = 0.5^2 \) or \( \alpha = 0.5 \)

12. (B) In previous solution \( P_c = \frac{2}{3} P \). If carrier is suppressed then \( \frac{2}{3} P \) or 66% power will be saved.

13. (A) \( P_i = P_c \left( 1 + \frac{\alpha_1^2}{2} + \frac{\alpha_2^2}{2} \right) = 10 \left( 1 + \frac{0.3^2}{2} + \frac{0.4^2}{2} \right) \) 
    \( = 112.5 \) kW

14. (D) \( x(t) = A_c \left( 1 + \alpha \cos 2\pi f_m t \right) \cos 2\pi f_c t \) 
   Here \( A_c(1 + \alpha) = 2A_c \). Thus \( \alpha = 1 \), therefor modulation index is 1 or 100% modulation.

15. (A) If modulation index \( \alpha \) is 0, then 
    \( P_{i1} = \frac{A_c^2}{2} \left( 1 + 0^2 \right) = \frac{A_c^2}{2} \) 
    If modulation index is 1 then 
    \( P_{i2} = \frac{A_c^2}{2} \left( 1 + 1^2 \right) = \frac{3 A_c^2}{4} \) 
    \( P_{i2} = \frac{3}{2} P_{i1} \) 
    Thus \( P_{i2} = 15P_{i1} \) and \( P_{i2} \) is increases by 50% 

16. (A) \( f_m = 10 \) kHz, \( R = 10000 \Omega \), \( C = 10000 \) pF 
   Hence \( 2\pi f_m RC = 2\pi \times 10^4 \times 10^3 \times 10^{-8} = 0.628 \) 
   \( \alpha_{\text{max}} = \left( 1 + (0.628)^2 \right)^{\frac{1}{2}} = 0.847 \)

17. (B) \( \frac{1}{f_c} \leq RC \leq \frac{1}{BW_m} \), Here \( f_c = 1 \) MHz 
   Signal Bandwidth \( BW_m = 2f_m = 2 \times 2 \times 10^3 = 4 \) kHz 
   Thus \( \frac{1}{10^6} \leq RC \leq \frac{1}{4 \times 10^7} \) or \( 10^{-6} \leq RC \leq 250 \mu \text{s} \) 
   Thus appropriate value is 20 \( \mu \text{sec} \)

18. (A) \( P_t = \frac{A_c^2}{2} \left[ 1 + \frac{\alpha^2 |m(t)|^2}{2} \right] \) 
   Here modulation index \( \alpha = 1 \). Thus 
   \( P_t = \frac{500^2}{2} \left[ 1 + \frac{0.8^2}{2} \right] = 165 \) kW

19. (C) \( c(t) = \sin 2\pi f_c t, \quad f_c = 1000 \) kHz, \( x(t) = c(t)m(t) \) 
   Expressing square wave as modulating signal \( m(t) \) 
   \( m(t) = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_m(2n-1)] \) 
   The modulated output 
   \( x(t) = 4 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_m(2n-1)] \sin(2\pi 1000 \times 10^3 t) \) 
   So frequency component \( 10^6 \pm f_m(2n-1) \) will be present where \( n = 1, 2, 3, \ldots \) 
   For \( f_m = 10 \) kHz and \( n = 1 \& 2 \) frequency present is 990, 970, 1030 kHz. Thus 1020 kHz will be absent.

20. (B) \( f_c + f_m = 705 \) kHz, 
   \( BW = 2f_m = 10 \) kHz or \( f_m = 5 \) kHz 
   \( f_c = 705 - 5 = 700 \) kHz

21. (C) \( x(t) = m(t)c(t) = A\text{sinc}(t) + \text{sinc}^2(t) \cos(2\pi f_c t) \) 
   Taking the Fourier transform of both sides, we obtain 
   \( X(f) = \frac{A}{2} [\Pi(f) + A(f)] \left( \delta(f \pm f_c) + \delta(f \mp f>c) \right) \) 
   \( = \frac{A}{2} [\Pi(f - f_c) + A(f - f_c) + \Pi(f + f_c) + A(f + f_c)] \) 
   Since \( \Pi(f-f_c) \neq 0 \) for \( |f - f_c| < \frac{1}{2} \) whereas \( \Pi(f-f_c) = 0 \) for \( |f - f_c| > \frac{1}{2} \) 
   Hence, the bandwidth of the bandpass filter is 2.

22. (C) \( x(t) = m(t)c(t) \) 
   \( = 100[\cos(2\pi 1000 t) + 2\cos(2\pi 20000 t)\cos(2\pi f_c t)] + 200 \cos(2\pi 20000 t)\cos(2\pi f_c t) \) 
   \( = \frac{100}{2} [\cos(2\pi f_c t + 1000 t) + \cos(2\pi f_c t - 1000 t)] + 200 \frac{1}{2} [\cos(2\pi f_c t + 2000 t) + \cos(2\pi f_c t - 2000 t)] \) 
   Thus, the upper sideband (USB) signal is 
   \( x_u(t) = 50 \cos[2\pi (f_c + 1000) t] + 100(2\pi f_c t + 2000) t) \)

23. (B) When USSB is employed the bandwidth of the modulated signal is the same with the bandwidth of the message signal. Hence \( W_{\text{USSB}} = W = 10^4 \) Hz
37. (B) Sidebands are \((6 \times 10^6 \pm 5000)\) and 
\((6 \times 10^6 \pm 9000)\)
Thus \(6 \times 10^6 \times 5, 5995 \times 10^6, 5991 \times 10^6\) or \(5991 \times 10^6, 6005 \times 10^6\) and \(6009 \times 10^6\)

38. (B) \(x(t)\) can be written as
\[x(t) = (40 + 8 \cos 40 \pi t) \cos 400 \pi t\]
modulation index \(\alpha = \frac{8}{40} = 0.2\)
\[P_c = \frac{1}{2}(40)^2 = 800 \text{ W}\]
The components at 180 Hz and 220 Hz are side band
\[P_{sb} = \frac{1}{2}(4)^2 + \frac{1}{2}(4)^2 = 16 \text{ W},\]
\[E_{eff} = \frac{P_{sb}}{P_c + P_{sb}} = \frac{40}{100} \text{ or } \frac{P_{sb}}{100 + P_{sb}} = 0.4\]
\[P_{sb} = 66.67 \text{ W}, \quad P_{sb} = \frac{1}{2}B^2 + \frac{1}{2}B^2 = 66.67 \text{ or } B = 8.161\]

39. (A) Carrier power \(P_c = \frac{A^2}{2} = 100 \text{ W}, A = 14.14\)
\[E_{eff} = \frac{P_{sb}}{P_c + P_{sb}} = \frac{40}{100} \text{ or } \frac{P_{sb}}{100 + P_{sb}} = 0.4\]
\[P_{sb} = 66.67 \text{ W}, \quad P_{sb} = \frac{1}{2}B^2 + \frac{1}{2}B^2 = 66.67 \text{ or } B = 8.161\]

40. (D) The Hilbert transform of \(\cos(2\pi1000t)\) is \(\sin(2\pi1000t)\), whereas the Hilbert transform of \(\sin(2\pi1000t)\) is \(\cos(2\pi1000t)\)
Thus \(\dot{m}(t) = \sin(2\pi1000t) - \cos(2\pi1000t)\)

41. (D) The expression for the LSSB AM signal is.
\[x_1(t) = A_m m(t) \cos(2\pi f_c t) + A_m m(t) \sin(2\pi f_w t)\]
Substituting
\[A_m = 100, \quad m(t) = \cos(2\pi1000t) + 2\sin(2\pi1000t)\]
and \(\dot{m}(t) = \sin(2\pi1000t) - \cos(2\pi1000t)\)
we obtain
\[x_1(t) = 100[\cos(2\pi1000t) + 2\sin(2\pi1000t) \cos(2\pi f_c t)] + 100[\sin(2\pi1000t) - 2\cos(2\pi1000t) \sin(2\pi f_c t)] = 100[\cos(2\pi1000t) \cos(2\pi f_c t) + \sin(2\pi1000t) \sin(2\pi f_c t)] + 200[\cos(2\pi f_c t) \sin(2\pi1000t) - \sin(2\pi1000t) \cos(2\pi f_c t)] = 100 \cos(2\pi f_c t - 1000t) - 200 \sin(2\pi f_c t - 1000t)\]

42. (D) \(y(t) = 4(m(t) + \cos \omega_0 t) + 10(m(t) + \cos \omega_0 t)^2\)
\[= 4m(t) + 4 \cos \omega_0 t + 10m(t) \cos \omega_0 t + 5 + 5 \cos 2\omega_0 t\]
The AM signal is, \(x_1(t) = 4(1 + 5m(t)) \cos \omega_0 t\)
\[m(t) = M \sin(2\pi f_c t)\]
\[x_1(t) = 4(1 + 5M \sin(2\pi f_c t)) \cos \omega_0 t\]
\[5M = 0.8 \text{ or } M = 0.16\]

43. (C) The filter characteristic is shown in fig.S7.4.43
\[f_c < W > 2W \text{ or } f_c > 3W,\]
\[f_c + W < 2f \text{ or } f_c > W\]
Therefore \(f_c > 3W\)

44. (B) Since High-side tuning is used
\[f_{LO} = f_m + f_{IF} = 500 \text{ kHz},\]
\[f_{LOL} = 5 + 0.5 = 5.5 \text{ MHz},\]
\[f_{LOU} = 10 + 0.5 = 10.5 \text{ MHz}\]

45. (B) \(f_{image} = f_L + 2f_{IF} = 2400 = 3310 \text{ kHz}\)

46. (D) The given circuit is a ring modulator. The output is DSB-SC signal. So it will contain \(m(t) \cos(n \omega_0 t)\) where \(n = 1, 2, 3, \ldots\) Therefore there will be only \((1 MHz \pm 400 \text{ Hz})\) frequency component.

47. (A) \(f_{max} = 1650 + 450 = 2100 \text{ kHz}\)
\[f_{min} = 550 + 450 = 1000 \text{ kHz}. \quad \text{or } f = \frac{1}{2\pi\sqrt{LC}}\]
frequency is minimum, capacitance will be maximum
\[R = \frac{C_{max}}{C_{min}} = \frac{f_{max}^2}{f_{min}^2} = (2.1)^2 \text{ or } R = 4.41\]
\[f_c = f_i + 2f_{IF} = 700 + 2(455) = 1600 \text{ kHz}\]

48. (B) Since \(X(f)\) has spectral peak at 1 kHz so at the output of first modulator spectral peak will be at \((10 + 1)\) kHz and \((10 - 1)\) kHz. After passing the HPF frequency component of 11 kHz will remain. The output of 2nd modulator will be \((13 \pm 11)\) kHz. So \(Y(f)\) has spectral peak at 2 kHz and 24 kHz.

49. (C) \(m(t)s(t) = y(t)\)
\[= \frac{2 \sin(2\pi t) \cos(200\pi t)}{t} - \frac{\sin(202\pi t) - \sin(198\pi t)}{t}\]
\[y_1(t) + n(t) = y_2(t) = \frac{\sin(202\pi t) - \sin(198\pi t)}{t} + \frac{\sin(198\pi t)}{t}\]
\[y_2(t)s(t) = y(t)\]
\[= \frac{\sin(202\pi t) - \sin(198\pi t) + \sin(199\pi t) \cos(200\pi t)}{t}\]
After filtering
\[ y(t) = \frac{\sin(2\pi t) + \sin(2\pi t) - \sin(\pi t)}{2t} = \frac{\sin(2\pi t) + 2\sin(0.5t)\cos(15\pi t)}{2t} \]
\[ = \frac{\sin(2\pi t) + \sin(0.5t)\cos(15\pi t)}{2} \]
\[ = \frac{\sin(2\pi t) + \sin(0.5\pi t)}{2} \cos(15\pi t) \]

50. (D) The total signal bandwidth = 5 \times 12 = 60 kHz
There would be 11 guard band between 12 signal. So guard band width = 11 kHz
Total band width = 60 + 11 = 71 kHz

51. (D) \( x_1(t) = g(t)\cos(2000\pi t) \)
\[ = x(t)\sin(2000\pi t)\cos(2000\pi t) = \frac{1}{2} x(t)\sin(4000\pi t) \]
\[ X_1(j\omega) = \frac{1}{4j} X(j(\omega - 4000\pi)) - X(j(\omega + 4000\pi)) \]
This implies that \( X_1(j\omega) \) is zero for \( |\omega| \leq 2000\pi \) because \( \omega < 2\pi f_m = 2\pi 1000 \). When \( x_1(t) \) is passed through a LPF with cutoff frequency 2000\pi, the output will be zero.

52. (A) \( y(t) = g(t)\sin(400\pi t) = x(t)\sin^2(400\pi t) \)
\[ = (\sin(200\pi t) + 2\sin(400\pi t)\left(\frac{1 - \cos(800\pi t)}{2}\right) \]
\[ = \frac{1}{2} [\sin(200\pi t) - \sin(200\pi t)\cos(800\pi t) + 2\sin(400\pi t) \]
\[ - \sin(400\pi t)\cos(800\pi t) \]
\[ - \frac{1}{2} \sin(200\pi t) - \sin(1000\pi t) - (\sin(600\pi t) \]
\[ + \sin(400\pi t) - \frac{1}{4} [\sin(1200\pi t) - \sin(400\pi t)] \]
If this signal is passed through LPF with frequency 400\pi and gain 2, the output will be sin(200\pi t)

53. (A) Message signal BW \( f_m = 5 \) kHz
Noise power density is \( 10^{-18} \) W/Hz
Total noise power is \( 10^{-18} \times 5 \times 10^3 = 5 \times 10^{-15} \) W
Input signal-to-noise ratio
\[ \text{SNR}= \frac{10^{-10}}{5 \times 10^{-15}} = 2 \times 10^4 \] or 43 dB

54. (C) Average side band power is
\[ \frac{A_s^2 e^{-2}}{4} = \frac{10^2 (0.5)^2}{4} = 6.25 \text{ W} \]
CHAPTER 7.6
DIGITAL TRANSMISSION

Statement for Question 1-5
Fig. P7.6.1-5 shows fourier spectra of signal $x(t)$ and $y(t)$. Determine the Nyquist sampling rate for the given function in question.

1. $x(t)$
   - (A) 100 kHz
   - (B) 200 kHz
   - (C) 300 kHz
   - (D) 50 kHz

2. $y(t)$
   - (A) 50 kHz
   - (B) 75 kHz
   - (C) 150 kHz
   - (D) 300 kHz

3. $x^2(t)$
   - (A) 100 kHz
   - (B) 150 kHz
   - (C) 250 kHz
   - (D) 400 kHz

4. $y^3(t)$
   - (A) 100 kHz
   - (B) 300 kHz
   - (C) 900 kHz
   - (D) 120 kHz

5. $x(t) y(t)$
   - (A) 250 kHz
   - (B) 500 kHz
   - (C) 50 kHz
   - (D) 100 kHz

Statement for Question 6-7
A signal $x(t)$ is multiplied by rectangular pulse train $c(t)$ shown in fig.P7.6.6-7.

6. $x(t)$ would be recovered from the product $x(t) c(t)$ by using an ideal LPF if $X(j\omega) = 0$ for
   - (A) $\omega > 2000\pi$
   - (B) $\omega > 1000\pi$
   - (C) $\omega < 1000\pi$
   - (D) $\omega < 2000\pi$

7. If $X(j\omega)$ satisfies the constraints required, then the pass band gain $A$ of the ideal lowpass filter needed to recover $x(t)$ from $e(t) x(t)$ is
   - (A) 1
   - (B) 2
   - (C) 4
   - (D) 8

8. Consider a set of 10 signals $x_i(t), i = 1, 2, 3, \ldots, 10$. Each signal is band limited to 1 kHz. All 10 signals are to be time-division multiplexed after each is multiplied by a carrier $e(t)$ shown in Figure. If the period $T$ of $e(t)$ is chosen the have the maximum allowable value, the largest value of $\Delta$ would be

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9. A compact disc recording system samples a signal with a 16-bit analog-to-digital convertor at 44.1 kbits/s. The CD can record an hour's worth of music. The approximate capacity of CD is
(A) 705.6 M Bytes  (B) 317.5 M Bytes
(C) 2.54 M Bytes    (D) 5.43 M Bytes

10. An analog signal is sampled at 36 kHz and quantized into 256 levels. The time duration of a bit of the binary coded signal is
(A) 5.78 μs  (B) 3.47 μs
(C) 6.43 ms    (D) 7.86 ms

11. An analog signal is quantized and transmitted using a PCM system. The tolerable error in sample amplitude is 0.5% of the peak-to-peak full scale value. The minimum binary digits required to encode a sample is
(A) 5  (B) 6
(C) 7    (D) 8


Ten telemetry signals, each of bandwidth 2 kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 1% extra bits.

12. The minimum possible data rate must be
(A) 272.64 kbits/sec  (B) 436.32 kbits/sec
(C) 936.32 kbits/sec   (D) None of the above

13. The minimum transmission bandwidth is
(A) 218.16 kHz  (B) 468.32 kHz
(C) 136.32 kHz    (D) None of the above

14. A television signal is sampled at a rate of 20% above the Nyquist rate. The signal has a bandwidth of 6 MHz. The samples are quantized into 1024 levels. The minimum bandwidth required to transmit this signal would be
(A) 72 M bits/sec  (B) 144 M bits/sec
(C) 72 k bits/sec   (D) 144 k bits/sec

15. A CD record audio signals digitally using PCM. The audio signal bandwidth is 15 kHz. The Nyquist samples are quantized into 32678 levels and then binary coded. The minimum number of binary digits required to encode the audio signal
(A) 450 k bits/sec  (B) 900 k bits/sec
(C) 980 340 k bits/sec (D) 490 170, k bits/sec

16. The American Standard Code for Information Interchange has 128 characters, which are binary coded. If a certain computer generates 1,000,000 character per second, the minimum bandwidth required to transmit this signal will be
(A) 1.4 M bits/sec  (B) 14 M bits/sec
(C) 7 M bits/sec    (D) 0.7 M bits/sec

17. A binary channel with capacity 36 k bits/sec is available for PCM voice transmission. If signal is band limited to 3.2 kHz, then the appropriate values of quantizing level L and the sampling frequency will be
(A) 32, 3.6 kHz  (B) 64, 7.2 kHz
(C) 64, 3.6 kHz    (D) 32, 7.2 kHz

18. Fig.P7.4.18 shows a PCM signal in which amplitude level of +1 volt and -1 volt are used to represent binary symbol 1 and 0 respectively. The code word used consists of three bits. The sampled version of analog signal from which this PCM signal is derived is

19. A PCM system uses a uniform quantizer followed by a 8-bit encoder. The bit rate of the system is equal to \(10^8\) bits/s. The maximum message bandwidth for which the system operates satisfactorily is
(A) 25 MHz  (B) 6.25 MHz
(C) 12.5 MHz    (D) 50 MHz

20. Twenty-four voice signals are sampled uniformly at a rate of 8 kHz and then time-division multiplexed. The sampling process uses flat-top samples with 1 ms duration. The multiplexing operating includes provision...
for synchronization by adding an extra pulse of 1 \( \mu s \) duration. The spacing between successive pulses of the multiplexed signal is
(A) 4 \( \mu s \)       (B) 6 \( \mu s \)
(C) 7.2 \( \mu s \)       (D) 8.4 \( \mu s \)

21. A linear delta modulator is designed to operate on speech signals limited to 3.4 kHz. The sampling rate is 10 times the Nyquist rate of the speech signal. The step size is 100 mV. The modulator is tested with a test signal required to avoid slope overload is
(A) 2.04 V       (B) 1.08 V
(C) 4.08 V       (D) 2.16 V

Statement for Question 22-23:

Consider a linear DM system designed to accommodate analog message signals limited to bandwidth of 3.5 kHz. A sinusoidal test signal of amplitude \( A_{\text{max}} = 1 \) V and frequency \( f_m = 800 \) Hz is applied to the system. The sampling rate of the system is 64 kHz.

22. The minimum value of the step size to avoid overload is
(A) 240 mV       (B) 120 mV
(C) 670 mV       (D) 78.5 mV

23. The granular-noise power would be
(A) \( 1.68 \times 10^{-3} \) W  (B) \( 2.86 \times 10^{-4} \) W
(C) \( 2.48 \times 10^{-3} \) W  (D) \( 1.12 \times 10^{-4} \) W

24. The SNR will be
(A) 298       (B) 1.75 \times 10^3
(C) 4.46 \times 10^3       (D) 201

25. The output signal-to-quantization-noise ratio of a 10-bit PCM was found to be 30 dB. The desired SNR is 42 dB. It can be increased by increasing the number of quantization levels. In this way the fractional increase in the transmission bandwidth would be (assume \( \log_{10} 10 = 0.3 \))
(A) 20%       (B) 30%
(C) 40%       (D) 50%

Statement for Question 26-27:

A signal has a bandwidth of 1 MHz. It is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels using a \( \mu \)-law quantizer with \( \mu = 225 \).

26. The signal-to-quantization-noise ratio is
(A) 34.91 dB  (B) 38.06 dB
(C) 42.05 dB  (D) 48.76 dB

27. It was found that a sampling rate 20% above the rate would be adequate. So the maximum SNR, that can be realized without increasing the transmission bandwidth, would be
(A) 60.4 dB  (B) 70.3 dB
(C) 50.1 dB  (D) None of the above

28. For a PCM signal the compression parameter \( \mu = 100 \) and the minimum signal to quantization-noise ratio is 50 dB. The number of bits per sample would be
(A) 8       (B) 10
(C) 12       (D) 14

29. A sinusoidal message signal \( m(t) \) is transmitted by binary PCM without compression. If the signal-to-quantization-noise ratio is required to be at least 48 dB, the minimum number of bits per sample will be
(A) 8       (B) 10
(C) 12       (D) 14

30. A speech signal has a total duration of 20 sec. It is sampled at the rate of 8 kHz and then PCM encoded. The signal-to-quantization noise ratio is required to be 40 dB. The minimum storage capacity needed to accommodate this signal is
(A) 1.12 KBytes       (B) 140 KBytes
(C) 168 KBytes       (D) None of the above

31. The input to a linear delta modulator having a step-size \( \Delta = 0.628 \) is a sine wave with frequency \( f_m \) and peak amplitude \( E_m \). If the sampling frequency \( f_s = 40 \) kHz, the combination of the sinc-wave frequency and the peak amplitude, where slope overload will take place is
\( E_m \) \( f_m \)
(A) 0.3 V 8 kHz
(B) 1.5 V 4 kHz
(C) 1.5 V 2 kHz
(D) 3.0 V 1 kHz
32. A sinusoidal signal with peak-to-peak amplitude of 1.536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is
(A) 0.768 V  (B) 48 \times 10^{-6} V^2
(C) 12 \times 10^{-6} V^2  (D) 3.072 V

33. A signal is sampled at 8 kHz and is quantized using 8 bit uniform quantizer. Assuming $\text{SNR}_q$ for a sinusoidal signal, the correct statement for PCM signal with a bit rate of $R$ is
(A) $R = 32$ kbps, $\text{SNR}_q = 25.8$ dB
(B) $R = 64$ kbps, $\text{SNR}_q = 49.8$ dB
(C) $R = 64$ kbps, $\text{SNR}_q = 55.8$ dB
(D) $R = 32$ kbps, $\text{SNR}_q = 49.8$ dB

34. A 1.0 kHz signal is flat-top sampled at the rate of 180 samples/sec and the samples are applied to an ideal rectangular LPF with cutoff frequency of 1100 Hz, then the output of the filter contains
(A) only 800 Hz component
(B) 800 and 900 Hz components
(C) 800 Hz and 1000 Hz components
(D) 800 Hz, 900 and 1000 Hz components

35. The Nyquist sampling interval, for the signal $\text{sinc}(700t) + \text{sinc}(500t)$ is
(A) $\frac{1}{350}$ sec  (B) $\frac{\pi}{350}$ sec
(C) $\frac{1}{700}$ sec  (D) $\frac{\pi}{175}$ sec

36. A signal $x(t) = 100 \cos(24\pi \times 10^3 t)$ is ideally sampled with a sampling period of 50 $\mu$sec and then passed through an ideal lowpass filter with cutoff frequency of 15 kHz. Which of the following frequencies is/are present at the filter output
(A) 12 KHz only
(B) 8 KHz only
(C) 12 KHz and 9 KHz
(D) 12 KHz and 8 KHz

37. In a PCM system, if the code word length is increased from 6 to 8 bits, the signal to quantization noise ratio improves by the factor.
(A) 8/6  (B) 12
(C) 16  (D) 8

38. Four signals $g_1(t)$, $g_2(t)$, $g_3(t)$ and $g_4(t)$ are to be multiplexed and transmitted. $g_1(t)$ and $g_4(t)$ have a bandwidth of 4 kHz, and the remaining two signals have bandwidth of 8 kHz. Each sample requires 8 bit for encoding. What is the minimum transmission bit rate of the system.
(A) 512 kbps  (B) 16 kbps
(C) 192 kbps  (D) 384 kbps

39. Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is
(A) 115.2 kbps  (B) 28.8 kbps
(C) 57.6 kbps  (D) 38.4 kbps

40. The minimum sampling frequency (in samples/sec) required to reconstruct the following signal form its samples without distortion would be

\[ x(t) = 5 \left( \frac{\sin 2\pi 1000t}{\pi t} \right)^3 + 7 \left( \frac{\sin 2\pi 1000t}{\pi t} \right)^2 \]

(A) $2 \times 10^3$  (B) $4 \times 10^3$
(C) $6 \times 10^3$  (D) $8 \times 10^3$

41. The minimum step-size required for a Delta-Modulator operating at 32 K samples/sec to track the signal (here $u(t)$ is the unit function) $x(t) = 125t(u(t) - u(t - 1)) + (250 - 125t)(u(t - 1) - u(t - 2))$ so that slope overload is avoided, would be
(A) $2^{-10}$  (B) $2^{-8}$
(C) $2^{-6}$  (D) $2^{-4}$

42. Four signals each band limited to 5 kHz are sampled at twice the Nyquist rate. The resulting PAM samples are transmitted over a single channel after time division multiplexing. The theoretical minimum transmissions bandwidth of the channel should be equal to.
(A) 5 kHz  (B) 20 kHz
(C) 40 kHz  (D) 80 kHz

43. Four independent messages have bandwidths of 100 Hz, 100 Hz, 200 Hz and 400 Hz respectively. Each is
sampled at the Nyquist rate, time division multiplexed and transmitted. The transmitted sample rate, in Hz, is given by
(A) 200 (B) 400  
(C) 800 (D) 1600

44. The Nyquist sampling rate for the signal \( g(t) = 10 \cos(50\pi t) \cos(150\pi t) \). Where ‘t’ is in seconds, is
(A) 150 samples per second  
(B) 200 samples per second  
(C) 300 samples per second  
(D) 350 samples per second

45. A TDM link has 20 signal channels and each channel is sampled 8000 times/sec. Each sample is represented by seven binary bits and contains an additional bit for synchronization. The total bit rate for the TDM link is
(A) 1180 K bits/sec  
(B) 1280 K bits/sec  
(C) 1180 M bits/sec  
(D) 1280 M bits/sec

46. In a CD player, the sampling rate is 44.1 kHz and the samples are quantized using a 16-bit/sample quantizer. The resulting number of bits for a piece of music with a duration of 50 minutes is
(A) \( 1.39 \times 10^9 \)  
(B) \( 4.23 \times 10^9 \)  
(C) \( 8.46 \times 10^9 \)  
(D) \( 12.23 \times 10^9 \)

47. Four voice signals, each limited to 4 kHz and sampled at Nyquist rate are converted into binary PCM signal using 256 quantization levels. The bit transmission rate for the time-division multiplexed signal will be
(A) 8 kbps  
(B) 64 kbps  
(C) 256 kbps  
(D) 512 kbps

48. Analog data having highest harmonic at 30 kHz generated by a sensor has been digitized using 6 level PCM. What will be the rate of digital signal generated?
(A) 120 kbps  
(B) 200 kbps  
(C) 240 kbps  
(D) 180 kbps

49. In a PCM system, the number of quantization levels is 16 and the maximum signal frequency is 4 kHz.; the bit transmission rate is
(A) 32 bits/s  
(B) 16 bits/s  
(C) 32 kbits/s  
(D) 64 dbits/s

50. A speech signal occupying the bandwidth of 300 Hz to 3 kHz is converted into PCM format for use in digital communication. If the sampling frequency is 8 kHz and each sample is quantized into 256 levels, then the output bit rate will be
(A) 3 kb/s  
(B) 8 kb/s  
(C) 64 kb/s  
(D) 256 kb/s

51. If the number of bits in a PCM system is increased from \( n \) to \( n + 1 \), the signal-to-quantization noise ratio will increase by a factor.
(A) \( \frac{n+1}{n} \)  
(B) \( \frac{(n+1)^2}{n^2} \)  
(C) 2  
(D) 4

52. In PCM system, if the quantization levels are increased from 2 to 8, the relative bandwidth requirement will.
(A) remain same  
(B) be doubled  
(C) be tripled  
(D) become four times

53. Assuming that the signal is quantized to satisfy the condition of previous question and assuming the approximate bandwidth of the signal is \( W \). The minimum required bandwidth for transmission of a binary PCM signal based on this quantization scheme will be.
(A) 5 \( W \)  
(B) 10 \( W \)  
(C) 20 \( W \)  
(D) None of the above
16. (C) 128 = 2^7. We need 7 bits/character. For 1,000,000 character we need 7 Mbits/second. Thus minimum bandwidth = 7 Mbits/sec.

17. (D) \( f_s > 2f_m = 6400 \) Hz, \( n f_s \leq 63000 \)

\[
\frac{n \cdot 6400}{6400} = 5.63, \quad n = 5, \quad L = 2^n = 32, \quad f_s = \frac{36000}{5} = 7.2 \text{ kHz}.
\]

18. (D) The transmitted code word are

```
  0 0 1 0 1 1 0 0 1 1 1 1 0 0 1 0 1
```

In 1st word 001(1)
In 2nd word 010(2)
In 3rd word 011(3)
In 4th word 100(4)
In 5th word 101(5)

19. (B) Message bandwidth=W, Nyquist rate = 2W

Bandwidth = 2W × 8 = 16W bit/s

16W = 10^8, or \( W = \frac{10^8}{16} = 6.25 \text{ MHz} \)

20. (A) Sampling interval \( T_s = \frac{1}{8k} = 125 \mu s \). There are 24 channels and 1 sync pulse, so the time allotted to each channel is \( T_s = \frac{1}{32} = 5 \mu s \). The pulse duration is 1 \mu s. So the time between pulse is 4 \mu s.

21. (B) \( A_{\text{max}} = \frac{\delta f_s}{\omega_m} = \frac{0.1 \times 68}{2 \pi \times 10^5} = 108 \text{ V} \)

22. (D) \( A_{\text{max}} = \frac{\delta f_s}{\omega_m} \) or \( \delta = \frac{A_{\text{max}} \omega_m}{f_s} = \frac{1 \times 2 \pi \times 800}{50 \times 10^5} = 78.5 \text{ mV} \)

23. (D) \( N_s = \frac{\delta B}{3f_s} = \frac{(0.0785) \times 3500}{3 \times 64000} = 1.122 \times 10^{-4} \text{ W} \)

24. (C) \( S_{\text{rms}} = \frac{0.5}{112 \times 10^{-4}} = 4.46 \times 10^3 \)

25. (A) \( \frac{S_{\text{rms}}}{N_s} \propto L^2, \quad L = 2^n \left( \frac{S_{\text{rms}}}{N_s} \right)_{\text{dB}} = 10 \log(2^{2n}) \)

= \log C + 20 \log 2 = \alpha + 6n \text{ dB. This equation shows that increasing n by one bits increase the by 6 dB. Hence an increase in the SNR by 12 dB can be accomplished by increasing 9is form 10 to 12, the transmission bandwidth would be increased by 20%}

26. (B) \( \frac{S_{\text{rms}}}{N_s} = \frac{3L^2}{(\ln(\mu + 1))^2} = 6394 = 38.06 \text{ dB} \)

27. (C) Nyquist Rate = 2 MHz

5% higher rate = 3 MHz, \( L = 256 = 2^8 \)

Thus transmission bandwidth is 3 MHz × 8 = 24 Mbits/s.

New sampling rate is at 20% above the Nyquist rate.

Sampling rate = 12 × 2 = 24 MHz.

bits per second = \( \frac{24 \times 10^6}{2.4 \times 10^6} = 10 \text{ bits} \)

Level = \( 2^{10} = 1024 \), \( \frac{S_{\text{rms}}}{N_s} = \frac{3(1024)^2}{(\ln(256))^2} = 102300 \approx 50.1 \text{ dB} \)

28. (B) \( \frac{S_{\text{rms}}}{N_s} = \frac{3L^2}{(\ln(\mu + 1))^2} \geq 50 \text{ dB, } \mu = 100 \)

\[
\frac{3L^2}{(\ln 101)^2} = 100000 \text{ or } L = 842.6
\]

Because \( L \) is power of 2, we select \( L = 1024 = 2^{10} \).

Thus 10 bits are required.

29. (A) \( S_{\text{rms}} = m^2(t), \quad N_s = \frac{3m^2}{3L^2}, \quad \frac{S_{\text{rms}}}{N_s} = \frac{3L^2m^2(t)}{m^2} \)

since signal is sinusoidal \( \frac{m^2(t)}{m^2} = \frac{1}{2} \)

\[
\frac{3L^2}{2} = 48 \text{ dB} = 63096, \quad L = 205.09
\]

Since \( L \) is power of 2, so we select \( L = 256 \)

Hence \( 256 = 2^8 \), So 8 bits per sample is required.

30. (B) \( (SNR)_q = 176 + 6.02(n) = 40 \text{ dB}, \quad n = 6.35 \)

We take the \( n = 7 \).

Capacity = \( 20 \times 8k \times 7 = 112 \text{ Mbits} = 140 \text{ Kbytes} \)

31. (B) For slope overload to take place \( E_{\text{rms}} \geq \frac{\delta f_s}{2\pi f_m} \)

This is satisfied with \( E_{\text{rms}} = 15 \text{ V} \) and \( f_m = 4 \text{ kHz} \).

32. (C) Step size \( \delta = \frac{2m}{L} = \frac{1536}{128} = 0.012 \text{ V} \)

quantization noise power

\[
\frac{\delta^2}{12} = \frac{(0.012)^2}{12} = 12 \times 10^{-6} \text{ V}^2
\]

33. (B) Bit Rate = \( 8k \times 8 = 64 \text{ kbps} \)

\( (SNR)_q = 176 + 6.02n \text{ dB} = 176 + 6.02 \times 8 = 49.8 \text{ dB} \)

34. (B) \( f_s = 1800 \text{ samples/sec, } \quad f_m = \frac{1800}{2} = 900 \text{ Hz} \)
Since the sampling rate is 1800 samples/sec the highest frequency that can be recovered is 900 Hz.

35. (C) \( x(t) = \text{sinc} 700t + \text{sinc} 500t \)
\[ = \frac{1}{\pi t} [\sin 700 \pi t + \sin 500 \pi t] \]
\( x(t) \) is band limited with \( f_m = 350 \) Hz. Thus Nyquist rate is \( 2f_m = 700 \) Hz, Sampling interval = \( \frac{1}{700} \) sec

36. (D) \( f_s = \frac{1}{T} = \frac{1}{50 \times 10^3} = 20 \text{ kHz}, \ f_s = 12 \text{ kHz} \)
The frequency passed through LPF are \( f_s, f_s - f_m \) or 12 kHz, 8 kHz

37. (C) \( P = \frac{(SNR)_l}{(SNR)_h} = \frac{2^{2n_1}}{2^{2n_2}}, \) Here \( n = \) code word length,
\( n_1 = 6, \ n_2 = 8, \) Thus rate = \( \frac{2^{16}}{2^8} = 16 \)

38. (D) Signal \( g_1(t), g_2(t), g_3(t) \) and \( g_4(t) \) will have 8 k, 8 k, 16 k and 16 k sample/sec at Nyquist rate. Thus 48000 sample/sec bit rate 48000 x 8 = 384 kbps

39. (C) Analog signals, having bandwidth 1200 Hz, 600 Hz and 600 Hz have 2400, 1200 samples/sec at Nyquist rate. Hence 48000 sample/sec bit rate = 48000 sample/sec \times 12 = 57.6 \text{ kbps}

40. (C) \( x(t) = 5 \left( \frac{\sin 2\pi 1000t}{\pi t} \right)^3 + 7 \left( \frac{\sin 2\pi 1000t}{\pi t} \right)^2 \)
Maximum frequency component = \( 3 \times 1000 = 3 \text{ kHz} \)
Sampling rate = \( 2f_m = 6 \text{ kHz} \)

41. (B) Here \( f_s = 32 \text{ k sample/sec} \)
\[ E_m = 125, \quad f_m = \frac{1}{T} = \frac{1}{2} \]
For slope-overload to be averted \( E_m \geq \frac{f_s}{f_m} \)
\[ \Delta \leq \frac{E_m f_m}{f_s} \text{ or } \Delta \leq \frac{125(\frac{1}{2})}{32 \times 10^6} \text{ or } \Delta \leq \frac{(128)(\frac{1}{2})}{32 \times 1024} \text{ or } \Delta \leq 2^{-8} \]

42. (D) \( f_m = 5 \text{ kHz}, \) Nyquist Rate = \( 2 \times 5 = 10 \text{ kHz} \)
Since signal are sampled at twice the Nyquist rate so sampling rate = \( 2 \times 10 = 20 \text{ kHz} \).
Total transmission bandwidth = \( 4 \times 20 = 80 \text{ kHz} \)

43. (D) Signal will be sampled 200, 200, 400 and 800 sample/sec thus 1600 sample per second,

44. (D) \( g(t) = 10 \cos 50\pi t \left( \frac{1 + \cos 300\pi t}{2} \right) \)
\[ = 5 \cos 50\pi t + 5 \cos 50\pi t \cos 300\pi t \]
The maximum frequency component will be 150 + 25 = 175 Hz.
Thus \( f_s = 2 \times 175 = 350 \) sample per second.

45. (B) Total sample = \( 8000 \times 20 = 160 \text{ k sample/sec} \)
Bit for each sample = \( 7 + 1 = 8 \)
Bit Rate = 160k x 8 = 1280 x 10^3 \text{ bits/sec} \]

46. (B) The sampling rate is \( f_s = 44100 \) meaning that we take 44100 samples per second. Each sample is quantized using 16 bits so the total number of bits per second is 44100 x 16. For a music piece of duration 50 min = 3000 sec the resulting number of bits per channel (left and right) is 44100 x 16 x 3000 = \( 2.1168 \times 10^9 \) and the overall number of bits is \( 2.1168 \times 10^9 \times 2 = 4.2336 \times 10^9 \)

47. (C) Nyquist Rate = \( 2 \times 4 \times k = 8 \text{ kHz} \)
Total sample = \( 4 \times 8 = 32 \text{ k sample/sec} \)
256 = \( 2^8 \), so that 8 bits are required
Bit Rate = 32k x 8 = 256 kbps

48. (D) Nyquist Rate = \( 2 \times 30k = 60 \text{ kHz} \)
\( 2^n \geq 6 \) Thus \( n = 3, \) Bit Rate = \( 60 \times 3 = 18 \text{ kHz} \)

49. (C) Nyquist rate= \( 2 \times 4 = 8 \text{ kHz} \)
\( 2^n = 16 \) or \( n = 4, \) Bit Rate = \( 4 \times 8 = 32 \text{ kbits/sec} \)

50. (C) \( f_s = 8 \text{ kHz}, \) 2\( ^n \geq 256 \Rightarrow n = 8 \)
Bit Rate = \( 8 \times 8k = 64 \text{ kb/x} \)

51. (D) \( \frac{S}{N_o} \propto 2^{2n}, \) If PCM is increased form \( n \) to \( n + 1, \) the ratio will increase by a factor 4. Which is independent of \( n. \)

52. (C) If \( L = 2, \) then \( 2 = 2^n \) or \( n = 1 \) ND If \( L = 8, \) then \( 8 = 2^n \) or \( n = 3. \) So relative bandwidth will be tripled.

53. (B) The minimum bandwidth requirement for transmission of a binary PCM signal is \( \text{BW}= vW. \) Since \( v = 10, \) we have \( \text{BW} = 10 \text{ W} \)

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Chapter 7.8

Spread Spectrum

Statement for Question 1-3:

A pseudo-noise (PN) sequence is generated using a feedback shift register of length \( m = 4 \). The chip rate is 10^7 chips per second.

1. The PN sequence length is
   (A) 10  (B) 12  (C) 15  (D) 18

2. The chip duration is
   (A) 1 ms  (B) 0.1 ms  (C) 0.1 ms  (D) 1 ms

3. The period of PN sequence is
   (A) 15 \( \mu \)s  (B) 15 \( \mu \)s  (C) 6.67 ns  (D) 0.67 ns

Statement for Question 4-5:

A direct sequence spread binary phase-shift-keying system uses a feedback shift register of Length 19 for the generation of PN sequence. The system is required to have an average probability of symbol error due to externally generated interfering signals that does not exceed \( 10^{-5} \).

4. The processing gain of system is
   (A) 37 dB  (B) 43 dB  (C) 57 dB  (D) 93 dB

5. The Antijam margin is
   (A) 47.5 dB  (B) 93.8 dB  (C) 86.9 dB  (D) 12.6 dB

6. A slow FH/MFSK system has the following parameters.
   Number of bits per MFSK symbol = 4
   Number of MFSK symbol per hop = 5
   The processing gain of the system is
   (A) 13.4 dB  (B) 37.8 dB  (C) 6 dB  (D) 26 dB

7. A fast FH/MFSK system has the following parameters.
   Number of bits per MFSK symbol = 4
   Number of pops per MFSK symbol = 4
   The processing gain of the system is
   (A) 0 dB  (B) 7 dB  (C) 9 dB  (D) 12 dB

Statement for Question 8-9:

A rate 1/2 convolution code with \( \text{dfrec} = 10 \) is used to encode a data sequence occurring at a rate of 1 kbps. The modulation is binary PSK. The DS spread spectrum sequence has a chip rate of 10 MHz.

8. The coding gain is
   (A) 7 dB  (B) 12 dB  (C) 14 dB  (D) 24 dB
9. The processing gain is
   (A) 14 dB        (B) 37 dB
   (C) 58 dB        (D) 104 dB

10. A total of 30 equal-power users are to share a common communication channel by CDM. Each user transmit information at a rate of 10 kbps via DS spread spectrum and binary PSK. The minimum chip rate to obtain a bit error probability of $10^{-5}$ is
   (A) $1.3 \times 10^6$ chips/sec   (B) $2.9 \times 10^5$ chips/sec
   (C) $19 \times 10^5$ chips/sec   (D) $1.3 \times 10^5$ chips/sec

11. A CDMA system is designed based on DS spread spectrum with a processing gain of 1000 and BPSK modulation scheme. If user has equal power and the desired level of performance of an error probability of $10^{-6}$, the number of user will be
   (A) 89        (B) 117
   (C) 147        (D) 216

12. In previous question if processing gain is changed to 500, then number of users will be
   (A) 27 users        (B) 38 users
   (C) 42 users        (D) 45 users

Statement for Question 13-15:

A DS spread spectrum system transmit at a rate of 1 kbps in the presence of a tone jammer. The jammer power is 20 dB greater than the desired signal, and the required $\varepsilon_b/J_0$ to achieve satisfactory performance is 10 dB.

13. The spreading bandwidth required to meet the specifications is
   (A) $10^7$ Hz        (B) $10^5$ Hz
   (C) $10^6$ Hz        (D) $10^3$ Hz

14. If the jammer is a pulse jammer, then pulse duty cycle that results in worst case jamming is
   (A) 0.14        (B) 0.05
   (C) 0.07        (D) 0.10

15. The correspond probability of error is
   (A) $4.9 \times 10^{-3}$        (B) $6.3 \times 10^{-3}$
   (C) $9.4 \times 10^{-4}$        (D) $8.3 \times 10^{-3}$

Statement for question 16-18:

A CDMA system consist of 15 equal power user that transmit information at a rate of 10 kbps, each using a DS spread spectrum signal operating at chip rate of 1 MHz. The modulation scheme is BPSK.

16. The Processing gain is
   (A) 0.01        (B) 100
   (C) 0.1        (D) 10

17. The value of $\varepsilon_b/J_0$ is
   (A) 8.54 dB        (B) 7.14 dB
   (C) 17.08 dB        (D) 14.28 dB

18. How much should the processing gain be increased to allow for doubling the number of users with affecting the autopad SNR
   (A) 1.46 MHz        (B) 2.07 MHz
   (C) 4.93 MHz        (D) 2.92 MHz

19. A DS/BPSK spread spectrum signal has a processing gain of 500. If the desired error probability is $10^{-5}$ and $(\varepsilon_b/J_0)$ required to obtain an error probability of $10^{-5}$ for binary PSK is 9.5 dB, then the Jamming margin against a containers tone jammer is
   (A) 23.6 dB        (B) 17.5 dB
   (C) 117.4 dB        (D) 109.0 dB

Statement for Question 20-21:

An $m=10$ ML shift register is used to generate the pre hdarandlm sequence in a DS spread spectrum system. The chip duration is $T_c = 1 \mu$s and the bit duration is $T_b = NT_c$, where N is the length (period of the m sequence).

20. The processing gain of the system is
   (A) 10 dB        (B) 20 dB
   (C) 30 dB        (D) 40 dB

21. If the required $\varepsilon_b/J_0$ is 10 and the jammer is a tone jammer with an average power $J_{av}$, then jamming margin is.
   (A) 10 dB        (B) 20 dB
   (C) 30 dB        (D) 40 dB

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Statement for Question 22-23 :

An FH binary orthogonal FSK system employs an \( m = 15 \) stage linear feedback shift register that generates an ML sequence. Each state of the shift register selects one of \( L \) non overlapping frequency bands in the hopping pattern. The bit rate is 100 bits/s. The demodulator employ non coherent detection.

22. If the hop rate is one per bit, the hopping bandwidth for this channel is
(A) 6.5534 MHz  
(B) 9.4369 MHz  
(C) 2.6943 MHz  
(D) None of the above

23. Suppose the hop rate is increased to 2 hops/bit and the receiver uses square law combining the signal over two hops. The hopping bandwidth for this channel is
(A) 3.2767 MHz  
(B) 13.1068 MHz  
(C) 26.2136 MHz  
(D) 1.6384 MHz

Statement for Question 24-25 :

In a fast FH spread spectrum system, the information is transmitted via FSK with non coherent detection. Suppose there are \( N = 3 \) hops/bit with hard decision decoding of the signal in each hop. The channel is AWGN with power spectral density \( \frac{1}{20} \) and an SNR 20-13 dB (total SNR over the three hops)

24. The probability of error for this system is
(A) 0.013  
(B) 0.0013  
(C) 0.049  
(D) 0.0049

25. In case of one hop per bit the probability of error is
(A) \( 1.96 \times 10^{-5} \)  
(B) \( 1.96 \times 10^{-7} \)  
(C) \( 2.27 \times 10^{-5} \)  
(D) \( 2.27 \times 10^{-7} \)

Statement for Question 26-29 :

A slow FH binary FSK system with non coherent detection operates at \( \frac{v_o}{J_o} = 10 \), with hopping bandwidth of 2 GHz, and a bit rate of 10 kbps.

26. The processing gain of this system is
(A) 23 dB  
(B) 43 dB  
(C) 43 dB  
(D) 53 dB

27. If the jammer operates as a partial band jammer, the bandwidth occupancy for worst case jamming is
SOLUTION

1. (C) The PN sequence length is
   \[ N = 2^m - 1 = 2^4 - 1 = 15 \]

2. (B) The chip duration is
   \[ T_c = \frac{1}{10^7} s = 0.1 \text{ ms} \]

3. (A) The period of the PN sequence is
   \[ T = NT_c = 15 \times 0.1 = 1.5 \mu s \]

4. (C) \( m = 19 \)
   \[ n = 2^m - 1 = 2^{19} - 1 = 2^{19} \]
   The processing gain is \( 10 \log_{10} N = 10 \log_{10} 2^{19} = 190 \times 0.3 \) or 57 dB

5. (A) Antijam margin = (Processing gain) - 10\log_{10}\left(\frac{E_b}{N_0}\right)
   
   The probability of error is
   \[ P_e = \frac{1}{2} \text{erfc} \left( \frac{E_b}{\sqrt{N_0}} \right) \]

   With \( P_e = 10^{-5} \), we have \( E_b / N_0 = 9 \).

   Hence, Antijam margin = \( 57 - 10 \log_{10} 9 = 57 - 9.5 \) or =47.5 dB

6. (D) The precessing gain (PG) is
   \[ \text{PG} = \frac{\text{FH Bandwidth}}{\text{Symbol Rate}} = \frac{W_c}{R_s} = 5 \times 4 = 20 \]

   Hence, expressed in decibels, PG = 10\log_{10} 20 = 26 dB

7. (D) The processing gain is
   \[ \text{PG} = 4 \times 4 = 16 \]

   Hence, in decibels,
   \[ \text{PG} = 10 \log_{10} 16 = 12 \text{ dB} \]

8. (A) The coding gain is \( R_{cd\min} = \frac{1}{2} \times 10 = 5 \) or 7 dB

9. (B) The processing gain
   \[ \frac{W}{R} = \frac{10^7}{2 	imes 10^3} = 5 \times 10^3 \text{ or } 37 \text{ dB} \]

10. (C) We assume that the interference is characterized as a zero-mean AWGN process with power spectral density \( J_o \). To achieve an error probability of \( 10^{-5} \), the required \( \varepsilon_b / J_o = 10 \) we have

   \[ \frac{W}{R} = 10^6 \text{ or } \frac{W}{J_o} = \frac{10^6}{14} = 7.14 \text{ or } 8.54 \text{ dB} \]
18. (B) With \( N_a = 30 \) and \( \varepsilon / J_o = 7.14 \), the processing gain should be increased to

\[ W/R = (7.14)(29) = 207 \]

\[ W = 207 \times 104 = 207 \text{ MHz} \]

Hence the bandwidth must be increased to 2.07 MHz

19. (B) The processing gain is given as

\[ W/R = 500 \text{ or } 27 \text{ dB} \]

The \( (\varepsilon / J_o) \) required to obtain an error probability of \( 10^{-5} \) for binary PSK is 9.5 dB. Hence, the jamming margin is

\[ \left( \frac{J_{aw}}{P_{aw}} \right)_{dB} = \left( \frac{W}{R} \right)_{dB} - \left( \frac{\varepsilon}{J_o} \right)_{dB} = 27.95 \text{ or } 17.5 \text{ dB} \]

20. (C) The period of the maximum length shift register sequence is

\[ N = 2^{10} - 1 = 1023 \]

Since \( T_b = NT_c \) then the processing gain is

\[ N \frac{T_b}{T_c} = 1023 \text{ or } 30 \text{ dB} \]

21. (B) A Jamming margin is

\[ \left( \frac{J_{aw}}{P_{aw}} \right)_{dB} = \left( \frac{W}{R} \right)_{dB} - \left( \frac{\varepsilon}{J_o} \right)_{dB} = 30 - 10 = 20 \text{ dB} \]

where \( J_{aw} = J_w \approx J_o/T_c = J_o \times 10^6 \)

22. (A) The length of the shift-register sequence is

\[ L = 2^{n} - 1 = 1215 = 32767 \text{ bits} \]

For binary FSK modulation, the minimum frequency separation is \( 2/T \), where \( 1/T \) is the symbol (bit) rate. The hop rate is 100 hops/sec. Since the shift register has \( L = 32767 \) states and each state utilizes a bandwidth of \( 2/T = 200 \text{ Hz} \), then the total bandwidth for the FH signal is 6.5534 MHz.

23. If the hopping rate is 2 hops/bit and the bit rate is 100 bits/sec, then, the hop rate is 200 hops/sec. The minimum frequency separation for orthogonality \( 2/T = 400 \text{ Hz} \). Since there are \( N = 32767 \) states of the shift register and for each state we select one of two frequencies separated by 400 Hz, the hopping bandwidth is 13.1068 MHz.

24. (B) The total SNR for three hops is \( 20 - 13 \text{ dB} \). Therefore the SNR per hop is 20/3. The probability of a chip error with non-coherent detection is

\[ P = \frac{1}{2} e^{\frac{-\varepsilon}{2N_o}} \]

where \( \varepsilon / N_o = 20 / 3 \). The probability of a bit error is

\[ P_b = 1 - (1 - p)^2 = 1 - (1 - 2p + p^2) = 2p - p^2 \]

\[ = e^{\frac{-\varepsilon}{2N_o}} - \frac{1}{2} e^{\frac{\varepsilon}{N_o}} = 0.0013 \]

25. (C) In the case of one hop per bit, the SNR per bit is

\[ 20, \text{ Hence, } P_b = \frac{1}{2} e^{\frac{\varepsilon}{2N_o}} = \frac{1}{2} e^{-10} = 2.27 \times 10^{-5} \]

26. (D) We are given a hopping bandwidth of 2 GHz and a bit rate of 10 kbs.

Hence, \[ W/R = 2 \times 10^3 / 10^4 = 2 \times 10^3 \text{ or } 53 \text{ dB} \]

27. (A) The bandwidth of the worst partial-band jammer is \( \alpha \cdot W \), where

\[ \alpha \cdot W = 2/(\varepsilon / J_o) = 0.2 \]

Hence \( \alpha \cdot W = 0.4 \text{ GHz} \)

28. (C) The probability of error with worst-case partial-band jamming is

\[ P_e = \frac{e^{-1}}{\varepsilon / J_o} = \frac{e^{-1}}{10} = 3.68 \times 10^{-2} \]

29. (D) \( d = 5 \text{ miles} = 8050 \text{ meters} \)

\[ \Delta d = 2 \times 8050 = 16100 \]

\[ \Delta d = x \times t \quad \text{ or } \quad t = \frac{\Delta d}{x} \]

\[ \Rightarrow t = \frac{\Delta d}{x} = \frac{16100}{3 \times 10^5} = 5.367 \times 10^{-5} \]

\[ f = \frac{1}{t} = \frac{1}{5.367 \times 10^{-5}} = 18.63 \text{ kHz} \]

*************
10. A field is given as

\[ \mathbf{G} = \frac{13}{x^2 + y^2} (y\mathbf{u}_y + 3\mathbf{u}_z + x\mathbf{u}_x) \]

The field at point \((-2, 3, 4)\) is
(A) \(13\mathbf{u}_x + 3\mathbf{u}_y + 4\mathbf{u}_z\)  (B) \(-2\mathbf{u}_x + 3\mathbf{u}_y + 4\mathbf{u}_z\)
(C) \(13\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z\)  (D) \(3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z\)

11. A field is given as \(\mathbf{F} = y\mathbf{u}_x + z\mathbf{u}_y + x\mathbf{u}_z\). The angle between \(\mathbf{G}\) and \(\mathbf{u}_x\) at point \((2, 2, 0)\) is
(A) \(45^\circ\)  (B) \(30^\circ\)  (C) \(60^\circ\)  (D) \(90^\circ\)

12. A vector field is given as

\[ \mathbf{G} = 12xy\mathbf{u}_x + 6(x^2 + 2)\mathbf{u}_y + 18z^2\mathbf{u}_z \]

The equation of the surface \(\mathbf{M}\) on which \(|\mathbf{G}| = 60\) is
(A) \(4x^2y^2 + 4x^4 + 9z^4 + 2x^2 = 96\)
(B) \(2x^2y^2 + x^4 + 9z^4 + 2x^2 = 96\)
(C) \(2x^2y^2 + 4x^4 + 9z^4 + 2x^2 = 96\)
(D) \(4x^2y^2 + x^4 + 9z^4 + 2x^2 = 96\)

13. A vector field is given by

\[ \mathbf{E} = 4xy^2\mathbf{u}_x + 2y\sin 2x\mathbf{u}_y + y^2\sin 2x\mathbf{u}_z \]

The surface on which \(\mathbf{E}_y = 0\) is
(A) Plane \(y = 0\)  (B) Plane \(x = 0\)
(C) Plane \(x = \frac{3\pi}{2}\)  (D) all

14. The vector field \(\mathbf{E}\) is given by

\[ \mathbf{E} = 6zy^2\cos 2x\mathbf{u}_x + 4xy\sin 2x\mathbf{u}_y + y^2\sin 2x\mathbf{u}_z \]

The region in which \(\mathbf{E} = 0\) is
(A) \(y = 0\)  (B) \(x = 0\)
(C) \(z = 0\)  (D) \(x = \frac{n\pi}{2}\)

15. Two vector fields are \(\mathbf{F} = -10\mathbf{u}_x + 20x(y - 1)\mathbf{u}_y\) and \(\mathbf{G} = 2x^2y\mathbf{u}_x - 4\mathbf{u}_x + 2\mathbf{u}_z\). At point \((2, 3, -4)\) a unit vector in the direction of \(\mathbf{F} - \mathbf{G}\) is
(A) \(0.18\mathbf{u}_x + 0.98\mathbf{u}_y - 0.05\mathbf{u}_z\)
(B) \(-0.18\mathbf{u}_x - 0.98\mathbf{u}_y + 0.05\mathbf{u}_z\)
(C) \(-0.37\mathbf{u}_x + 0.92\mathbf{u}_y + 0.02\mathbf{u}_z\)
(D) \(0.37\mathbf{u}_x - 0.92\mathbf{u}_y - 0.02\mathbf{u}_z\)

16. A field is given as

\[ \mathbf{G} = \frac{25}{x^2 + y^2} (x\mathbf{u}_x + y\mathbf{u}_y) \]

The unit vector in the direction of \(\mathbf{G}\) at \((3, 4, -2)\) is
(A) \(0.6\mathbf{u}_x - 0.8\mathbf{u}_y\)  (B) \(0.8\mathbf{u}_x + 0.6\mathbf{u}_y\)
(C) \(0.6\mathbf{u}_x + 0.8\mathbf{u}_y\)  (D) \(0.6\mathbf{u}_x + 0.6\mathbf{u}_y\)

17. A field is given as \(\mathbf{F} = xy\mathbf{u}_x + yz\mathbf{u}_y + zx\mathbf{u}_z\). The value of the double integral \(I = \iint \mathbf{F} \cdot \mathbf{u}_z dzdx\) in the plane \(y = 7\) is
(A) 128  (B) 56  (C) 190  (D) 0

18. Two vector extending from the origin are given as \(\mathbf{R}_1 = 4\mathbf{u}_x + 3\mathbf{u}_y - 2\mathbf{u}_z\) and \(\mathbf{R}_2 = 3\mathbf{u}_x - 4\mathbf{u}_y - 6\mathbf{u}_z\). The area of the triangle defined by \(\mathbf{R}_1\) and \(\mathbf{R}_2\) is
(A) 12.47  (B) 20.15  (C) 10.87  (D) 15.46

19. The four vertices of a regular tetrahedron are located at \(O(0, 0, 0)\), \(A(0, 1, 0)\), \(B(0.5 \sqrt{3}, 0.5, 0)\) and \(C(\frac{2\sqrt{3}}{3}, \frac{1}{3}, \frac{2\sqrt{2}}{3})\). The unit vector perpendicular (outward) to the face ABC is
(A) \(0.41\mathbf{u}_x + 0.71\mathbf{u}_y + 0.29\mathbf{u}_z\)
(B) \(0.47\mathbf{u}_x + 0.82\mathbf{u}_y + 0.33\mathbf{u}_z\)
(C) \(-0.47\mathbf{u}_x - 0.82\mathbf{u}_y - 0.33\mathbf{u}_z\)
(D) \(-0.41\mathbf{u}_x - 0.71\mathbf{u}_y - 0.29\mathbf{u}_z\)

20. The two vector are \(\mathbf{R}_{AM} = 20\mathbf{u}_x + 18\mathbf{u}_y - 18\mathbf{u}_z\) and \(\mathbf{R}_{AN} = -10\mathbf{u}_x + 8\mathbf{u}_y + 15\mathbf{u}_z\). The unit vector in the plane of the triangle that bisects the interior angle at A is
(A) \(0.168\mathbf{u}_x + 0.915\mathbf{u}_y + 0.367\mathbf{u}_z\)
(B) \(0.729\mathbf{u}_x + 0.134\mathbf{u}_y - 0.672\mathbf{u}_z\)
(C) \(0.729\mathbf{u}_x + 0.134\mathbf{u}_y + 0.672\mathbf{u}_z\)
(D) \(0.168\mathbf{u}_x + 0.915\mathbf{u}_y - 0.367\mathbf{u}_z\)

21. Two points in cylindrical coordinates are \(A(\rho = 5, \phi = 70^\circ, z = -3)\) and \(B(\rho = 2, \phi = 30^\circ, z = 1)\). A unit vector at A towards B is
(A) \(0.03\mathbf{u}_x - 0.82\mathbf{u}_y + 0.57\mathbf{u}_z\)
(B) \(0.03\mathbf{u}_x + 0.82\mathbf{u}_y + 0.57\mathbf{u}_z\)
(C) \(-0.82\mathbf{u}_x + 0.003\mathbf{u}_y + 0.57\mathbf{u}_z\)
(D) \(0.003\mathbf{u}_x - 0.82\mathbf{u}_y + 0.57\mathbf{u}_z\)
22. A field in cartesian form is given as

\[ \mathbf{D} = x\mathbf{u}_x + y\mathbf{u}_y \]

In cylindrical form it will be

(A) \( \mathbf{D} = \frac{x}{\rho} \mathbf{u}_\rho + \frac{y}{\rho} \mathbf{u}_\phi \)

(B) \( \mathbf{D} = \frac{x}{\rho} \mathbf{u}_\rho + \frac{y}{\rho} \mathbf{u}_\phi + \cos \phi \mathbf{u}_z \)

(C) \( \mathbf{D} = \rho \mathbf{u}_\rho + \cos \phi \mathbf{u}_z \)

23. A vector extends from \( A(\rho = 4, \phi = 40^\circ, z = -2) \) to \( B(\rho = 5, \phi = -110^\circ, z = 1) \). The vector \( \mathbf{R}_{AB} \) is

(A) \( 4.77\mathbf{u}_x + 7.30\mathbf{u}_y + 4\mathbf{u}_z \)

(B) \( -4.77\mathbf{u}_x - 7.30\mathbf{u}_y + 4\mathbf{u}_z \)

(C) \( -7.30\mathbf{u}_x - 4.77\mathbf{u}_y + 4\mathbf{u}_z \)

(D) \( 7.30\mathbf{u}_x + 4.77\mathbf{u}_y + 4\mathbf{u}_z \)

24. The surface \( \rho = 3, \phi = 100^\circ, \phi = 130^\circ, z = 3 \) and \( z = 4.5 \) define a closed surface. The enclosed volume is

(A) \( 480 \)

(B) \( 5.46 \)

(C) \( 360 \)

(D) \( 6.28 \)

25. The surface \( \rho = 2, \phi = 45^\circ, \phi = 135^\circ, z = 3 \) and \( z = 4 \) define a closed surface. The total area of the enclosing surface is

(A) \( 34.29 \)

(B) \( 20.7 \)

(C) \( 32.27 \)

(D) \( 16.4 \)

26. The surface \( \rho = 3, \phi = 5, \phi = 100^\circ, \phi = 130^\circ, z = 3 \) and \( z = 4.5 \) define a closed volume. The length of the longest straight line that lies entirely within the volume is

(A) \( 3.21 \)

(B) \( 3.13 \)

(C) \( 4.26 \)

(D) \( 4.21 \)

27. A vector field \( \mathbf{H} \) is

\[ \mathbf{H} = \rho z^2 \sin \phi \mathbf{u}_\rho + e^z \sin \left( \frac{\phi}{2} \right) \mathbf{u}_\phi + \rho^2 \mathbf{u}_z \]

At point \( \left( 2, \frac{\pi}{3}, 0 \right) \) the value of \( \mathbf{H} \cdot \mathbf{u}_z \) is

(A) \( 0.25 \)

(B) \( 0.433 \)

(C) \( -0.433 \)

(D) \( -0.25 \)

28. A vector is \( \mathbf{A} = y\mathbf{u}_x + (x + z)\mathbf{u}_z \). At point \( P(-2, 6, 3) \) \( \mathbf{A} \) in cylindrical coordinate is

(A) \( -0.949\mathbf{u}_\rho - 6.008\mathbf{u}_\phi \)

(B) \( 0.949\mathbf{u}_\rho - 6.008\mathbf{u}_\phi \)

(C) \( -6.008\mathbf{u}_\rho - 0.949\mathbf{u}_\phi \)

(D) \( 6.008\mathbf{u}_\rho + 0.949\mathbf{u}_\phi \)

29. The vector

\[ \mathbf{B} = \frac{10}{r} \mathbf{u}_r + r \cos \theta \mathbf{u}_\theta + \mathbf{u}_z \]

in cartesian coordinates at (-3, 4, 0) is

(A) \( -2\mathbf{u}_x + \mathbf{u}_y \)

(B) \( -2\mathbf{u}_x - \mathbf{u}_y \)

(C) \( 1.36\mathbf{u}_x + 2.72\mathbf{u}_y \)

(D) \( -2.72\mathbf{u}_x + 1.36\mathbf{u}_y \)

30. The two point have been given \( A(20, 30^\circ, 45^\circ) \) and \( B(30, 115^\circ, 160^\circ) \). The \( \mathbf{R}_{AB} \) is

(A) \( 22.2 \)

(B) \( 44.4 \)

(C) \( 11.1 \)

(D) \( 33.3 \)

31. The surface \( z = 2 \) and \( 4, \theta = 30^\circ \) and \( 60^\circ, \phi = 20^\circ \) and \( 80^\circ \) identify a closed surface. The enclosed volume is

(A) \( 11.45 \)

(B) \( 7.15 \)

(C) \( 6.14 \)

(D) \( 8.26 \)

32. The surface \( z = 2 \) and \( 4, \theta = 30^\circ \) and \( 50^\circ \) and \( \phi = 20^\circ \) and \( 60^\circ \) identify a closed surface. The total area of the enclosing surface is

(A) \( 6.31 \)

(B) \( 18.91 \)

(C) \( 25.22 \)

(D) \( 12.61 \)

33. At point \( P(r = 4, \theta = 0.2\pi, \phi = 0.8\pi) \), \( \mathbf{u}_\rho \) in cartesian component is

(A) \( 0.48\mathbf{u}_x + 0.35\mathbf{u}_y + 0.81\mathbf{u}_z \)

(B) \( 0.48\mathbf{u}_x - 0.35\mathbf{u}_y - 0.81\mathbf{u}_z \)

(C) \( -0.48\mathbf{u}_x + 0.35\mathbf{u}_y + 0.81\mathbf{u}_z \)

(D) \( -0.48\mathbf{u}_x - 0.35\mathbf{u}_y - 0.81\mathbf{u}_z \)

34. The expression for \( \mathbf{u}_\rho \) in spherical coordinates at \( P(4, r = 0.2\pi, \phi = 0.8\pi) \) is

(A) \( 0.48\mathbf{u}_x + 0.35\mathbf{u}_y - 0.81\mathbf{u}_z \)

(B) \( 0.35\mathbf{u}_x + 0.48\mathbf{u}_y - 0.81\mathbf{u}_z \)

(C) \( -0.48\mathbf{u}_x + 0.35\mathbf{u}_y - 0.81\mathbf{u}_z \)

(D) \( -0.35\mathbf{u}_x + 0.48\mathbf{u}_y - 0.81\mathbf{u}_z \)

35. Given a vector field

\[ \mathbf{D} = r \sin \phi \mathbf{u}_r - \frac{1}{r} \sin \theta \cos \phi \mathbf{u}_\theta + r^2 \mathbf{u}_r \]

The component of \( \mathbf{D} \) tangential to the spherical surface \( r = 10 \) at \( P(10, 150^\circ, 330^\circ) \) is
36. The circulation of \( \mathbf{F} = x^2 \mathbf{u}_x - xz \mathbf{u}_y - y^2 \mathbf{u}_z \) around the path shown in fig. P8.1.36 is

\[
\begin{align*}
\text{(A)} & \quad 0.043 \mathbf{u}_x + 100 \mathbf{u}_y \\
\text{(B)} & \quad -0.043 \mathbf{u}_x - 100 \mathbf{u}_y \\
\text{(C)} & \quad 110 \mathbf{u}_x + 0.043 \mathbf{u}_y \\
\text{(D)} & \quad 0.043 \mathbf{u}_x - 100 \mathbf{u}_y
\end{align*}
\]

37. The circulation of \( \mathbf{A} = \rho \cos \phi \mathbf{u}_x + z \sin \phi \mathbf{u}_z \) around the edge \( L \) of the wedge shown in Fig. P8.1.37 is

\[
\begin{align*}
\text{(A)} & \quad -\frac{1}{3} \\
\text{(B)} & \quad \frac{1}{6} \\
\text{(C)} & \quad -\frac{1}{6} \\
\text{(D)} & \quad \frac{1}{3}
\end{align*}
\]

38. The gradient of field \( f = y^2 x + xyz \) is

\[
\begin{align*}
\text{(A)} & \quad y(y+z) \mathbf{u}_x + x(2y+z) \mathbf{u}_y + xy \mathbf{u}_z \\
\text{(B)} & \quad y(2x+z) \mathbf{u}_x + x(x+z) \mathbf{u}_y + xy \mathbf{u}_z \\
\text{(C)} & \quad y^2 \mathbf{u}_x + 2y \mathbf{u}_y + xy \mathbf{u}_z \\
\text{(D)} & \quad y(2y+z) \mathbf{u}_x + x(2y+z) \mathbf{u}_y + xy \mathbf{u}_z
\end{align*}
\]

39. The gradient of the field \( f = \rho^2 z \cos 2\phi \) at point \((1, 45^\circ, 2)\) is

\[
\begin{align*}
\text{(A)} & \quad 4 \mathbf{u}_\phi \\
\text{(B)} & \quad 4\sqrt{2} \mathbf{u}_\phi \\
\text{(C)} & \quad -4 \mathbf{u}_\phi \\
\text{(D)} & \quad -4\sqrt{2} \mathbf{u}_\phi
\end{align*}
\]
47. The flux of $\mathbf{D} = \rho^2 \cos^2 \phi \mathbf{u}_\rho + 3\sin \phi \mathbf{u}_\phi$ over the closed surface of the cylinder $0 \leq z < 3$, $\rho = 3$ is
(A) $324$  (B) $81\pi$
(C) $81$  (D) $64$

48. The curl of vector $\mathbf{A} = e^\theta \mathbf{u}_z + \sin x y \mathbf{u}_x + \cos^2 x z \mathbf{u}_z$
is
(A) $y e^\theta \mathbf{u}_z + x \cos x y \mathbf{u}_x - 2x \sin 2x z \mathbf{u}_z$
(B) $z \sin 2x y \mathbf{u}_x + (y \cos x y - x e^\theta) \mathbf{u}_z$
(C) $z \sin 2x y \mathbf{u}_x + (x \cos x y - x e^\theta) \mathbf{u}_z$
(D) $x e^\theta \mathbf{u}_z + x \cos x y \mathbf{u}_x - 2x \sin 2x z \mathbf{u}_z$

49. The curl of vector field $\mathbf{A} = \rho \cos \theta \mathbf{u}_\rho - 1 \sin \theta \mathbf{u}_\phi + 2r^2 \sin \theta \mathbf{u}_z$ is
(A) $0$  (B) $12 \mathbf{u}_z$
(C) $6 \mathbf{u}_z$  (D) $5 \mathbf{u}_z$

50. The curl of vector field $\mathbf{A} = r \cos \theta \mathbf{u}_r - \frac{1}{r} \sin \theta \mathbf{u}_\phi + 2r^2 \sin \theta \mathbf{u}_z$ is
(A) $\cos \theta \mathbf{u}_r - \frac{1}{r} \cos \theta \mathbf{u}_\phi$
(B) $2r^2 \cos \theta \mathbf{u}_r - 4r \sin \theta \mathbf{u}_\phi + \left(\frac{1}{r^2} \sin \theta - r \sin \theta \right) \mathbf{u}_z$
(C) $4r \cos \theta \mathbf{u}_r - 6 \sin \theta \mathbf{u}_\phi + \sin \theta \mathbf{u}_z$
(D) $0$

51. If $\mathbf{A} = (3y^2 - 2z) \mathbf{u}_z - 2x^2 \mathbf{u}_x + (x + y) \mathbf{u}_y$, the value of $\nabla \times \nabla \times \mathbf{A}$ at $P(-2, 3, -1)$ is
(A) $-6 \mathbf{u}_z + 4 \mathbf{u}_x$  (B) $8 (\mathbf{u}_x + \mathbf{u}_y)$
(C) $-8 (\mathbf{u}_x + \mathbf{u}_y)$  (D) $0$

52. The $\text{grad} \cdot \nabla \times \mathbf{A}$ of a vector field $\mathbf{A} = x^2 y \mathbf{u}_x + y^2 z \mathbf{u}_y - 2xz \mathbf{u}_z$ is
(A) $2xy + 2yz - 2x$
(B) $x^2 y + y^2 z - 2xz$
(C) $2x^2 y + 2y^2 z - 2xz$
(D) $0$

53. If $V = xy - x^2 y + y^2 z$, the value of the $\text{div grad} V$ is
(A) $0$
(B) $z + x^2 + 2y^2 z$
(C) $2y(z^2 - yz - x)$
(D) $2(z^2 - y^2 - y)$

54. If $V = x^2 y^2 z^2$, the laplacian of the field $V$ is
(A) $2(xy^2 + yz^2 + 2x^2)$
(B) $2(x^2 y^2 + y^2 z^2 + z^2 x^2)$
(C) $(x^2 y^2 + y^2 z^2 + z^2 x^2)$
(D) $0$

55. The value of $\nabla^2 V$ at point $P(3, 60^\circ, -2)$ is if $V = \rho^2 z (\cos \phi + \sin \phi)$
(A) $-8.2$  (B) $12.3$
(C) $-12.3$  (D) $0$

56. If the scalar field $V = r^2 (1 + \cos \theta \sin \phi)$ then $\nabla^2 V$ is
(A) $1 + 2(1 - r^2) \cos \theta \sin \phi$
(B) $6 + 4 \cos \theta \sin \phi - \cot \theta \cos \theta \sin \phi$
(C) $2 + 2(1 - r^2) \cos \theta \sin \phi$
(D) $0$

57. $\nabla \ln \rho$ is equal to
(A) $\nabla \times (\mathbf{u}_\rho)$  (B) $\nabla \times (z \mathbf{u}_z)$
(C) $\nabla \times (\rho \mathbf{u}_\rho)$

58. If $r = x \mathbf{u}_x + y \mathbf{u}_y + z \mathbf{u}_z$ then $(\mathbf{r} \cdot \nabla) r^2$ is equal to
(A) $2r^2$  (B) $3r^2$
(C) $4r^2$  (D) $0$

59. If $r = x \mathbf{u}_x + y \mathbf{u}_y + z \mathbf{u}_z$ is the position vector of point $P(x, y, z)$ and $r = |\mathbf{r}|$ then $\nabla \cdot \mathbf{r} \mathbf{r}$ is equal to
(A) $nr^n$  (B) $(n + 3)r^n$
(C) $(n + 2)r^n$  (D) $0$

60. If $\mathbf{F} = x^2 y \mathbf{u}_x - y \mathbf{u}_y$, the circulation of vector field $\mathbf{F}$ around closed path shown in fig. P8.1.60 is,

(A) $\frac{7}{3}$  (B) $-\frac{7}{6}$
(C) $\frac{7}{6}$  (D) $-\frac{7}{3}$
31. If \( \mathbf{A} = \rho \sin \phi \mathbf{u}_\rho + \rho^2 \mathbf{u}_\rho \), and \( L \) is the contour of fig. P8.1.61, then circulation \( \oint_L \mathbf{A} \cdot d\mathbf{L} \) is

\[
\begin{align*}
\text{Fig. P8.1.61} \\
(A) \quad 7\pi + 2 & (B) \quad 7\pi - 2 \\
(C) \quad 7\pi & (D) \quad 0
\end{align*}
\]

32. The surface integral of vector \( \mathbf{F} = 2\rho^2 \mathbf{z} \cdot \mathbf{u}_\rho + \rho \cos^2 \phi \mathbf{u}_\phi \) over the region defined by \( 2 \leq \rho \leq 5 \), \(-1 < z < 1\), \( 0 < \phi < 2\pi \) is

(A) 44 \quad (B) 176 \quad (C) 88 \quad (D) 352

33. If \( \mathbf{D} = xy \mathbf{u}_x + yz \mathbf{u}_y + zx \mathbf{u}_z \), then the value of \( \iiint_S \mathbf{A} \cdot d\mathbf{S} \) is, where \( S \) is the surface of the cube defined by \( 0 \leq x \leq 1 \), \( 0 \leq y \leq 1 \), \( 0 \leq z \leq 1 \)

(A) 0.5 \quad (B) 3 \quad (C) 0 \quad (D) 1.5

34. If \( \mathbf{D} = 2\rho \mathbf{x} \mathbf{u}_x + 3z \sin \phi \mathbf{u}_z - 4\rho \cos \phi \mathbf{u}_\phi \) and \( S \) is the surface of the wedge \( 0 < \rho < 2 \), \( 0 < \phi < 45^\circ \), \( 0 < z < 5 \), then the surface integral of \( \mathbf{D} \) is

(A) 24.89 \quad (B) 131.57 \quad (C) 63.26 \quad (D) 0

35. If the vector field \( \mathbf{F} = (\alpha xy + \beta z^3) \mathbf{u}_x + (3x^2 - yz) \mathbf{u}_y + (3xz^2 - y) \mathbf{u}_z \) is irrotational, the value of \( \alpha, \beta \) and \( \gamma \) is

(A) \( \alpha = \beta = \gamma = 1 \) \quad (B) \( \alpha = \beta = 1, \gamma = 0 \) \\
(C) \( \alpha = 0, \beta = \gamma = 1 \) \quad (D) \( \alpha = \beta = \gamma = 0 \)

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SOLUTIONS

1. (D) \( d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \)
   \[= \sqrt{(4 - 2)^2 + (6 - 3)^2 + (3 - (-1))^2} = \sqrt{4 + 81 + 16} = \sqrt{101} \]

2. (A) \( \mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A \)
   \[= (3\mathbf{u}_x + 0\mathbf{u}_y + 2\mathbf{u}_z) - (5\mathbf{u}_x - \mathbf{u}_y + 0\mathbf{u}_z) \]
   \[-2\mathbf{u}_x + \mathbf{u}_y + 2\mathbf{u}_z \]
   \[|\mathbf{R}_{AB}| = \sqrt{2^2 + 1^2 + 2^2} = 3 \]
   \[\mathbf{u}_R = \frac{-2}{3}\mathbf{u}_x + \frac{1}{3}\mathbf{u}_y + \frac{2}{3}\mathbf{u}_z \]

3. (C) The component of \( \mathbf{F} \) parallel to \( \mathbf{G} \) is
   \[= \frac{\mathbf{F} \cdot \mathbf{G}}{\mathbf{G}^2} \mathbf{G} = \frac{(10, -6, 5) \cdot (0.1, 0.2, 0.3)}{0.1^2 + 0.2^2 + 0.3^2} (0.1, 0.2, 0.3) \]
   \[= 0.93(0.1, 0.2, 0.3) = (0.09, 0.18, 0.279) \]

4. (C) The vector component of \( \mathbf{F} \) perpendicular to \( \mathbf{G} \) is
   \[= \mathbf{F} - \frac{\mathbf{F} \cdot \mathbf{G}}{\mathbf{G}^2} \mathbf{G} \]
   \[=(3, 2, 1) - \frac{(3, 2, 1) \cdot (4, 4, -2)}{4^2 + 4^2 + 2^2} (4, 4, -2) \]
   \[= (3, 2, 1) - (2, 2, -1) = (1, 0, 2) = \mathbf{u}_1 + 2\mathbf{u}_2 \]

5. (C) \( \mathbf{R} = 3\mathbf{u}_x + 4\mathbf{M} - \mathbf{N} \)
   \[= 3\mathbf{u}_x + 4(2\mathbf{u}_x + 3\mathbf{u}_y - 4\mathbf{u}_z) - (-4\mathbf{u}_x + 4\mathbf{u}_y + 3\mathbf{u}_z) \]
   \[= 15\mathbf{u}_x + 8\mathbf{u}_y - 19\mathbf{u}_z \]
   \[|\mathbf{R}| = \sqrt{15^2 + 8^2 + 19^2} = 25.5 \approx 25.5 \]

6. (B) \( \mathbf{R} = -\mathbf{M} + 2\mathbf{N} \)
   \[= -(8\mathbf{u}_x + 4\mathbf{u}_y - 8\mathbf{u}_z) + 2(8\mathbf{u}_x + 6\mathbf{u}_y - 2\mathbf{u}_z) \]
   \[= 8\mathbf{u}_x + 8\mathbf{u}_y + 4\mathbf{u}_z \]
   \[\mathbf{u}_R = \frac{8\mathbf{u}_x + 8\mathbf{u}_y + 4\mathbf{u}_z}{\sqrt{8^2 + 8^2 + 4^2}} \]
   \[= \frac{2}{3}\mathbf{u}_x + \frac{2}{3}\mathbf{u}_y + \frac{1}{3}\mathbf{u}_z = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \]

- \( \mathbf{M} + 2\mathbf{N} = -8, 4, -8) + 2(8, 6, -2) = (8, 8, 4) \)

\[\mathbf{u}_R = \frac{(8, 8, 4)}{\sqrt{8^2 + 8^2 + 4^2}} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \]

\[\mathbf{u}_R = \frac{2}{3}\mathbf{u}_x + \frac{2}{3}\mathbf{u}_y + \frac{1}{3}\mathbf{u}_z \]

7. (C) Mid point is \(\frac{(1, 7, -6 - 2)}{2}, \frac{4 + 0}{2}) = (4, -4, 2) \)

\[\mathbf{u}_R = \frac{(4, -4, 2)}{\sqrt{4^2 + 4^2 + 2^2}} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \]

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10. (D) \( G = \frac{13}{(-2)^2 + (3)^2} (3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z) \)
\[ = 3\mathbf{u}_x + 4\mathbf{u}_y - 2\mathbf{u}_z \]

11. (A) Let \( \theta \) be the angle between \( \mathbf{F} \) and \( \mathbf{u}_x \).
Magnitude of \( \mathbf{F} \) is \( |\mathbf{F}| = \sqrt{x^2 + z^2 + x^2} \)
\[ \mathbf{F} \cdot \mathbf{u}_x = (\mathbf{F})(1) \cos \theta = y \]
[\( \cos \theta = \frac{y}{\sqrt{x^2 + z^2 + x^2}} = \frac{2}{\sqrt{2^2 + 2^2}} = \frac{1}{\sqrt{2}} \)]
[\( \theta = 45^\circ \)]

12. (D) \( |G| = 60 \)
\[ \sqrt{(12xy)^2 + (6(x^2 + 2)) + (18z^2)^2} = 60 \]
\[ \Rightarrow \quad 144x^2y^2 + 36(x^4 + 4x^2 + 4) + 324z^4 = 3600 \]
\[ \Rightarrow \quad 4x^2y^2 + (x^4 + 4x^4 + 4) + 9z^4 = 100 \]
\[ \Rightarrow \quad 4x^2y^2 + 9z^4 + 4x^2 = 96 \]

13. (D) For \( E_x = 0, 2y \sin 2x = 0 \) \( \Rightarrow \) \( y = 0 \)
\( \sin 2x = 0, \Rightarrow \) \( 2x = 0, \pi, 3\pi \), \( \Rightarrow \) \( x = 0, \frac{3\pi}{2} \)
Hence (D) is correct.

14. (A)
\[ \mathbf{E} = (6yz \cos 2x)\mathbf{u}_x + 4x \sin 2x \mathbf{u}_y + (y \sin 2x) \mathbf{u}_z \]
Hence in plane \( y = 0, \quad \mathbf{E} = 0 \).

15. (C) \( \mathbf{R} = \mathbf{F} - \mathbf{G} \)
\[ = (-10\mathbf{u}_x + 20(y-1)\mathbf{u}_z) - (2x^2y \mathbf{u}_x - 4 \mathbf{u}_y + 2 \mathbf{u}_z) \]
At \( P(2, 3, -4) \),
\[ \mathbf{R} = \mathbf{F} - \mathbf{G} = (-10\mathbf{u}_x + 80\mathbf{u}_z) - (24\mathbf{u}_z - 4 \mathbf{u}_y + 2 \mathbf{u}_z) \]
\[ = -34\mathbf{u}_x + 84\mathbf{u}_y - 2\mathbf{u}_z \]
\[ \mathbf{u}_R = -34\mathbf{u}_x + 84\mathbf{u}_y - \mathbf{u}_z \]
\[ \sqrt{34^2 + 84^2 + 2^2} = -0.37\mathbf{u}_x + 0.92\mathbf{u}_y - 0.02\mathbf{u}_z \]

16. (A) At \( P(3, 4, -2) \)
\[ \mathbf{G} = \frac{25}{3^2 + 4^2 + z^2} (3\mathbf{u}_x + 4\mathbf{u}_y) = 3\mathbf{u}_x + 4\mathbf{u}_y \]
\[ \mathbf{u}_G = \frac{3u_x + 4u_y}{\sqrt{3^2 + 4^2}} = 0.6u_x + 0.8u_y \]

17. (B) \( \mathbf{F} \cdot \mathbf{u}_y = F_y = yz \)
\[ I = \int \int_0^{\frac{\pi}{4}} yzdx \, dz = \int_0^{\frac{\pi}{4}} \int_0^2 yz \, dz \, dx = \int_0^2 2ydx = 2(4)y = 8y \]
At \( y = 7, I = 8(7) = 56 \)

18. (B) Area \( A = \frac{1}{2} |\mathbf{R}_1 \times \mathbf{R}_2| = \frac{1}{2} \begin{vmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 4 & 3 & -2 \\ 3 & -4 & -6 \end{vmatrix} \)
\[ = u_x(18 - 8) - u_y(-24 + 6) + u_z(-16 - 9) \]
\[ = -26u_x + 18u_y - 25u_z \]
\[ |\mathbf{R}_1 \times \mathbf{R}_2| = \sqrt{26^2 + 18^2 + 25^2} = 40.31 \]
\[ \text{area} = \frac{40.31}{2} = 20.15 \]

19. (B) \( \mathbf{R}_{LA} = (0, 1, 0) - (0.5 \sqrt{3}, 0.5, 0) = (-0.5 \sqrt{3}, 0.5, 0) \)
\[ \mathbf{R}_{BC} = \left( \frac{0.5}{\sqrt{3}}, 0.5, \frac{\sqrt{2}}{3} \right) \]
\[ \text{Area} = \frac{1}{2} \begin{vmatrix} \mathbf{u}_x & \mathbf{u}_y & \mathbf{u}_z \\ 0.5 & 0.5 & 0 \\ 1 & 0 & \frac{\sqrt{2}}{3} \end{vmatrix} \]
\[ \mathbf{R}_{LA} \times \mathbf{R}_{BC} = \mathbf{u}_x \left( \frac{\sqrt{2}}{3} - u_y (0.5 \sqrt{2}) \right) + \mathbf{u}_z \left( 0.5 \right) \]
\[ = 0.41u_x - 0.71u_y + 0.29u_z \]
The required unit vector is \( \mathbf{u} \)
\[ = \frac{0.41u_x + 0.71u_y + 0.29u_z}{\sqrt{0.41^2 + 0.71^2 + 0.29^2}} \]
\[ = 0.47u_x + 0.81u_y + 0.33u_z \]

20. (A) The non-unit vector in the required direction is \( \frac{1}{2}(\mathbf{u}_{AV} + \mathbf{u}_{AW}) \)
\[ \mathbf{u}_{AW} = \frac{(-10, 8, 15)}{\sqrt{100 + 64 + 225}} \]
\[ = (-0.507, 0.406, 0.761) \]
21. (D) In cartesian coordinates
A (5 cos 70°, 5 sin 70°, −3) = A (171, 4.70, −3)
B (2 cos (−30°), 2 sin (−30°), −3) = B (173, −1, 1)
\[ \mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A = (-173, -1, -1) \]
\[ = (-0.02, -5.70, 4) \]
\[ \mathbf{u}_{AB} = \frac{(-0.02, -5.70, 4)}{\sqrt{(-0.02)^2 + (-5.70)^2 + 4^2}} = (0.003, -0.82, 0.57) \]

Hence (A) is correct.

22. (A) \( x = \rho \cos \phi, \ y = \rho \sin \phi \)
\[ \mathbf{D} = \frac{\rho \cos \phi \mathbf{u}_x + \rho \sin \phi \mathbf{u}_y}{\rho^2 \cos^2 \phi + \rho^2 \sin^2 \phi} = \frac{\mathbf{u}_x \cos \phi + \mathbf{u}_y \sin \phi}{\rho} \]
\[ D_x = D \cdot \mathbf{u}_x = \frac{1}{\rho} \left[ \cos \phi (\mathbf{u}_x \cdot \mathbf{u}_x) + \sin \phi (\mathbf{u}_y \cdot \mathbf{u}_x) \right] \]
\[ = \frac{1}{\rho} \left[ \cos^2 \phi + \sin^2 \phi \right] = \frac{1}{\rho} \]
\[ D_y = D \cdot \mathbf{u}_y = \frac{1}{\rho} \left[ \cos \phi (\mathbf{u}_x \cdot \mathbf{u}_y) + \sin \phi (\mathbf{u}_y \cdot \mathbf{u}_y) \right] \]
\[ = \frac{1}{\rho} \left[ \cos \phi (\sin \phi) + \sin \phi (\cos \phi) \right] = 0 \]
Therefore \( \mathbf{D} = \frac{1}{\rho} \mathbf{u}_x \)

23. (B) \( A (4 \cos 40°, 4 \sin 40°, -2) = A (306, 257, -2) \)
B (5 cos (−110°), 5 sin (−110°), 2) = B (−171, −4.7, 2)
\[ \mathbf{R}_{AB} = \mathbf{R}_B - \mathbf{R}_A = (-171, -4.7, 2) - (306, 257, -2) \]
\[ = (-477, -73, 4) \]

24. (D) \( \text{Vol} = \int_0^{4.5} \int_0^{130} \int_0^{5} \rho \, d\rho \, d\phi \, dz = 628 \)

25. (C) Area is
\[ = 2 \int_0^{\frac{135\pi}{4}} \int_0^{\frac{4.5\pi}{2}} \int_0^{\frac{4\pi}{2}} d\phi d\rho dz + 2 \int_0^{\frac{135\pi}{4}} \int_0^{\frac{4\pi}{2}} d\phi d\rho dz + 2 \int_0^{\frac{135\pi}{4}} \int_0^{\frac{4\pi}{2}} d\phi d\rho dz \]
\[ = 2 \left[ \frac{\rho^2}{2} \left( \frac{\pi}{2} \right) + \frac{2\rho^2}{2} \left( \frac{\pi}{2} \right) + \frac{2\rho^2}{2} \left( \frac{\pi}{2} \right) \right] = 32.27 \]

26. (A) \( A (\rho = 3, \ \phi = 100°, z = 3) = A (-0.52, 2.95, 3) \)
\( B (\rho = 5, \ \phi = 130°, z = 4.5) = B (-321, 3.83, 4.5) \)
\( \text{length} = |\mathbf{B} - \mathbf{A}| \)
\( |\mathbf{B} - \mathbf{A}| = \sqrt{(-321, 3.83, 4.5) - (-0.52, 2.95, 3)} \)
\( = (2.69, 0.88, 15) \)
\[ |\mathbf{B} - \mathbf{A}| = (2.69, 0.88, 15) = \sqrt{2.69^2 + 0.88^2 + 15^2} = 3.21 \]

27. (C) At \( P \left( 2, \frac{\pi}{3}, 0 \right) \)
\[ \mathbf{H} = 0.5 \mathbf{u}_x + 8 \mathbf{u}_z \]
\[ \mathbf{u}_x = \cos \phi \mathbf{u}_x - \sin \phi \mathbf{u}_y = \frac{1}{2} (\mathbf{u}_x - \sqrt{3} \mathbf{u}_y) \]
\[ \mathbf{H} \cdot \mathbf{u}_z = (0.5) \left(-\frac{\sqrt{3}}{2}\right) = -0.433 \]

28. (A) \[ \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \]
At \( P (-2, 6, 3) \)
\[ \mathbf{A} = 6 \mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z \]
\[ \phi = \tan^{-1} \left( \frac{6}{-2} \right) = 108.43° \]
\[ \cos \phi = -0.316, \ \sin \phi = 0.948 \]
\[ \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} -0.316 \\ -0.948 \\ 0 \end{bmatrix} \]
\[ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]
\[ A_x = 6(-0.316) + 0.948 = -0.949, \]
\[ A_y = 6(-0.948) - 0.316 = -6.008, \ A_z = 0 \]
Hence (A) is correct option.

29. (B) At \( P (-3, 4, 0) \)
\[ r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 0^2} = 5 \]
\[ \theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \frac{\pi}{2} \]
\[ \phi = \tan^{-1} \left( \frac{y}{x} \right) = \tan^{-1} \left( \frac{4}{-3} \right) = 126.87° \]
\[ \mathbf{B} = 2 \mathbf{u}_x + \mathbf{u}_z \]
\[ \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \theta \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 1 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \]
\[ = \begin{bmatrix} -0.6 & 0 & -0.8 \\ 0.8 & 0 & -0.6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \]
\[ B_x = 2(-0.6) + 0.8 = -2 \]
\[ B_y = 0 \]
\[ B_z = 0 \]
Along 3, \( C_3 = \frac{1}{2} \rho \cos \phi d\rho \bigg|_{460^\circ} = \frac{\rho^2}{2} = -1 \)

\[ \int_A \cdot dL = C_1 + C_2 + C_3 = 1 \]

38. (A) \( \nabla f = u_x \frac{\partial f}{\partial x} + u_y \frac{\partial f}{\partial y} + u_z \frac{\partial f}{\partial z} = y(y + z)u_x + x(2y + z)u_y + xyu_z \)

39. (C) \( \nabla f = u_r \frac{\partial f}{\partial r} + u_\theta \frac{\partial f}{\partial \theta} + u_\phi \frac{\partial f}{\partial \phi} = 2 \rho^2 z \cos 2\phi u_r - 2\rho z \sin 2\phi u_\theta + \rho^2 \cos 2\phi u_\phi \)

At point P(1, 45°, 2), \( \nabla f = -4u_r \)

40. (B) \( r \sin \theta \cos \phi = x, \ r \sin \theta \sin \phi = y, \ r \cos \theta = z \)

\( G = r^3 \sin \theta \sin \phi \cos \phi \)

\( = 4(r \sin \theta \cos \phi (r \sin \theta \sin \phi) (r \cos \theta)) = 4xyz \)

\( \nabla G = u_x \frac{\partial (4xyz)}{\partial x} + u_y \frac{\partial (4xyz)}{\partial y} + u_z \frac{\partial (4xyz)}{\partial z} = 4yzu_x + 4xzu_y + 4xyu_z \)

At point \( P \left(1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \) \( \nabla G = u_x + u_y + u_z \)

41. (C) \( \nabla \Phi = (y + z)u_x + (x + z)u_y + (x + y)u_z \)

At point P(3, -3, -3),

\( \nabla \Phi = -6u_x, \) \( \mathbf{R}_{pq} = (4, -1, -1) - (3, -3, -3) = (1, 2, 2) \)

\( \nabla \Phi \cdot \mathbf{u}_r = \frac{\nabla \Phi \cdot (u_x + 2u_y + 2u_z)}{3} = -2 \)

42. (C) \( \nabla T = u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} = 2xyu_x + 2yu_y - 4zu_z \)

At point P(2, 2, 1), \( \nabla T = 4u_x + 4u_y - 4u_z \)

43. (A) Let \( f = x^2y + z - 3, \ g = x \ln z - y^2 + 4 \)

\( \nabla f = 2xu_x + x^2u_y + u_z, \)

\( \nabla g = ln z \mathbf{u}_x - 2yu_y + \frac{x}{z} \mathbf{u}_z \)

At point P(-1, 2, 1),

\( \mathbf{u}_f = \frac{-4u_x + u_y + u_z}{\sqrt{18}}, \) \( \mathbf{u}_s = \frac{-4u_y - u_z}{\sqrt{17}} \)

\( \cos \theta = \pm \frac{\mathbf{u}_f \cdot \mathbf{u}_s}{\sqrt{18} \times \sqrt{17}} = 0.286 \)

\( \frac{\cos^{-1} 0.286 = 73.4^\circ}{} \)

44. (D) \( \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + 4x + 0 \)

At point P(1, -2, 3), \( \nabla \cdot \mathbf{A} = 4 \)

45. (A)

\( \nabla \cdot \mathbf{A} = \frac{1}{\rho^2} \frac{\partial (\rho^2 A_r)}{\partial \rho} + \frac{\partial (\sin \theta A_\theta)}{\partial \theta} + \frac{\partial (\sin \theta A_\phi)}{\partial \phi} \)

\( = \frac{1}{\rho^2} (6\rho^2 \cos \phi \cos \theta) + 0 + 0 \)

At point P(1, 30°, 60°), \( \nabla \cdot \mathbf{A} = 6(\cos 30^\circ \cos 60^\circ) = 2.6 \)

46. (C) \( \nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial (\rho A_r)}{\partial \rho} + \frac{\partial (\rho A_\theta)}{\partial \theta} + \frac{\partial (A_\phi)}{\partial \phi} \)

\( = \frac{1}{\rho} (\rho \rho^2 \cos \phi \cos \theta) + \frac{\rho \sin^2 \phi}{\cos \phi} = 2z^2 \cos \phi + \sin^2 \phi \)

47. (B) The flux is \( \int \mathbf{D} \cdot dS, \) By divergence theorem

\( \int \mathbf{D} \cdot dS = \int \mathbf{D} \cdot dv \)

\( \mathbf{D} = \frac{1}{\rho} \frac{\partial (\rho^2 \cos^2 \phi)}{\partial \rho} + \frac{1}{\rho} \frac{\partial (z \sin \phi)}{\partial \rho} = 3 \rho \cos^2 \phi + \frac{3}{\rho} \sin \phi \)

\( \int \mathbf{D} \cdot dv = \int \left[ 3 \rho \cos^2 \phi + \frac{z}{\rho} \sin \phi \right] d\phi dz d\rho \)

\( = \frac{3}{\rho} \int_0^{2\pi} \int_0^1 \int_0^{2\pi} \left[ 3 \rho \cos^2 \phi + \frac{z}{\rho} \sin \phi \right] d\phi dz d\rho \)

48. (B) \( \nabla \times \mathbf{A} = \begin{bmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \)

49. (D) \( \nabla \times \mathbf{A} = \frac{1}{\rho} \begin{bmatrix} u_\rho & u_\phi & u_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & A_\phi & A_z \end{bmatrix} \)

\( = \frac{1}{\rho} \begin{bmatrix} u_\rho & u_\phi & u_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & 2\rho^2 z^2 \cos \phi & 0 \end{bmatrix} \)

\( = \frac{1}{\rho} \begin{bmatrix} u_\rho \left(-6 \rho^2 z \cos \phi \right) - u_\phi \left(-\rho \sin \phi \right) \frac{1}{\rho} u_\phi \left(6 \rho z^2 \cos \phi - \rhoz \cos \phi \right) \end{bmatrix} \)

At point P(5, 90°, 1), \( \nabla \times \mathbf{A} = 5u_\phi \)
50. (C) $\nabla \times A = \frac{1}{r^2 \sin \theta} \left[ \begin{array}{c} \partial_x r u_\theta - \partial_\theta r u_x - \partial_\phi r u_y \\ \partial_n r u_\theta - \partial_\phi r u_x + \partial_\theta r u_y \\ \partial_\phi r u_x - \partial_\theta r u_y + \partial_\phi r u_n \end{array} \right]$

$$= \frac{1}{r^2 \sin \theta} \left[ \begin{array}{c} \frac{\partial}{\partial x} (r \sin \theta) \\ \frac{\partial}{\partial y} (r \sin \theta) \\ \frac{\partial}{\partial z} (r \sin \theta) \end{array} \right] = \frac{1}{r^2 \sin \theta} \left[ \begin{array}{c} \partial_x r u_\theta - \partial_\theta r u_x - \partial_\phi r u_y \\ \partial_n r u_\theta - \partial_\phi r u_x + \partial_\theta r u_y \\ \partial_\phi r u_x - \partial_\theta r u_y + \partial_\phi r u_n \end{array} \right]$$

$$= \frac{1}{r^2 \sin \theta} (2r^2 \sin \theta - 0) - \frac{1}{r^2 \sin \theta} u_y (6r^2 \sin^2 \theta - 0) + \frac{1}{r} u_x (0 + r \sin \theta)$$

$$= 4r \cos \theta u_x - 6r \sin \theta u_y + \sin \theta u_n$$

51. (A) $\nabla \times A = \left[ \begin{array}{c} \frac{\partial}{\partial x} u_x \\ \frac{\partial}{\partial y} u_x \\ \frac{\partial}{\partial z} u_x \\ (3y^2 - 2z) - (2x^2) \end{array} \right]$

$$= u_x (2 + 2x^2) - u_y (1 - (-2)) + u_z (-4x - 6y)$$

$\nabla \times \nabla \times A = \left[ \begin{array}{c} \frac{\partial}{\partial x} u_z \\ \frac{\partial}{\partial y} u_z \\ \frac{\partial}{\partial z} u_z \\ (2 + 2x^2) \end{array} \right]$

$$= u_y (-6) - u_z (-4z) + u_x (0) = -6u_x + 4z u_y$$

At P(-2, 3, -1),

$\nabla \times \nabla \times A = -6u_x - 4u_y = (6u_x + 4u_y)$

52. (D) $\nabla \times A = \left[ \begin{array}{c} \frac{\partial}{\partial x} u_y \\ \frac{\partial}{\partial y} u_y \\ \frac{\partial}{\partial z} u_y \\ (x^2 y + y^2 z - 2xz) \end{array} \right]$

$$= -y^2 u_x + 2z u_y - x^2 u_z$$

$\nabla \times \nabla \times A = 0$

53. (D) $\nabla \times \nabla = u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}$

$= (z - 2xy) u_x + (2y^2 - x^2) u_y + (x - 2y^2 z) u_z$

$\nabla \times \nabla \times A = \frac{\partial}{\partial x} (z - 2xy) + \frac{\partial}{\partial y} (2y^2 - x^2) + \frac{\partial}{\partial z} (x - 2y^2 z)$

$$= -2y + 2x^2 - 2y^2 = 2(x^2 - y^2 - y)$$

54. (B) $\nabla \times \nabla \times A = \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial x} \right)$

$$= 2(y^2 z + 2x^2 z + x^2 y^2) = 2(x^2 y^2 + y^2 z^2 + z^2 x^2)$$

55. (A) $\nabla \times \nabla \times A = \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial x} \right)$

$$= 4x (\cos \phi + \sin \phi) - z (\cos \phi + \sin \phi) + 0 = 3x (\cos \phi + \sin \phi)$$

At P(3, 60°, -2), $\nabla \times \nabla \times A = 3(-2) \left( \frac{1}{2} + \frac{\sqrt{3}}{2} \right) = -8.2$

56. (B)

$$\nabla \times \nabla \times A = \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{\partial}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{\partial}{\partial x} \right)$$

$$= 2 \left( \frac{x^2}{4} - \frac{3}{2} \right) - \frac{2}{3} \left( \frac{1}{4} \right) = \frac{1}{4} + \frac{1}{4} = \frac{7}{6}$$
61. (C) \[ \oint_{c} \mathbf{A} \cdot d\mathbf{L} = \left( \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \right) \mathbf{A} \cdot d\mathbf{L} \]

Along \( ab \), \( d\phi = 0 \), \( \phi = 0 \),
\[ \mathbf{A} \cdot d\mathbf{L} = 0, \quad \oint_{a} \mathbf{A} \cdot d\mathbf{L} = 0 \]

Along \( bc \), \( d\rho = 0 \), \( \mathbf{A} \cdot d\mathbf{L} = \rho^3 d\phi \)
\[ \oint_{b} \mathbf{A} \cdot d\mathbf{L} = \frac{2}{0} \rho^3 d\phi = (2)^3 \pi = 8\pi \]

Along \( cd \), \( d\phi = 0 \), \( \phi = \pi \), \( \mathbf{A} \cdot d\mathbf{L} = 0 \)
\[ \oint_{c} \mathbf{A} \cdot d\mathbf{L} = 0 \]

Along \( da \), \( d\rho = 0 \), \( \mathbf{A} \cdot d\mathbf{L} = \rho^3 d\phi \)
\[ \oint_{d} \mathbf{A} \cdot d\mathbf{L} = \rho^3 \left[ \frac{d\phi}{\pi} \right] = (1)^3 (-\pi ) = -\pi \]
\[ \oint_{d} \mathbf{A} \cdot d\mathbf{L} = 0 + 8\pi + 0 - \pi = 7\pi \]

62. (B) Using divergence theorem
\[ \oint_{s} \mathbf{F} \cdot d\mathbf{S} = \int_{v} \nabla \cdot \mathbf{F} dv \]
\[ \nabla \cdot \mathbf{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (2\rho^2 z^2) = 4z^2 = 4z^2 \]
\[ \int_{v} \nabla \cdot \mathbf{F} dv = \int_{v} \int_{-1}^{1} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 4z^2 \rho d\rho d\phi dz = 4 \frac{1}{2} z^2 dz \int_{-1}^{1} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \rho d\rho d\phi = 176 \]

63. (D) \[ \oint_{s} \mathbf{A} \cdot d\mathbf{S} = \oint_{v} (\nabla \cdot \mathbf{A}) dv \]
\[ \nabla \cdot \mathbf{A} = \frac{\partial (xy)}{\partial x} + \frac{\partial (yz)}{\partial y} + \frac{\partial (xz)}{\partial z} = y + z + x \]
\[ \oint_{s} \mathbf{A} \cdot d\mathbf{S} = \int_{0}^{1} \int_{0}^{1} (y + z + x) dx dy dz \]
\[ = 3 \left( \int_{0}^{1} dx \int_{0}^{1} dy \right) \int_{0}^{1} dz = 3 \left( \frac{1}{2} \right) = 15 \]

64. (B) \[ \nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial (\rho D_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial (D_\theta)}{\partial \theta} + \frac{\partial (D_z)}{\partial z} \]
\[ = 4z + \frac{3}{\rho} z \cos \phi \]

65. (A) \[ \mathbf{F} = \left[ \mathbf{u}_x, \mathbf{u}_y, \mathbf{u}_z \right] \]
\[ \Rightarrow \mathbf{u}_x = \frac{\partial}{\partial x}, \quad \mathbf{u}_y = \frac{\partial}{\partial y}, \quad \mathbf{u}_z = \frac{\partial}{\partial z} \]
\[ \Rightarrow \mathbf{F} = \frac{\partial}{\partial x} (ax + \beta z^2), \quad \frac{\partial}{\partial y} (3\beta z^2 - 3x^2) \]
\[ \mathbf{u}_x + (6x - \alpha y) \mathbf{u}_y \]
If \( \mathbf{F} \) is irrotational, \( \nabla \times \mathbf{F} = 0 \)
i.e. \( \alpha = 1 = \beta = \gamma \).
CHAPTER
8.2
ELECTROSTATICS

1. Let \( Q_1 = 4 \ \mu C \) be located at \( P_1(3, \ 11, \ 8) \) while \( Q_2 = -5 \ \mu C \) is at \( P_2(6, \ 15, \ 8) \). The force \( F_2 \) on \( Q_2 \) will be
   (A) \(-\frac{4.32}{\mu C} \mathbf{u}_x + \frac{5.76}{\mu C} \mathbf{u}_y \) N  
   (B) \(4.32 \mathbf{u}_x + \frac{5.76}{\mu C} \mathbf{u}_y \) N  
   (C) \(-\frac{4.32}{\mu C} \mathbf{u}_x + \frac{5.76}{\mu C} \mathbf{mN} \)  
   (D) \(4.32 \mathbf{u}_x + \frac{5.76}{\mu C} \mathbf{mN} \)

2. Four 5 nC positive charge are located in the \( z = 0 \) plane at the corners of a square 8 mm on a side. A fifth 5 nC positive charge is located at a point 8 mm distant from each of the other charge. The magnitude of the total force on this fifth charge is
   (A) \(2 \times 10^{-4} \) N  
   (B) \(4 \times 10^{-4} \) N  
   (C) 0.014 N  
   (D) 0.01 N

3. Four 40 nC are located at A (1, 0, 0), B (–1, 0, 0), C (0, 1, 0) and D (0, –1, 0) in free space. The total force on the charge at A is
   (A) \(24.6 \mathbf{u}_y \) \( \mu \)N  
   (B) \(-24.6 \mathbf{u}_y \) \( \mu \)N  
   (C) \(-13.6 \mathbf{u}_y \) \( \mu \)N  
   (D) \(13.6 \mathbf{u}_y \) \( \mu \)N

4. A point charge of 41 nC be located at \( P_1(4, \ -2, \ 7) \) and a charge 45 nC be at \( P_2(-3, \ 4, \ -2) \). The electric field \( \mathbf{E} \) at \( P_3(1, \ 2, \ 3) \) will be
   (A) \(0.13 \mathbf{u}_x + 0.33 \mathbf{u}_y + 0.12 \mathbf{u}_z \)  
   (B) \(-0.13 \mathbf{u}_x - 0.33 \mathbf{u}_y - 0.12 \mathbf{u}_z \)  
   (C) \(115 \mathbf{u}_x + 293 \mathbf{u}_y + 109 \mathbf{u}_z \)  
   (D) \(-115 \mathbf{u}_x - 293 \mathbf{u}_y - 109 \mathbf{u}_z \)

5. Let a point charge 25 nC be located at \( P_1(4, \ -2, \ 7) \) and a charge 60 nC be at \( P_2(-3, \ 4, \ -2) \). The point, at which on the \( y \) axis, is \( E_y = 0 \) is
   (A) \(-7.46 \)  
   (B) \(-22.11 \)  
   (C) \(-6.89 \)  
   (D) (B) and (C)

6. A 2 \( \mu C \) point charge is located at A (4, 3, 5) in free space. The electric field at P(8, 12, 2) is
   (A) \(13.1 \mathbf{u}_x + 159.7 \mathbf{u}_y - 49.4 \mathbf{u}_z \)  
   (B) \(159.7 \mathbf{u}_x + 27.4 \mathbf{u}_y - 49.4 \mathbf{u}_z \)  
   (C) \(13.1 \mathbf{u}_x + 27.4 \mathbf{u}_y - 49.4 \mathbf{u}_z \)  
   (D) \(159.7 \mathbf{u}_x + 137.1 \mathbf{u}_y - 49.4 \mathbf{u}_z \)

7. A point charge of \(-10 \) nC is located at \( P_1(0, \ 0, \ 0.5) \) and a charge of \( 2 \mu C \) at the origin. The \( \mathbf{E} \) at P(0, 2, 1) is
   (A) \(68.83 \mathbf{u}_x + 14.85 \mathbf{u}_y \)  
   (B) \(68.83 \mathbf{u}_x + 64.01 \mathbf{u}_y \)  
   (C) \(68.83 \mathbf{u}_x - 14.85 \mathbf{u}_y \)  
   (D) \(68.83 \mathbf{u}_x - 64.01 \mathbf{u}_y \)

8. Charges of 20 nC and \(-20 \) nC are located at (3, 0, 0) and \(-3, \ 0, \ 0 \) and \(-3, \ 0, \ 0 \), respectively. The magnitude of \( \mathbf{E} \) at y axis is
   (A) \(1080 \) \( \mathbf{u} \)  
   (B) \(1080 \) \( \mathbf{u} \)  
   (C) \(1080 \) \( \mathbf{u} \)  
   (D) \(1080 \) \( \mathbf{u} \)

9. A charge \( Q_0 \) located at the origin in free space produces a field for which \( E_z = 1 \) kV/m at point P(–2, 2, –1). The charge \( Q_0 \) is
   (A) \(2 \mu C \)  
   (B) \(-3 \mu C \)  
   (C) \(3 \mu C \)  
   (D) \(-2 \mu C \)

10. The volume charge density \( \rho_e = \rho_e e^{-|\mathbf{x}|} \) exist over all free space. The total charge present is
    (A) \(2 \rho_e \)  
    (B) \(4 \rho_e \)  
    (C) \(8 \rho_e \)  
    (D) \(3 \rho_e \)
11. A uniform volume charge density of 0.2 μC/m² is present throughout the spherical shell extending from \( r = 3 \text{ cm} \) to \( r = 5 \text{ cm} \). If \( \rho = 0 \) elsewhere, the total charge present throughout the shell will be
(A) 41.05 pC (B) 257.92 pC (C) 82.1 pC (D) 129.0 pC

12. If \( \rho_z = \frac{1}{r^2} 5e^{-0.1r}(\pi + \phi) \) μC/m³ in the region \( 0 \leq \rho \leq 10, -\pi < \phi < \pi \) and all \( z \), and \( \rho_z = 0 \) elsewhere, the total charge within the region is
(A) 1.29 mC (B) 0.57 C (C) 0.46 C (D) 0.23 C

13. The region in which \( 4 < r < 5, 0 < \theta < 25^\circ \), and \( 0.9 < \phi < 1.1 \pi \) contains the volume charge density of \( \rho_z = 10(r - 4)(r - 5) \sin \theta \sin \phi \). Outside the region, \( \rho_z = 0 \). The charge within the region is
(A) 0.657 C (B) 0.68 C (C) 0.46 C (D) 0.23 C

14. A uniform line charge of 5 nC/m is located along the line defined by \( y = -2, z = 5 \). The electric field \( \mathbf{E} \) at \( P(1, 2, 3) \) is
(A) \(-9\mathbf{u}_y + 45\mathbf{u}_z\) (B) \(9\mathbf{u}_y - 45\mathbf{u}_z\) (C) \(-18\mathbf{u}_y + 9\mathbf{u}_z\) (D) \(18\mathbf{u}_y - 9\mathbf{u}_z\)

15. A uniform line charge of 6.25 nC/m is located along the line defined by \( y = -2, z = 5 \). The \( \mathbf{E} \) at that point in the \( z = 0 \) plane where the direction of \( \mathbf{E} \) is given by \( \left( \frac{1}{3}\mathbf{u}_y - \frac{2}{3}\mathbf{u}_z \right) \) is
(A) \(4.5\mathbf{u}_y + 9\mathbf{u}_z\) (B) \(4.5\mathbf{u}_y - 9\mathbf{u}_z\) (C) \(9\mathbf{u}_y - 18\mathbf{u}_z\) (D) \(18\mathbf{u}_y - 36\mathbf{u}_z\)

16. Uniform line charge of 20 nC/m and −20 nC/m are located in the \( x = 0 \) plane at \( y = 3 \) and \( y = -3 \) m respectively. The \( \mathbf{E} \) at \( P(6, 0, 6) \) will be
(A) \(-24\mathbf{u}_y, \text{V/m}\) (B) \(48\mathbf{u}_y, \text{V/m}\) (C) \(-48\mathbf{u}_y, \text{V/m}\) (D) \(24\mathbf{u}_y, \text{V/m}\)

17. Uniform line charges of 100 nC/m lie along the entire extent of the three coordinate axes. The \( \mathbf{E} \) at \( P(-3, 2, -1) \) is
(A) \(-19.2\mathbf{u}_x + 2\mathbf{u}_y - 1.08\mathbf{u}_z, \text{kV/m}\) (B) \(-9.6\mathbf{u}_x + \mathbf{u}_y - 0.54\mathbf{u}_z, \text{kV/m}\) (C) \(0.96\mathbf{u}_x - \mathbf{u}_y + 0.54\mathbf{u}_z, \text{kV/m}\) (D) \(-19.2\mathbf{u}_x - 2\mathbf{u}_y + 1.08\mathbf{u}_z, \text{kV/m}\)

18. Two identical uniform charges with \( \rho_i = 80 \text{nC/m} \) are located in free space at \( x = 0, y = \pm 3 \text{ m} \). The force per unit length acting on the line at positive \( y \) arising from the charge at negative \( y \) is
(A) 9.375\( \mathbf{u}_y, \mu\text{N}\) (B) 37.5\( \mathbf{u}_y, \mu\text{N}\) (C) 19.17\( \mathbf{u}_y, \mu\text{N}\) (D) 75\( \mathbf{u}_y, \mu\text{N}\)

19. A uniform surface charge density of 10 nC/m² is present in the region \( x = 0, -2 < y < 2 \) and all \( z \) if \( \varepsilon = \varepsilon_0 \), the electric field at \( P(3, 0, 0) \) has
(A) \(x\) component only (B) \(y\) component only (C) \(x\) and \(y\) component (D) \(x, y,\) and \(z\) component

20. The surface charge density is \( \rho_s = 5 \text{nC/m}^2 \), in the region \( \rho < 0.2, z = 0 \), and is zero elsewhere. The electric field \( \mathbf{E} \) at \( A(\rho = 0, z = 0.5) \) is
(A) 5.4 V/m (B) 10.1 V/m (C) 10.5 V/m (D) 20.2 V/m

21. Three infinite charge sheet are positioned as follows: 10 nC/m² at \( x = -3, \) 40 nC/m² at \( y = 4 \) and 50 nC/m² at \( z = 2 \). The \( \mathbf{E} \) at \( (4, 3, -2) \) is
(A) \(0.56\mathbf{u}_x + 2.23\mathbf{u}_y - 2.8\mathbf{u}_z, \text{kV/m}\) (B) \(-0.56\mathbf{u}_x - 2.23\mathbf{u}_y + 2.8\mathbf{u}_z, \text{kV/m}\) (C) \(0.56\mathbf{u}_x + 2.23\mathbf{u}_y + 2.8\mathbf{u}_z, \text{kV/m}\) (D) \(-0.56\mathbf{u}_x - 2.23\mathbf{u}_y - 2.8\mathbf{u}_z, \text{kV/m}\)

22. Let \( \mathbf{E} = 5x^2\mathbf{u}_x - 15x^2y\mathbf{u}_y \). The equation of the stream line that passes through \( P(4, 2, 1) \) is
(A) \(y = \frac{128}{x^2}\) (B) \(x = \frac{128}{y^2}\) (C) \(y = \frac{64}{x^2}\) (D) \(x = \frac{64}{y^2}\)

23. A point charge 10 nC is located at origin. Four uniform line charge are located in the \( x = 0 \) plane as follows : 40 nC/m at \( y = 1 \) and \(-5 \text{ m}\), \(-60 \text{ nC/m} \) at \( y = -2 \) and \(-4 \text{ m}\). The \( \mathbf{D} \) at \( P(0, -3, 4) \) is
(A) \(-19.1\mathbf{u}_x + 25.5\mathbf{u}_z, \text{pC/m}^2\) (B) \(-19.1\mathbf{u}_x - 25.5\mathbf{u}_z, \text{pC/m}^2\) (C) \(-16.4\mathbf{u}_x + 21.9\mathbf{u}_z, \text{pC/m}^2\) (D) \(-16.4\mathbf{u}_x - 21.9\mathbf{u}_z, \text{pC/m}^2\)
24. A point charge 20 nC is located at origin. Four uniform line charge are located as follows 40 nC/m at \( y = \pm 1 \) and 50 nC/m at \( y = \pm 2 \). The electric flux that leaves the surface of a sphere, 4 m in radius, centered at origin is
(A) 1.33 nC  
(B) 1.89 \( \mu \)C  
(C) 1.33 \( \mu \)C  
(D) 1.89 \( \mu \)C

25. The cylindrical surface \( \rho = 8 \) contains the surface charge density, \( \rho_s = 5e^{-20t} \) nC/m². The flux that leaves the surface \( \rho = 8 \) cm, \( 1 \leq z < 5 \) cm \( 30^\circ < \phi < 90^\circ \) is
(A) 270.7 nC  
(B) 9.45 nC  
(C) 270.7 pC  
(D) 9.45 pC

26. Let \( \mathbf{D} = 4xy \mathbf{u}_x + 2(x^2 + z^2) \mathbf{u}_y + 4yz \mathbf{u}_z \) C/m². The total charge enclosed in the rectangular parallelepiped \( 0 < x < 2, 0 < y < 3, 0 < z < 5 \) m is
(A) 360 C  
(B) 180 C  
(C) 100 C  
(D) 560 C

27. Volume charge density is located in free space as \( \rho_v = 2e^{-1000r} \) nC/m³ for \( 0 < r < 1 \) mm and \( \rho_v = 0 \) elsewhere. The value of \( D_r \) on the surface \( r = 1 \) mm is
(A) 1.28 pC/m²  
(B) 0.28 pC/m²  
(C) 0.78 pC/m²  
(D) 0.32 pC/m²

28. Spherical surfaces at \( r = 2 \) and 4 carry uniform charge densities of 20 nC/m² and \(-4 \) nC/m². The \( D_r \) at \( 2 < r < 4 \) is
(A) \(-\frac{16}{r^2} \) nC/m²  
(B) \(\frac{16}{r^2} \) nC/m²  
(C) \(\frac{80}{r} \) nC/m²  
(D) \(-\frac{80}{r^2} \) nC/m²

29. Given the electric flux density, \( \mathbf{D} = 2xy \mathbf{u}_x + x^2 \mathbf{u}_y + 6z^2 \mathbf{u}_z \) C/m². The total charge enclosed in the volume \( 0 < x, y, z < a \) is
(A) \(6a^5 + \frac{5}{3}a^4 \)  
(B) \(a^5 + 6a^4 \)  
(C) \(6a^5 + a^4 \)  
(D) \(\frac{5}{3}a^5 + 6a^4 \)

30. Let \( \mathbf{D} = 5x^4y^4z^4 \mathbf{u}_z \). The flux enclosed by volume \( x = 3 \) and 3.1, \( y = 1 \) and 1.1, and \( z = 2 \) and 2.1 is
(A) 49.6  
(B) 24.8  
(C) 35.4  
(D) 36.4

31. A spherical surface of radius of 3 mm is centered at P(4, 1, 5) in free space. If \( \mathbf{D} = xu_1 \) C/m² the net electric flux leaving the spherical surface is
(A) 113.1 \( \mu \)C  
(B) 339.3 nC  
(C) 113.1 nC  
(D) 452.4 nC

32. The electric flux density is
\( \mathbf{D} = \frac{1}{z^2}[10xyu_x + 5x^2u_y + (2z^3 - 5x^2y)u_z] \)
The volume charge density \( \rho_v \) at \((-2, 3, 5)\) is
(A) 6.43  
(B) 8.96  
(C) 10.4  
(D) 7.86

33. If \( \mathbf{D} = 2ru_r \) C/m², the total electric flux leaving the surface of the cube, \( 0 < x, y, z < 0.4 \) is
(A) 0.32  
(B) 0.34  
(C) 0.38  
(D) 0.36

34. If \( \mathbf{E} = 4u_x - 3u_y + 5u_z \) in the neighborhood of point P(6, 2, -3). The incremental work done in moving 5 C charge a distance of 2 m in the direction \( u_x + u_y + u_z \) is
(A) -60 J  
(B) 34.64 J  
(C) -34.64 J  
(D) 60 JJ

35. If \( \mathbf{E} = 100u_p \) V/m , the incremental amount of work done in moving a 60 \( \mu \)C charge a distance of 2 mm from P(1, 2, 3) toward Q(2, 1, 4) is
(A) -5.4 \( \mu \)J  
(B) 3.1 \( \mu \)J  
(C) -3.1 \( \mu \)J  
(D) 0

36. A 10 C charge is moved from the origin to P(3, 1, -1) in the field \( \mathbf{E} = 2xu_x - 3y^2u_y + 4u_z \) V/m along the straight line path \( x = -3y, y = x + 2z \). The amount of energy required is
(A) -40 J  
(B) 20 J  
(C) -20 J  
(D) 40 J

37. A uniform surface charge density of 30 nC/m² is present on the spherical surface \( r = 6 \) mm in free space. The \( V_{ab} \) between A \((r = 2 \) cm, \( \theta = 35^\circ, \phi = 55^\circ)\) and B \((r = 3 \) cm, \( \theta = 40^\circ, \phi = 90^\circ)\) is
(A) 2.03 V  
(B) 10.17 V  
(C) 4.07 mV  
(D) -10.17 V
38. A point charge is located at the origin in free space. The work done in carrying a charge 10 C from point A $(r = 4, \theta = \pi/6, \phi = \pi/4)$ to B $(r = 4, \theta = \pi/3, \phi = \pi/6)$ is  
(A) 0.45 $\mu$J  
(B) 0.32 $\mu$J  
(C) $-0.45 \mu$J  
(D) 0

39. Let a uniform surface charge density of $5 \text{nC/m}^2$ be present at the $z = 0$ plane, a uniform line charge density of $8 \text{nC/m}$ be located at $x = 0, z = 4$ and a point charge of $2 \text{nC}$ be present at $P(2, 0, 0)$. If $V = 0$ at $A(0, 0, 5)$, the $V$ at $B(1, 2, 3)$ is  
(A) 10.46 kV  
(B) 1.98 kV  
(C) 0.96 kV  
(D) 3.78 kV

40. A non uniform linear charge density, $\rho_L = 6/\left(z^2 + 1\right)$ nC/m lies along the $z$ axis. The potential at $P(r = 1, 0, 0)$ in free space is $(V_x = 0)$  
(A) 0 V  
(B) 216 V  
(C) 144 V  
(D) 108 V

41. The annular surface, $1 \text{cm} < r < 3 \text{cm}$ carries the nonuniform surface charge density $\rho_s = 5\pi \text{nC/m}^2$. The $V$ at $P(0, 0, 2 \text{cm})$ is  
(A) 81 mV  
(B) 90 mV  
(C) 63 mV  
(D) 76 mV

42. If $V = 2x y^2 z^3 + 3\ln(x^2 + 2y^2 + 3z^2)$ in free space the magnitude of electric field $E$ at $P(3, 2, -1)$ is  
(A) 72.6 V/m  
(B) 79.6 V/m  
(C) 75 V/m  
(D) 70.4 V/m

43. It is known that the potential is given by $V = 70r^{0.6}$ V. The volume charge density at $r = 0.6 m$ is  
(A) 1.79 nC/m$^3$  
(B) $-1.79 \text{nC/m}^3$  
(C) 1.22 nC/m$^3$  
(D) $-1.22 \text{nC/m}^3$

44. The potential field $V = 80r^2 \cos \theta$ V. The volume charge density at point $P(2.5, \theta = 30^\circ, \phi = 60^\circ)$ in free space is  
(A) $-2.45 \text{nC/m}^3$  
(B) 1.42 nC/m$^3$  
(C) $-1.42 \text{nC/m}^3$  
(D) 2.45 nC/m$^3$

45. Within the cylinder $\rho = 2, \ 0 < z < 1$ the potential is given by $V = 100 + 50 \rho + 150 \rho \sin \phi$ V. The charge lies within the cylinder is  
(A) $-4.94 \text{nC}$  
(B) $-4.86 \text{nC}$  
(C) $-5.56 \text{nC}$  
(D) $-3.68 \text{nC}$

46. A dipole having $Q d = 100 \text{nC} \cdot \text{m}$ is located at the origin in free space and aligned so that its moment is in the $u_z$ direction. The $E$ at point $(r = 1, 45^\circ, \phi = 0)$ is  
(A) 158.11 V/m  
(B) 194.21 V/m  
(C) 146.21 V/m  
(D) 167.37 V/m

47. A dipole located at the origin in free space has a moment $p = 2 \times 10^{-3} \text{u}_z \text{C.m}$. The points at which $E = 1 \text{mV/m}$ on line $y = z, x = 0$ are  
(A) $y = z = \pm 23.35$  
(B) $y = z = \pm 16.5$, $x = 0$  
(C) $y = z = 16.5$  
(D) $y = 0, z = 23.35, x = 0$

48. A dipole having a moment $p = 3\text{u}_z - 5\text{u}_y + 10\text{u}_x \text{nC.m}$ is located at $P(1, 2, -4 \text{)}$ in free space. The $V$ at $Q (2, 3, 4)$ is  
(A) 1.31 V  
(B) 1.26 V  
(C) 2.62 V  
(D) 2.52 V

49. A potential field in free space is expressed as $V = 40/\text{xyz}$. The total energy stored within the cube $1 < x, y, z < 2$ is  
(A) 1548 pJ  
(B) 0  
(C) 774 pJ  
(D) 387 pJ

50. Four $1.2 \text{nC}$ point charge are located in free space at the corners of a square $4 \text{cm}$ on a side. The total potential energy stored is  
(A) 1.75 $\mu$J  
(B) 2 $\mu$J  
(C) 3.5 $\mu$J  
(D) 0

51. Given the current density  
$$J = 10^4[\sin(2x) e^{-2} \text{u}_x + \cos(2x) e^{-2} \text{u}_y] \text{kA/m}^2$$  
The total current crossing the plane $y = 1$ in the $\text{u}_y$ direction in the region $0 < x < 1, 0 < z < 2$ is  
(A) 0  
(B) 12.3 mA  
(C) 24.6 mA  
(D) 6.15 mA
54. In a certain region where the relative permittivity is 2.4, \( \mathbf{D} = 2\mathbf{u}_x - 4\mathbf{u}_y + 5\mathbf{u}_z \) nC/m². The polarization is 
(A) 2.8u_x - 5.6u_y + 7u_z nC/m²  
(B) 3.4u_x - 6.9u_y + 8.6u_z nC/m²  
(C) 1.2u_x - 2.3u_y + 2.9u_z nC/m²  
(D) 3.89u_x - 6.43u_y + 8.96u_z nC/m²

55. Medium 1 has the electrical permittivity \( \varepsilon_1 = 15\varepsilon_0 \) and occupies the region to the left of \( x = 0 \) plane. Medium 2 has the electrical permittivity \( \varepsilon_2 = 25\varepsilon_0 \) and occupies the region to the right of \( x = 0 \) plane. If \( \mathbf{E}_1 \) in medium 1 is \( \mathbf{E}_1 = (2\mathbf{u}_x - 3\mathbf{u}_y + \mathbf{u}_z) \) V/m then \( \mathbf{E}_2 \) in medium 2 is 
(A) (2.0u_x - 1.8u_y + 0.6u_z) V/m  
(B) (1.67u_x - 3u_y + u_z) V/m  
(C) (1.2u_x - 3u_y + u_z) V/m  
(D) (1.2u_x - 1.8u_y + 0.6u_z) V/m

56. Two perfect dielectrics have relative permittivities \( \varepsilon_1 = 2 \) and \( \varepsilon_2 = 8 \). The planner interface between them is the surface \( x - y + 2z = 5 \). The origin lies in region 1. If \( \mathbf{E}_1 = 100\mathbf{u}_x + 200\mathbf{u}_y - 50\mathbf{u}_z \) V/m then \( \mathbf{E}_2 \) is 
(A) 400u_x + 800u_y - 200u_z V/m  
(B) 400u_x + 200u_y - 50u_z V/m  
(C) 25u_x + 200u_y - 50u_z V/m  
(D) 125u_x + 175u_y V/m

57. The two spherical surfaces \( r = 4 \) cm and \( r = 9 \) cm are separated by two perfect dielectric shells, \( \varepsilon_1 = 2 \) for \( 4 < r < 6 \) and \( \varepsilon_2 = 5 \) for \( 6 < r < 9 \). If \( \mathbf{E}_1 = \frac{1000}{\pi r^2} \mathbf{u}_r \) then \( \mathbf{E}_2 \) is 
(A) \( \frac{5000}{r^2} \mathbf{u}_r \) V/m  
(B) \( \frac{400}{r^2} \mathbf{u}_r \) V/m  
(C) \( \frac{2500}{r^2} \mathbf{u}_r \) V/m  
(D) \( \frac{2000}{r^2} \mathbf{u}_r \) V/m

58. The surface \( x = 0 \) separate two perfect dielectric. For \( x > 0 \), let \( \varepsilon_1 = 3 \), while \( \varepsilon_2 = 5 \) where \( x < 0 \). If \( \mathbf{E}_1 = 80\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z \) V/m then \( \mathbf{E}_2 \) is 
(A) \( 133.3\mathbf{u}_x - 100\mathbf{u}_y - 66.7\mathbf{u}_z \) V/m  
(B) \( 133.3\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z \) V/m  
(C) \( 48\mathbf{u}_x - 36\mathbf{u}_y - 24\mathbf{u}_z \) V/m  
(D) \( 48\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z \) V/m

59. A potential field exists in a region where \( \varepsilon = f(x) \). If \( \rho_s = 0 \), the \( \nabla^2 V \) is 
(A) \( -\frac{1}{f(x)} \frac{df}{dx} \frac{\partial V}{\partial x} \)  
(B) \( f(x) \frac{d^2 f}{dx^2} \frac{\partial V}{\partial x} \)  
(C) \( \frac{1}{f(x)} \frac{d^2 f}{dx^2} \frac{\partial V}{\partial x} \)  
(D) \( -f(x) \frac{df}{dx} \frac{\partial V}{\partial x} \)

60. If \( V(x, y) = 4e^{2x} + f(x) - 3y^2 \) in a region of free space where \( \rho_s = 0 \). It is know that both \( E_x \) and \( V \) are zero at the origin. The \( V(x, y) \) is 
(A) \( 3(x^2 - y^2) \)  
(B) \( 3(y^2 - x^2) \)  
(C) \( 4x^2 - 3y^2 \)  
(D) \( 4y^2 - 3x^2 \)
SOLUTIONS

1. (C) \( \mathbf{F}_2 = \frac{Q_1 Q_2}{4 \pi \varepsilon_0} \frac{R_{23}}{|R_{12}|^3} \)
\[ = \frac{(4 \times 10^{-6})(-5 \times 10^{-6})}{4 \pi \varepsilon_0} \times \frac{(3u_x + 4u_y)}{5^3} \]
\[ = (4.32u_x + 5.76u_y) \text{ mN} \]

2. (D) Arranging the charge in the xy plane at location (4, 4), (-4, 4), (4, -4), (-4, 4), the fifth charge will be on the z-axis at location \( z = 4 \sqrt{2} \). By symmetry, the force on the fifth charge will be \( z \) directed, and will be four times the \( z \) component
\[ F = \frac{4}{\sqrt{2}} \times \frac{q^2}{4 \pi \varepsilon_o d^2} \]
\[ = \frac{4}{\sqrt{2}} \times \frac{(5 \times 10^{-6})^2}{4 \pi \times (8.85 \times 10^{-12})(8 \times 10^{-3})^2} = 10^{-7} \text{ N} \]

3. (D) The force will be
\[ F = \frac{(40 \times 10^{-6})^2}{4 \pi \varepsilon_o} \left[ \frac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|^3} + \frac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3} + \frac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3} \right] \]
where \( \mathbf{R}_{CA} = \mathbf{u}_x - \mathbf{u}_y \), \( \mathbf{R}_{BA} = \mathbf{u}_x + \mathbf{u}_y \), \( R_{BA} = 2u_x \)
\[ F = \frac{4}{(4 \pi \varepsilon_o) \times 65 \times 10^{-6}} \left[ \frac{\mathbf{u}_x - \mathbf{u}_y + \mathbf{u}_x + \mathbf{u}_y + 2\mathbf{u}_x}{2\sqrt{2}} + \frac{2\mathbf{u}_x}{2\sqrt{2}} \right] \]
\[ = 13.76u_x \text{ mN} \]

4. (C) \( \mathbf{E} = \frac{10^{-9}}{4 \pi \varepsilon_o} \left[ \frac{41\mathbf{R}_{12}}{|\mathbf{R}_{12}|^2} + \frac{45\mathbf{R}_{23}}{|\mathbf{R}_{23}|^2} \right] \]
\( \mathbf{R}_{12} = -3u_x + 4u_y - 4u_x, \quad \mathbf{R}_{23} = 4u_x - 2u_y + 5u_z \)
\( E = 9 \times 10^{-3} \left[ \frac{41(-3u_x + 4u_y - 4u_x)}{(41)^{1/2}} + \frac{454u_y - 2u_z + 5u_x}{(45)^{1/2}} \right] \]
\[ = 1152u_x + 293u_y + 1089u_z \]

5. (D) The point is \( P_3(0, y, 0) \)
\( \mathbf{R}_{12} = -4u_x + (y + 2)u_y - 7u_z \),
\( \mathbf{R}_{13} = 3u_x + (y - 4)u_y + 2u_z \)
\( E_x = 10^{-9} \left[ \frac{25(-4)}{(65 + (y + 2)^2)^{3/2}} + \frac{60 \times 3}{(13 + (y - 4)^2)^{3/2}} \right] \)
To obtain \( E_x = 0 \), \( 0.48y^2 + 1392y + 7312 = 0 \)
which yields the two value \( y = -6.89, -22.11 \)

6. (B) \( \mathbf{E}_p = \frac{2 \times 10^{-6}}{4 \pi \varepsilon_o} \frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3} \)
\[ = \frac{2 \times 10^{-6}}{4 \pi \varepsilon_o} \left[ \frac{4u_x + 9u_y - 3u_z}{(106)^{3/2}} \right] \]
\[ = 65.9u_x + 148.3u_y - 49.4u_z \]
Then at point P, \( \rho = \sqrt{8^2 + 12^2} = 14.4 \),
\( \phi = \tan^{-1} \frac{12}{8} = 56.3^\circ, \quad z = 2 \)
\( E_p = \mathbf{E}_p \cdot \mathbf{u}_p = 65.9(u_x \cdot u_y) + 148.3(u_y \cdot u_p) \)
\[ = 65.9 \cos 56.3^\circ + 148.3 \sin 56.3 = 159.7 \]
\( E_4 = \mathbf{E}_p \cdot \mathbf{u}_4 = 65.9(u_x \cdot u_y) + 148.3(u_y \cdot u_p) \)
\[ = -65.9 \sin 56.3^\circ + 148.3 \cos 56.3 = 27.4, \]
\( E_z = -49.4 \)

7. (C) \( \mathbf{E}_p = \frac{2 \times 10^{-8}}{4 \pi \varepsilon_o} \left[ \frac{R_{12}}{|R_{12}|^3} + \frac{2R_{23}}{|R_{23}|^3} \right] \)
\( R_1 = (0, 2, 1) - (0, 0, 0.5) = (0, 2, 0.5) \)
\( R_2 = (0, 2, 1) - (0, 0, 0) = (0, 2, 1) \)
\( E_p = 9 \times 10^8 \times 10^{-8} \left[ \frac{-2(u_x + 0.5u_y)}{(425)^{3/2}} + \frac{2(2u_x + u_y)}{(5)^{3/2}} \right] \)
\( E_p = 54.9u_y + 44.1u_z \)
At \( P, \ r = \sqrt{5} \), \( \theta = \cos^{-1} \frac{1}{\sqrt{5}} = 63.4^\circ \) and \( \phi = 90^\circ \)
So \( E_x = \mathbf{E}_p \cdot \mathbf{u}_x = 54.9(u_x \cdot u_x) + 44.1(u_y \cdot u_x) \)
\[ = 54.9 \sin \theta \sin \phi + 44.1 \cos \theta = 68.83 \]
\( E_y = \mathbf{E}_p \cdot \mathbf{u}_y = 54.9(u_x \cdot u_y) + 44.1(u_x \cdot u_y) \)
\[ = 54.9 \cos \theta \sin \phi + 44.1(-\sin \theta) = -14.85 \]
\( E_z = \mathbf{E}_p \cdot \mathbf{u}_z = 54.9(u_x \cdot u_z) + 44.1(u_y \cdot u_z) = 54.9 \cos \phi = 0 \)

8. (A) Let a point on y axis be P(0, y, 0)
\( \mathbf{E}_p = \frac{20 \times 10^{-8}}{4 \pi \varepsilon_o} \left[ \frac{R_{12}}{|R_{12}|^3} + \frac{R_{23}}{|R_{23}|^3} \right] \)
\( R_1 = (0, y, 0) - (3, 0, 0) = (-3, y, 0) \)
\( R_2 = (0, y, 0) - (-3, 0, 0) = (3, y, 0) \)
\( |R_1| = |R_2| = \sqrt{9 + y^2} \)
\( E_p = 20 \times 10^8 \times 9 \times 10^{-8} \left[ \frac{-3u_x + yu_y}{(9 + y^2)^{3/2}} + \frac{3u_x + yu_y}{(\sqrt{9 + y^2})^3} \right] \)
\[ = \frac{-1080u_x}{(9 + y^2)^{3/2}}, \quad |E| = \frac{1080}{(9 + y^2)^{3/2}} \]

9. (B) The field at P will be
\( \mathbf{E}_p = \frac{Q_o}{4 \pi \varepsilon_o} \left[ \frac{-2u_x + 2u_y - u_z}{9^{3/2}} \right], \quad E_z = 1 \text{ kV/m} \)
\[ = -4 \pi \varepsilon_o \times 9^{3/2} \times 10^{-3} = -3 \mu \text{C} \]

10. (C) This will be 8 times the integral of \( \rho \) over the first octant
11. (C) \[ Q = \int \int \rho \rho e^{\beta^2 - y^2} dx dy dz = \frac{8 \rho_c}{3} \]

\[ \frac{4\pi(0.2)^3}{3} = 82.1 \text{ pC} \]

12. (A) \[ Q = \int \int \int 0.2 r^2 \sin \theta \; d\theta \; d\phi \]

\[ = \left[ \frac{4}{3} \pi (0.2)^3 \right] = 82.1 \text{ pC} \]

\[ Q = 5 \times 26.4 \times \int_{-\infty}^{\infty} \frac{\pi}{2} \; dx = 129 \text{ mC} \]

13. (A) \[ \phi = 10 \int \int (r - 4)(r - 5) \sin \theta \cdot \sin \phi \; r^2 \sin \theta \; d\theta \; d\phi \]

\[ = \frac{5 \times 10^{-9}}{2 \pi \rho_0} \times \frac{4 \mathbf{u}_x - 2 \mathbf{u}_y}{20} = 18 \mathbf{u}_x - 9 \mathbf{u}_y \]

14. (D) \[ \mathbf{E}_p = \frac{\rho_L}{2 \pi \rho_0} \mathbf{R} \frac{\mathbf{R}}{|\mathbf{R}|^3} \]

\[ \mathbf{R}_p = (1, 2, 3) - (1, -2, 5) = (0, 4, -2) \]

\[ |\mathbf{R}_p|^2 = 20, \]

\[ \mathbf{E}_p = \frac{5 \times 10^{-9}}{2 \pi \rho_0} \left[ \frac{4 \mathbf{u}_x - 2 \mathbf{u}_y}{20} \right] = 18 \mathbf{u}_x - 9 \mathbf{u}_y \]

15. (C) With \( z = 0 \), the general field is

\[ \mathbf{E}_{z=0} = \frac{\rho_L}{2 \pi \rho_0} \left[ \frac{(y + 2) \mathbf{u}_x - 5 \mathbf{u}_z}{(y + 2)^2 + 25} \right] \]

we require \[ |\mathbf{E}| = |2 \mathbf{E}| \]

So \[ 2(y + 2) = 5 \Rightarrow y = 1 \]

\[ \mathbf{E} = \frac{6.25 \times 25 \mathbf{u}_x - 5 \mathbf{u}_y}{6.25 + 25} = 9 \mathbf{u}_x - 18 \mathbf{u}_y \]

16. (C) \[ \mathbf{E}_p = \frac{\rho_L}{2 \pi \rho_0} \left[ \frac{\mathbf{R}_{x, \rho}}{|\mathbf{R}_{x, \rho}|^3} - \frac{\mathbf{R}_{y, \rho}}{|\mathbf{R}_{y, \rho}|^3} \right] \]

\[ \mathbf{R}_{x, \rho} = (6, 0, 6) - (6, -3, 0) = (6, 3, 0) \]

\[ \mathbf{R}_{y, \rho} = (6, 0, 6) - (0, -3, 6) = (6, 3, 0) \]

\[ \mathbf{E}_p = \frac{20 \times 10^{-9}}{2 \pi \rho_0} \left[ \frac{6 \mathbf{u}_x - 3 \mathbf{u}_y}{36 + 9} - \frac{6 \mathbf{u}_x + 3 \mathbf{u}_y}{36 + 9} \right] \]

\[ = -48 \mathbf{u}_y \text{ V/m} \]

17. (B) \[ \mathbf{E}_p = \frac{\rho_L}{2 \pi \rho_0} \left[ \frac{\mathbf{R}_{x, \rho}}{|\mathbf{R}_{x, \rho}|^3} + \frac{\mathbf{R}_{y, \rho}}{|\mathbf{R}_{y, \rho}|^3} \right] \]

\[ \mathbf{R}_{x, \rho} = (0, 3, -1, 0) \]

\[ \mathbf{R}_{y, \rho} = (0, 2, -1, 0) \]

Similarly \[ \mathbf{R}_{x, \rho} = (-3, 0, -1, 0), \mathbf{R}_{y, \rho} = (-3, 2, 0) \]

\[ \mathbf{E}_p = 100 \times 10^{-9} \times 2 \times 9 \times 10^{-9} \times \left[ \frac{2 \mathbf{u}_x - \mathbf{u}_y + \mathbf{u}_z}{5} - \frac{3 \mathbf{u}_x - \mathbf{u}_y + \mathbf{u}_z}{10} - \frac{3 \mathbf{u}_x + \mathbf{u}_y}{13} \right] \]

\[ = -0.96 \mathbf{u}_x + \mathbf{u}_y - 0.54 \mathbf{u}_z \text{ kV/m} \]

18. (C) At \( y = 4 \), \[ \mathbf{E} = \frac{\rho_L}{2 \pi \rho_0} \mathbf{u}_y \]

\[ d\mathbf{F} = d\mathbf{q} \mathbf{E} = \rho_L \text{ d}\mathbf{z} \mathbf{E} \]

\[ \mathbf{F} = \frac{1}{2 \pi \rho_0} \int \mathbf{u}_y \text{ d}\mathbf{z} = 18.75 \mathbf{u}_y \mu \text{N} \]

19. (A) \[ \mathbf{E} = \frac{\rho_L}{2 \pi \rho_0} \int \mathbf{R} - \mathbf{R}' \text{ d}\mathbf{S} \]

\[ \text{where } \mathbf{R} = 3 \mathbf{u}_x \text{ and } \mathbf{R}' = y \mathbf{u}_y + \mathbf{u}_z \]

\[ \mathbf{E}_{PA} = \frac{\rho_L}{2 \pi \rho_0} \int \mathbf{u}_y \text{ d}\mathbf{y} \cdot \text{ d}\mathbf{z} \]

Due to odd function

\[ \mathbf{E}_{PA} = \frac{\rho_L}{2 \pi \rho_0} \int \mathbf{u}_y \text{ d}\mathbf{y} \cdot \text{ d}\mathbf{z} \]

So there is only \( x \) component.

20. (D) There will be \( z \) component of \( \mathbf{E} \)

\[ \mathbf{R} = 2 \mathbf{u}_z \]

\[ \mathbf{R}' = \mathbf{u}_y \]

\[ \mathbf{E}_{x, p} = \frac{\rho_L}{2 \pi \rho_0} \int \mathbf{R} \cdot \mathbf{R}' \text{ d}\mathbf{p} \]

\[ = 2 \pi \rho \frac{z^2}{4 \pi \rho_0} \left[ \frac{1}{\sqrt{z^2 + \rho^2}} \right] \]

\[ = \frac{1}{\sqrt{z^2 + \rho^2}} \]

\[ = \frac{z}{\sqrt{z^2 + 0.04}} \]

\[ \mathbf{E}_z = \frac{2 \pi \rho}{4 \pi \rho_0} \left[ \frac{1 - \frac{z}{\sqrt{z^2 + 0.04}}} \right] \]

At \( z = 0.5 \), \( \mathbf{E}_z = 20.2 \text{ V/m} \)

21. (A) Since charge sheet are infinite, the field magnitude associated with each one will be \( \rho_c / 2 \pi \rho_0 \), which is position independent. The field direction will depend on which side of a given sheet one is positioned.

\[ \mathbf{E}_A = \frac{2 \times 10^{-9}}{2 \pi \rho_0} \mathbf{u}_z - \frac{4 \times 10^{-9}}{2 \pi \rho_0} \mathbf{u}_y - \frac{5 \times 10^{-9}}{2 \pi \rho_0} \mathbf{u}_z \]

\[ = 0.56 \mathbf{u}_z + 2.23 \mathbf{u}_y - 2.8 \mathbf{u}_z \]

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22. (A) \( \frac{dy}{dx} = \frac{E_x}{E_y} = -15x^2y = -3y \frac{x}{x^3} \) 

\[ \Rightarrow \frac{dy}{y} = -3dx \Rightarrow \ln y = -3 \ln x = \ln C_y = \frac{C_1}{x^3} \]

At \( P \), \( C = \frac{C_1}{4^3} \Rightarrow C_1 = 128 \Rightarrow y = \frac{128}{x^3} \)

23. (A) This point lies in the center of a symmetric arrangement of line charges, whose field will all cancel at that point. Thus \( \mathbf{D} \) arises from the point charge alone

\( \mathbf{D} = 10 \times 10^{-9} \left(-3 \mathbf{u}_x + 4 \mathbf{u}_z \right) \)

\[ \mathbf{D} = -19.1 \mathbf{u}_x + 25.5 \mathbf{u}_z \text{ pC/m}^2 \]

24. (C) \( h_1 = 2\sqrt{4^2 - 1} = 7.75, \)

\[ h_2 = 2\sqrt{4^2 - 1} = 6.93 \]

\( Q_z = 2 \times 7.75 \times 40n + 2 \times 6.93 \times 50 + 20n = 1.33 \mu C \)

25. (D) \( Q = \int \int \int 5e^{-20y}(0.08) d\phi dz \) nC

\[ = \left( \frac{\pi}{2} - \frac{\pi}{6} \right) 5(0.08) \left[ \frac{-10}{20} \right]_{0.01}^{0.05} \]

\[ = 9.45 \times 10^{-3} \text{ nC} = 9.45 \text{ pC} \]

26. (A) Out of the 6 surface only 2 will contribute to the net outward flux. The \( y \) component of \( \mathbf{D} \) will penetrate the surface \( y = 0 \) and \( y = z \) and net flux will be zero. At \( x = 0 \) plane \( D_x = 0 \) and at \( z = 0 \) plane \( D_z = 0 \).

This leaves the 2 remaining surfaces at \( x = 2 \) and \( z = 5 \). The net outward flux become

\[ f = \frac{1}{2} \int \int \mathbf{D} \cdot \mathbf{u}_y dydz + \frac{1}{2} \int \int \mathbf{D} \cdot \mathbf{u}_z dzdy \]

\[ = 5 \int 4(2) dy dz + 3 \int 4(5) dz dy = 360 \text{ C.} \]

27. (D) \( Q = \int \int \int 2e^{-1000r^2} \sin \theta dr d\theta d\phi \)

\[ Q = 8 \pi \left[ -\frac{r^2 e^{-1000}}{1000} \sin \theta \right]_{0.001}^{0.001} + 2e^{-1000}(-1000r - 1) \left[ 0.001 \right]_0 \]

\[ Q = 4 \times 10^{-3} \text{ nC,} \]

\[ D_r = \frac{Q}{4 \pi \epsilon_0} = 4.0 \times 10^{-9} \left( \frac{4 \pi (0001)}{2} \right) = 0.32 \text{ C/m}^2 \]

28. (C) \( 4 \pi r^2 D_r = 4 \pi (2^2) 20 \times 10^{-9} \text{ C/m}^2, \)

\[ D_r = \frac{80}{\pi} \text{ nC/m}^2 \]

29. (C) \( Q = \frac{1}{2} \int \int \left[ \int 2ay dydz + \int -2(0) dydz \right] \)

\[ + \left[ \int -x^2 dx dz + \int x^2 dx dz + \int -6(0^5) dx dy + \int 6a^3 dx dy \right] \]

\[ = a^4 + 0 - \frac{a^4}{3} + \frac{a^4}{3} + 0 + 6a^5 = a^4 + 6a^5 \]

30. (C) \( \nabla \cdot \mathbf{D} = \frac{\partial D}{\partial y} = 20x^4 y^2 z^4 \)

Center of cube \( = (3.05, 1.05, 2.05) \) and Volume

\[ V = (0.1)^3 = 0.001, \]

\[ \phi = 20 (3.05)^4 (1.05)^3 (2.05)^3 (0.001) = 35.4 \]

31. (C) \( \phi = (\nabla \cdot \mathbf{D}) \Delta v \)

\[ = \frac{\partial (x)}{\partial x} \left( \frac{4}{3} \pi (0.003)^3 \right) = 113.1 \text{ nC} \]

32. (B) \( \rho_c = \nabla \cdot \mathbf{D} = \left[ \frac{10y}{x} + 0 + \left( 2 + 10x^2 y \right) \right] \)

\[ = 8.96 \]
37. (D) \( r_A = 2 \text{ cm}, \quad r_B = 3 \text{ cm} \)

\[
V(r) = \frac{Q}{4\pi\rho r} = \left(6 \times 10^{-3}\right)^2 \times 30 \times 10^{-9}
\]

\[
V(r) = \frac{0.122}{r}, \quad V_{AB} = V_A - V_B = 0.122 \quad 0.02 = 0.03 = 2.03
\]

38. (D) \( W = \frac{Q^2Q_2}{4\pi\epsilon_0 r_{AB}^2} \frac{dr}{r^2} = \frac{Q^2Q_2}{4\pi\epsilon_0} \left[ \frac{1}{r_A} - \frac{1}{r_B} \right] \)

\( r_A = r_B \quad W = 0 \)

39. (B) \( V_p(s) = \frac{Q}{4\pi\rho r} \)

\( V_p(r) = \int \frac{\rho_o d\rho}{2\pi\rho_o} + C_1 = \frac{\rho_L}{2\pi\rho_o} \ln \rho + C_1 \)

\( V(z) = \int \frac{\rho_o r^2}{2\epsilon_o} \, dz + C_2 = \frac{\rho z^2}{2\epsilon_o} + C_2 \)

\( V = \frac{Q}{4\pi\rho_o} - \frac{\rho_L}{2\pi\rho_o} \ln \rho - \frac{\rho_o}{2\pi\rho_o} - z + C \)

Here \( r, \rho, z \) are the scalar distance from the charge.

\( r = \sqrt{2^2 + 5^2} = \sqrt{29} \), \( \rho = \sqrt{(5 - 4)^2} = 1 \), \( z = 5 \)

By putting these values. \( C = 193 \times 10^3 \)

At point \( N, \quad r = \sqrt{(2 - 1)^2 + 2^2 + z^2} = \sqrt{14} \)

\( \rho = \sqrt{1^2 + 1^2} = \sqrt{2} \), \( z = 3 \), \( V_B = 198 \text{ kV} \)

40. (D) \( V_p = \int \int \frac{\rho_o dS}{4\pi\rho_o (\mathbf{R} - \mathbf{R}')^2} \)

\( \mathbf{R} = z \mathbf{u}_z, \quad \mathbf{R}' = \rho \mathbf{u}_\rho, \quad dS = \rho d\rho d\phi \)

\( V_p = \int_0^{2\pi} \int_0^{0.03} \left(5 \times 10^{-9}\right)^2 \rho^2 d\rho d\phi \)

\( V_p = \frac{5 \times 10^{-9}}{2\epsilon_o} \left[ \frac{\rho}{2} \right] \left[ \rho^2 + z^2 - \frac{z^2}{2} \ln \left( \rho + \sqrt{\rho^2 + z^2} \right) \right]^{0.03} \)

At \( z = 0.02, \quad V_p = 0.081 \text{ V} \)

41. (A) \( V = \int \int \frac{\rho_o dS}{4\pi\rho_o (\mathbf{R} - \mathbf{R}')} \)

\( \mathbf{R} = z \mathbf{u}_z, \quad \mathbf{R}' = \rho \mathbf{u}_\rho, \quad dS = \rho d\rho d\phi \)

\( V_p = \int_0^{2\pi} \int_0^{0.03} \left(5 \times 10^{-9}\right)^2 \rho^2 d\rho d\phi \)

\( V_p = \frac{5 \times 10^{-9}}{2\epsilon_o} \left[ \frac{\rho}{2} \right] \left[ \rho^2 + z^2 - \frac{z^2}{2} \ln \left( \rho + \sqrt{\rho^2 + z^2} \right) \right]^{0.03} \)

42. (C) \( \mathbf{E} = -\nabla V \)

\( = \left[ 2y^2 + \frac{6x}{x^2 + 2y^2 + 3z^2} \right] \mathbf{u}_x \)

\( - \left[ 4xyz^3 + \frac{12y}{x^2 + 2y^2 + 3z^2} \right] \mathbf{u}_y \)

\( \mathbf{E} = 7.1 \mathbf{u}_x + 22.8 \mathbf{u}_y, \quad -71.1 \mathbf{u}_z \text{ V/m}, \)

\( |\mathbf{E}| = 75 \text{ V/m} \)

43. (D) \( \mathbf{E} = -\nabla V = -\frac{dV}{dr} \mathbf{u}_r = -(0.6)(70)r^{-0.4} \mathbf{u}_r \)

\( = -42r^{-0.4} \mathbf{u}_r \text{ V/m} \)

\( \mathbf{D} = \varepsilon_o \mathbf{E} = -42r^{-0.4} \varepsilon_o \mathbf{u}_r \text{ C/m}^2 \)

\( \rho_c = \nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{d}{dr} \left( r^2 D_r \right) = \frac{1}{r^2} \frac{d}{dr} \left( -42 \varepsilon_o r^{1.6} \right) = -67.2 \frac{\varepsilon_o}{r^{1.4}} \)

At \( r = 0.6 \text{ m} \), \( \rho_c = -67.2 \times 8.85 \times 10^{-12} \)

\( = -122 \text{ nC/m}^3 \)

44. (A) \( \mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r} \mathbf{u}_r - \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{u}_\phi \)

\( = -160r \cos \theta \mathbf{u}_r + 80r \sin \theta \mathbf{u}_\theta \text{ V/m} \)

\( \mathbf{D} = \varepsilon_o \mathbf{E} = -80 \varepsilon_o (2r \cos \theta \mathbf{u}_r - r \sin \theta \mathbf{u}_\theta) \)

\( \rho_c = \nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{d}{dr} \left( r^2 D_r \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( D_r \sin \theta \right) \)

\( \rho_c = -80 \varepsilon_o \left[ \frac{1}{r^2} \left( 2 \times 3r^2 \cos \theta - 12 \sin \theta \cos \theta \right) \right] \quad \rho_c = -320 \varepsilon_o \cos \theta = -2.45 \text{ nC/m}^3 \)

45. (C) \( \mathbf{E} = -\nabla V = -\frac{\partial V}{\partial r} \mathbf{u}_r - \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{u}_\phi \)

\( = -(50 + 150 \sin \phi) \mathbf{u}_r - (150 \cos \phi) \mathbf{u}_\phi \text{ V/m} \)

\( \mathbf{D} = \varepsilon_o \mathbf{E} = -(50 + 150 \sin \phi) \mathbf{u}_r - (150 \cos \phi) \mathbf{u}_\phi \)

\( \rho_c = \nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho E_\phi) + \frac{1}{\rho} \frac{\partial}{\partial \phi} E_\phi \)

\( \rho_c = \varepsilon_o \left[ -(50 + 150 \sin \phi) + 150 \sin \phi \right] + -\frac{50 \varepsilon_o}{\rho} \text{ C/m}^2 \)

\( Q = \int \int \int \frac{50 \varepsilon_o}{\rho} \rho d\rho d\phi dz = -2 \pi (50) \varepsilon_o = -556 \text{ nC} \)

46. (A) \( V = \frac{Q d \cos \theta}{4\pi \rho_o r^2} = \frac{100 \cos \theta}{r^2} \)

\( \mathbf{E} = -\nabla V = \left( \frac{\partial V}{\partial r} \mathbf{u}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \mathbf{u}_\theta \right) \)

\( = \frac{100}{r^2} \left( 2 \cos \theta \mathbf{u}_r + \sin \theta \mathbf{u}_\theta \right) \)

\( |\mathbf{E}| = 100(4 \cos^2 \theta + \sin^2 \theta)^{1/2} = 100 \times \sqrt{2} = 158.11 \text{ V/m} \)

47. (A) \( \mathbf{E} = \frac{2 \times 10^{-9}}{4\pi \rho_o r^3} \left[ 2 \cos \theta \mathbf{u}_r + \sin \theta \mathbf{u}_\theta \right] \)
55. (A) \[ I = \int_S \mathbf{J} \cdot \mathbf{n} \mid_{r=0.2} \, dS \]
\[ = \frac{1}{8} \int_0^{0.4} 40 \rho d\rho d\phi = 40 \int_0^{0.4} \log(\rho^2 + (0.1)^2) \, d\rho \]
\[ = 40 \pi \log 17 = 356 \, \text{A} \]

56. (B) So \[ E_{al} = E_{st} = \frac{J_{al}}{\sigma_{al}} = J_{st} \Rightarrow J_{al} = \frac{\sigma_{st}}{\sigma_{al}} J_{st} \]
\[ I = \pi (2 \times 10^{-3}) J_{st} + \pi (4 \times 10^{-3})^2 - (2 \times 10^{-3})^2 J_{st} = 80 \]
Solving \[ J_{st} = 3.2 \times 10^5 \, \text{A/m}^2 \]

57. (B) \[ J = \frac{4}{2 \pi l} \mathbf{u}_y, \, \text{A/m}^2, \]
\[ E = \frac{J}{\sigma} \left( \frac{12.73}{\rho l} \mathbf{u}_y, \, \text{V/m} \right) \]
\[ V = -\frac{1}{3} E \cdot dL = \frac{1}{3} \frac{12.73}{\rho l} \mathbf{u}_y \cdot \mathbf{u}_y d\rho = 12.73 \ln \frac{5}{3} = 6.51 \, \text{V} \]
\[ R = \frac{V}{I} = \frac{6.51}{1.63 \, \Omega} = 4.0 \]

58. (E) \[ V = \left( \frac{\partial V}{\partial \rho} \mathbf{u}_y + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \mathbf{u}_y + \frac{\partial V}{\partial \mathbf{u}_y} \right) \]
\[ = -\mathbf{r}^2 \cos \phi \mathbf{u}_y + (\rho + 1) \mathbf{r}^2 \sin \phi \mathbf{u}_y - 2(\rho + 1) \mathbf{r} \cos \phi \mathbf{u}_y \]
\[ E = -1.82 \mathbf{u}_y + 14.5 \mathbf{u}_y - 2.67 \mathbf{u}_y \, \text{V/m} \]
\[ J_{al} = \frac{\varepsilon_4}{\varepsilon_0} \mathbf{E} \cdot \mathbf{E} \]
\[ J_{st} = \varepsilon_4 \sqrt{1.82^2 + 14.5^2 + 2.67^2} = 1315 \, \text{pC/m}^2 \]

59. (C) \[ V = \frac{40 \cos \frac{\pi}{3} \sin \frac{\pi}{2}}{2^{3}} = 2.5 \, \text{V} \]
So the equation of the surface is \[ 40 \cos \theta \sin \phi = 2.5, \quad 16 \cos \theta \sin \phi = r^3 \]

60. (A) \[ E = -\mathbf{V}, \quad D = \varepsilon_0 \mathbf{E} = -\varepsilon_0 \mathbf{V} \]
\[ = -\varepsilon_0 \left[ 200x \frac{\partial}{\partial x} \left( x^2 + 4 \right) \mathbf{u}_x + 200x \frac{\partial}{\partial x} \mathbf{u}_x \right] \quad \text{C/m}^2 \]
\[ \mathbf{D}_{x=0} = -\frac{200 \varepsilon_0 x}{x^2 + 4} \mathbf{u}_x, \, \text{C/m}^2, \]
\[ \rho_s = -\mathbf{D} \cdot \mathbf{u}_x \mid_{x=0} = -\frac{200 \varepsilon_0 x}{x^2 + 4} C/m^2, \]
\[ Q = \int_0 \int_0 \int_{-3}^{2} = -3 \cdot (200) \varepsilon_0 \frac{1}{2} \ln \left( x^2 + 4 \right) \, dx dy \]
\[ = -300 \ln 2 = -1.84 \, \text{nC} \]

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61. (B) \( \mathbf{E} = -\nabla V , \ -3xy^2z^3 \) satisfy this equation.

62. (D) The plane can be replaced by \(-60\ \text{nC}\) at \(Q (2, 5, -6)\).
\[ \mathbf{R} = (5, 3, 1) - (2, 4, 6) = (3, -1, -5) \]
\[ \mathbf{R}' = (5, 3, 1) - (2, 4, 6) = (3, -1, 7), \]
\[ |\mathbf{R}| = \sqrt{35} , \ |\mathbf{R}'| = \sqrt{59} \]
\[ V_p = \frac{q}{4\pi\varepsilon_0 R} - \frac{q}{4\pi\varepsilon_0 R'} = \frac{q}{4\pi\varepsilon_0} \left[ \frac{1}{\sqrt{35}} - \frac{1}{\sqrt{59}} \right] \]
\[ V_p = 60 \times 10^{-9} \times 9 \times 10^{-9} \left[ \frac{1}{\sqrt{35}} - \frac{1}{\sqrt{59}} \right] = 21 \ \text{V} \]

63. (A) Using method of images
\[ V_p = \frac{q}{2\varepsilon_0} \ln \left[ \frac{\text{Final Distance from the charge}}{\text{Initial Distance from the charge}} \right] \]
\[ V = 0, \]
\[ V_r = \frac{q}{2\varepsilon_0} \ln \left[ \frac{1}{2 \ln \sqrt{1 + (2 - 1)^2} - \ln \sqrt{1 + (2 - (-2))^2} - \ln \frac{1}{1} } \right] \]
\[ = 2.40 \ \text{kV} \]

64. (C) \( \mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \mathbf{D} - \frac{\mathbf{D}}{\varepsilon_r} = \frac{\mathbf{D}}{\varepsilon_r} (\varepsilon_r - 1) \]
\[ \mathbf{P} = (2\mathbf{u}_x - 4\mathbf{u}_y + 5\mathbf{u}_z) \left( \frac{(2.4 - 1) \times 10^{-9}}{2.4} \right) \]
\[ \mathbf{P} = 12\mathbf{u}_x - 23\mathbf{u}_y + 29\mathbf{u}_z \ \text{nC/m}^2 \]

65. (C) \( \mathbf{D}_{n1} = \mathbf{D}_{n2} \Rightarrow \varepsilon_0 \mathbf{E}_{N1} = \varepsilon_0 \mathbf{E}_{N2} \]
\[ = 2 \times 15\varepsilon_0 = 25\varepsilon_0 \mathbf{E}_{n2} \Rightarrow \mathbf{E}_{n2} = 12 \]
\[ \mathbf{E}_{n1} = \mathbf{E}_{n2} , \quad \mathbf{E}_2 = 1.2\mathbf{u}_x - 3\mathbf{u}_y + 1\mathbf{u}_z \]

66. (D) The unit vector that is normal to the surface is
\[ \mathbf{u}_N = \frac{\nabla F}{|\nabla F|} = \frac{\mathbf{u}_x - \mathbf{u}_y + 2\mathbf{u}_z}{\sqrt{6}}, \]
\[ E_{n1} = \mathbf{E}_1 \cdot \mathbf{u}_N = \frac{1}{\sqrt{6}} |100 - 200 - 100| = -81.7 \ \text{V/m} \]
\[ E_{n2} = \frac{1}{\sqrt{6}} |100 - 200 + 100| = -54.3 \ \text{V/m} \]
\[ = -33.33\mathbf{u}_x + 33.33\mathbf{u}_y - 66.67\mathbf{u}_z \ \text{V/m} \]
\[ \mathbf{E}_{n1} = \mathbf{E}_1 - \mathbf{E}_{n1} = 133.3\mathbf{u}_x + 166.7\mathbf{u}_y + 16.67\mathbf{u}_z \]
\[ \mathbf{E}_{n2} = \mathbf{E}_2 - \mathbf{E}_{n2} = 133.3\mathbf{u}_x + 166.7\mathbf{u}_y - 833\mathbf{u}_x + 833\mathbf{u}_y - 1667\mathbf{u}_z \]
\[ = 125\mathbf{u}_x + 175\mathbf{u}_y \ \text{V/m} \]

67. (B) \( \mathbf{D}_{n1} = \mathbf{D}_{n2} , \quad \varepsilon_0 \varepsilon_r \mathbf{E}_{n1} = \varepsilon_0 \varepsilon_r \mathbf{E}_{n2} \)

Since \( \mathbf{E} \) is normal to the surface
\[ E_{x2} = \frac{\varepsilon_0}{\varepsilon_r} E_{n1} = \frac{2}{5} \times \frac{1000}{r^2} \mathbf{u}_x = \frac{400}{r^2} \mathbf{u}_x \ \text{V/m} \]

68. (D) \( \mathbf{D}_{n1} = \mathbf{D}_{n2} \) and \( \mathbf{E}_{n1} = \mathbf{E}_{n2} , \quad \mathbf{D} = \varepsilon \mathbf{E} \)
\[ \mathbf{D}_{n1} = 80 \times 3\varepsilon_0 \mathbf{D}_{n2} = 5\varepsilon_0 \mathbf{E}_{n1} , \quad \mathbf{E}_{n1} = 48 \]
\[ \mathbf{E}_2 = 48\mathbf{u}_x - 60\mathbf{u}_y - 40\mathbf{u}_z \]

69. (A) \( \mathbf{D} = \varepsilon \mathbf{E} = -f(x)\mathbf{v} \mathbf{v}, \)
\[ \nabla \cdot \mathbf{D} = \rho_v = 0 = \nabla \cdot (-f(x)\mathbf{v} \mathbf{v}) \]
\[ 0 = \nabla \cdot (-f(x)\mathbf{v} \mathbf{v}) \]
\[ = \left[ \frac{dF}{\mathbf{v}} \frac{\partial V}{\partial x} + f(x) \frac{\partial^2 V}{\partial x^2} + f(x) \frac{\partial^2 V}{\partial y^2} + f(x) \frac{\partial^2 V}{\partial z^2} \right] \]
\[ = \left[ \frac{dF}{\mathbf{v}} \frac{\partial V}{\partial x} + f(x)V^2 \right] \]
\[ \Rightarrow \mathbf{v}^2 V = -\frac{1}{f(x)} \frac{dF}{\mathbf{v}} \frac{\partial V}{\partial x} \]

70. (A) \( \rho_v = 0 \quad \Rightarrow \mathbf{v}^2 V = 0, \]
\[ \mathbf{v}^2 V = 16e^{2x} + \frac{\partial^2 f}{\partial x^2} - 6 = 0 \]
\[ \frac{\partial f}{\partial x} = 16e^{2x} + 6 \quad \Rightarrow \frac{\partial f}{\partial x} = -8e^{2x} + 6x + C_1 \]
\[ E_x = \frac{\partial V}{\partial x} = 8e^{2x} + \frac{\partial f}{\partial x} \]
\[ E_x(0) = 8 + \frac{\partial f}{\partial x}_{x=0} = 0 \quad \Rightarrow \frac{\partial f}{\partial x}_{x=0} = -8 \]

It follows that \( C_1 = 0 \)

Integrating again
\[ f(x) = -4e^{2x} + 3x^2 + C , \quad f(0, 1) = -4 + C_2 \]
\[ V(0, 0) = 0 = 4 + f(0) \quad \Rightarrow C_2 = 0. \quad f(0) = -4 \]
\[ f(x) = -4e^{2x} + 3x^2, \]
\[ V(x, y) = 4e^{2x} - 4e^{2x} + 3x^2 - 3y^2 = 3(x^2 - y^2) \]
**Statement for Q.9–11:**

An infinite filament on the z-axis carries 10 mA in the \( \mathbf{u}_z \) direction. Three uniform cylindrical current sheets are also present at 400 mA/m at \( \rho = 1 \) cm, -250 mA/m at \( \rho = 2 \) cm and 300 mA/m at \( \rho = 3 \) cm.

9. The magnetic field \( H_z \) at \( \rho = 0.5 \) cm is
   (A) 0.32 A/m       (B) 0.64 A/m       (C) 1.36 mA/m       (D) 0

10. The magnetic field \( H_z \) at \( \rho = 15 \) cm is
    (A) 1.63 A/m       (B) 0.37 A/m       (C) 2.64 A/m       (D) 0

11. The magnetic field \( H_z \) at \( \rho = 3.5 \) cm is
    (A) 0.14 A/m       (B) 0.56 A/m       (C) 0.27 A/m       (D) 0.96 A/m

**Statement for Q.12–14:**

In the fig. P8.3.12–14 The region \( 0 \leq z \leq 2 \) is filled with an infinite slab of magnetic material (\( \mu_r = 2.5 \)). The surface of the slab at \( z = 0 \) and \( z = 2 \), respectively, carry surface current \( 30 \mathbf{u}_z \) A/m and \( -40 \mathbf{u}_z \) as shown in fig.

![Fig. P8.3.12–14](image)

12. In the region \( 0 < z < 2 \) the \( \mathbf{H} \) is
    (A) \(-35 \mathbf{u}_z \) A/m       (B) \(35 \mathbf{u}_z \) A/m       (C) \(-5 \mathbf{u}_z \) A/m       (D) \(5 \mathbf{u}_z \) A/m

13. In the region \( z < 0 \) the \( \mathbf{H} \) is
    (A) \(5 \mathbf{u}_z \) A/m       (B) \(-5 \mathbf{u}_z \) A/m       (C) \(10 \mathbf{u}_z \) A/m       (D) \(-10 \mathbf{u}_z \) A/m

14. In the region \( z > 2 \) the \( \mathbf{H} \) is
    (A) \(5 \mathbf{u}_z \) A/m       (B) \(-5 \mathbf{u}_z \) A/m       (C) \(35 \mathbf{u}_z \) A/m       (D) \(-35 \mathbf{u}_z \) A/m

**Statement for Q.15–16:**

In the cylindrical region

\[
H_z = \begin{cases} 
\frac{2}{\rho} & \text{for } \rho \leq 0.6 \\
\frac{3}{\rho} & \text{for } \rho > 0.6
\end{cases}
\]

15. The current density \( \mathbf{J} \) for \( \rho < 0.6 \) mm is
    (A) \(2 \mathbf{u}_z \) A/m       (B) \(-2 \mathbf{u}_z \) A/m       (C) \( \mathbf{u}_z \) A/m       (D) 0

16. The current density \( \mathbf{J} \) for \( \rho > 0.6 \) mm is
    (A) \(2 \mathbf{u}_z \) A/m       (B) \(-3 \mathbf{u}_z \) A/m       (C) \(3 \mathbf{u}_z \) A/m       (D) 0

17. An electron with velocity \( \mathbf{v} = (3 \mathbf{u}_x + 12 \mathbf{u}_y - 4 \mathbf{u}_z) \times 10^5 \) m/s experiences no net forces at a point in a magnetic field \( \mathbf{B} = \mathbf{u}_x + 2 \mathbf{u}_y + 3 \mathbf{u}_z \) mWb/m^2. The electric field \( \mathbf{E} \) at that point is
    (A) \(-4.4 \mathbf{u}_x + 1.3 \mathbf{u}_y + 0.6 \mathbf{u}_z \) kV/m
    (B) \(4.4 \mathbf{u}_x - 1.3 \mathbf{u}_y - 0.6 \mathbf{u}_z \) kV/m
    (C) \(-4.4 \mathbf{u}_x + 1.3 \mathbf{u}_y + 0.6 \mathbf{u}_z \) kV/m
    (D) \(4.4 \mathbf{u}_x - 1.3 \mathbf{u}_y - 0.6 \mathbf{u}_z \) kV/m

18. A point charge of \( 2 \times 10^{-16} \) C and \( 5 \times 10^{-26} \) kg is moving in the combined fields \( \mathbf{B} = -3 \mathbf{u}_x + 2 \mathbf{u}_y - \mathbf{u}_z \) mT and \( \mathbf{E} = 100 \mathbf{u}_x - 200 \mathbf{u}_y + 300 \mathbf{u}_z \) V/m. If the charge velocity at \( t = 0 \) is \( \mathbf{v}(0) = (2 \mathbf{u}_x - 3 \mathbf{u}_y - 4 \mathbf{u}_z) \times 10^5 \) m/s, the acceleration of charge at \( t = 0 \) is
    (A) \(600[3 \mathbf{u}_x + 2 \mathbf{u}_y - 3 \mathbf{u}_z] \times 10^5 \) m/s^2
    (B) \(400[6 \mathbf{u}_x + 6 \mathbf{u}_y - 3 \mathbf{u}_z] \times 10^5 \) m/s^2
    (C) \(400[6 \mathbf{u}_x - 6 \mathbf{u}_y + 3 \mathbf{u}_z] \times 10^5 \) m/s^2
    (D) \(800[6 \mathbf{u}_x + 6 \mathbf{u}_y - \mathbf{u}_z] \times 10^5 \) m/s^2

19. An electron is moving at velocity \( \mathbf{v} = 4.5 \times 10^7 \mathbf{u}_y \) m/s along the negative y-axis. At the origin, it encounters the uniform magnetic field \( \mathbf{B} = 2.5 \mathbf{u}_x \) mT, and remains in it up to \( y = 25 \) cm. If we assume that the electron remains on the y-axis while it is in the magnetic field, at \( y = 50 \) cm the x and z coordinate are respectively
    (A) 1.23 m, 0.23 m       (B) \(-1.23 \) m, \(-0.23 \) m
    (C) \(-11.7 \) cm, 0       (D) 11.7 cm, 0
Statement for Q.20–22:

A rectangular loop of wire in free space joins points A(1, 0, 1) to B(3, 0, 1) to C(3, 0, 4) to D(1, 0, 4) to A. The wire carries a current of 6 mA flowing in the $\mathbf{u}_x$ direction from B to C. A filamentary current of 15 A flows along the entire $z$ axis in the $\mathbf{u}_z$ direction.

20. The force on side BC is
(A) $-18 \mathbf{u}_x$ nN  
(B) $18 \mathbf{u}_x$ nN  
(C) $3.6 \mathbf{u}_x$ nN  
(D) $-3.6 \mathbf{u}_x$ nN

21. The force on side AB is
(A) $23.4 \mathbf{u}_y$ $\mu$N  
(B) $16.4 \mathbf{u}_y$ $\mu$N  
(C) $19.8 \mathbf{u}_x$ nN  
(D) $26.3 \mathbf{u}_x$ nN

22. The total force on the loop is
(A) $36 \mathbf{u}_x$ nN  
(B) $-36 \mathbf{u}_x$ nN  
(C) $54 \mathbf{u}_x$ nN  
(D) $-54 \mathbf{u}_x$ nN

23. Consider the rectangular loop on $z=0$ plane shown in Fig. P8.3.23. The magnetic flux density is $\mathbf{B} = 6 \mathbf{u}_x - 9 \mathbf{u}_y + 3 \mathbf{u}_z$ Wb/m$^2$. The total force experienced by the rectangular loop is

![Fig. P8.3.23](image)

(A) $30 \mathbf{u}_x$ N  
(B) $-30 \mathbf{u}_x$ N  
(C) $36 \mathbf{u}_x$ N  
(D) $-36 \mathbf{u}_x$ N

Statement for Q.24–25:

Three uniform current sheets are located in free space as follows: $8 \mathbf{u}_x$, A/m at $y=0$, $-4 \mathbf{u}_x$, A/m at $y=1$ and $-4 \mathbf{u}_x$, A/m at $y=-1$. Let $\mathbf{F}$ be the vector force per meter length exerted on a current filament carrying 7 mA in the $\mathbf{u}_L$ direction.

24. If the current filament is located at $x=0$, $y=0.5$ and $\mathbf{u}_L = \mathbf{u}_x$, then $\mathbf{F}$ is
(A) $35.2 \mathbf{u}_y$ nN/m  
(B) $-35.2 \mathbf{u}_y$ nN/m  
(C) $105.6 \mathbf{u}_y$ nN/m  
(D) 0

25. If the current filament is located at $y=0.5$, $z=0$, and $\mathbf{u}_L = \mathbf{u}_x$, then $\mathbf{F}$ is
(A) $35.2 \mathbf{u}_y$ nN/m  
(B) $68.3 \mathbf{u}_y$ nN/m  
(C) $105.6 \mathbf{u}_y$ nN/m  
(D) 0

Statement for Q.27–28:

A conducting current strip carrying $\mathbf{K} = 6 \mathbf{u}_x$, A/m lies in the $x=0$ plane between $y=0.5$ and $y=1.5$ m. There is also a current filament of $I=5$ A in the $\mathbf{u}_z$ direction on the $z$–axis.

27. The force exerted on the filament by the current strip is
(A) $12.2 \mathbf{u}_y$ $\mu$N/m  
(B) $6.6 \mathbf{u}_y$ $\mu$N/m  
(C) $-12.2 \mathbf{u}_y$ $\mu$N/m  
(D) $-6.6 \mathbf{u}_y$ $\mu$N/m

28. The force exerted on the strip by the filament is
(A) $-6.6 \mathbf{u}_y$ $\mu$N/m  
(B) $6.6 \mathbf{u}_y$ $\mu$N/m  
(C) $2.4 \mathbf{u}_y$ $\mu$N/m  
(D) $-2.4 \mathbf{u}_y$ $\mu$N/m

Statement for Q.29–32:

In a certain material for which $\mu_r = 6.5$,

$$\mathbf{H} = 10 \mathbf{u}_x + 25 \mathbf{u}_y - 40 \mathbf{u}_z$$, A/m

29. The magnetic susceptibility $\chi_m$ of the material is
(A) 5.5  
(B) 6.5  
(C) 7.5  
(D) None of the above

30. The magnetic flux density $\mathbf{B}$ is
(A) $82 \mathbf{u}_x + 204 \mathbf{u}_y - 327 \mathbf{u}_z$ $\mu$Wb/m$^2$  
(B) $82 \mathbf{u}_x + 204 \mathbf{u}_y - 327 \mathbf{u}_z$ $\mu$A/m  
(C) $82 \mathbf{u}_x + 204 \mathbf{u}_y - 327 \mathbf{u}_z$ mT  
(D) $82 \mathbf{u}_x + 204 \mathbf{u}_y - 327 \mathbf{u}_z$ mA/m

31. The magnetization $\mathbf{M}$ is
(A) $75 \mathbf{u}_x + 187.5 \mathbf{u}_y - 300 \mathbf{u}_z$ A/m$^2$  
(B) $75 \mathbf{u}_x + 187.5 \mathbf{u}_y - 300 \mathbf{u}_z$ A/m$^2$  
(C) $55 \mathbf{u}_x + 137.5 \mathbf{u}_y - 220 \mathbf{u}_z$ A/m$^2$  
(D) $55 \mathbf{u}_x + 137.5 \mathbf{u}_y - 220 \mathbf{u}_z$ A/m$^2$
32. The magnetic energy density is
(A) 19 mJ/m²  (B) 9.5 mJ/m²  (C) 16.3 mJ/m²  (D) 32.6 mJ/m²

Statement for Q.33–34:
For a given material magnetic susceptibility $\chi_m = 3.1$ and within which $B = 0.4 \mathbf{y}_z$, T.

33. The magnetic field $\mathbf{H}$ is
(A) 986.8$\mathbf{y}_z$ kA/m  (B) 151.6$\mathbf{y}_z$ kA/m  (C) 102.7$\mathbf{y}_z$ kA/m  (D) 77.6$\mathbf{y}_z$ kA/m

34. The magnetization $\mathbf{M}$ is
(A) 241$\mathbf{y}_z$ kA/m  (B) 318.2$\mathbf{y}_z$ kA/m  (C) 163$\mathbf{y}_z$ kA/m  (D) None of the above

35. In a material the magnetic field intensity is $H = 1200$ A/m when $B = 2$ Wb/m². When $H$ is reduced to 400 A/m, $B = 14$ Wb/m². The change in the magnetization $\mathbf{M}$ is
(A) 164 kA/m  (B) 326 kA/m  (C) 476 kA/m  (D) 238 kA/m

36. A particular material has $2.7 \times 10^{29}$ atoms/m³ and each atom has a dipole moment of $2.6 \times 10^{-36}$ $\mathbf{u}_z$ A·m². The $\mathbf{H}$ in material is ($\mu_r = 42$)
(A) 2.94$\mathbf{u}_z$ A/m  (B) 0.22$\mathbf{u}_z$ A/m  (C) 0.17$\mathbf{u}_z$ A/m  (D) 2.24$\mathbf{u}_z$ A/m

37. In a material magnetic flux density is 0.02 Wb/m² and the magnetic susceptibility is 0.003. The magnitude of the magnetization is
(A) 47.6 A/m  (B) 23.4 A/m  (C) 16.3 A/m  (D) 8.4 A/m

38. A uniform field $\mathbf{H} = -600\mathbf{u}_z$, A/m exist in free space. The total energy stored in spherical region 1 cm in radius centered at the origin in free space is
(A) 0.226 J/m³  (B) 1.452 J/m³  (C) 1.68 J/m³  (D) 0.84 J/m³

39. The magnetization curve for an iron alloy is approximately given by
$$B = \frac{1}{3} H + H^2 \mu \text{Wb/m}^2$$

If $H$ increases from 0 to 210 A/m, the energy stored per unit volume in the alloy is
(A) 6.2 MJ/m³  (B) 1.3 MJ/m³  (C) 2.3 kJ/m³  (D) 2.9 kJ/m³

40. If magnetization is given by $\mathbf{H} = \frac{\mathbf{z}}{a}(-\mathbf{y}_z + \mathbf{x}_z)$ in a cube of size $a$, the magnetization volume current density is
(A) $\frac{12}{a} \mathbf{y}_z$  (B) $\frac{6}{a} (x - y)$  (C) $\frac{6}{a} \mathbf{z}$  (D) $\frac{3}{a} (x - y)$

41. The point P(2, 3, 1) lies on the planner boundary separating region 1 from region 2. The unit vector $\mathbf{N} = \mathbf{y}_z$ is directed from region 1 to region 2. If $\mathbf{H}_1 = 100\mathbf{u}_z - 300\mathbf{u}_y + 200\mathbf{u}_x$ A/m, then $\mathbf{H}_2$ is
(A) 40.3$\mathbf{u}_x + 48.3\mathbf{u}_y - 178.9\mathbf{u}_z$, A/m  (B) 80.2$\mathbf{u}_z - 315.8\mathbf{u}_x + 178.9\mathbf{u}_z$, A/m
(C) 40.3$\mathbf{u}_z - 315.8\mathbf{u}_y - 178.9\mathbf{u}_z$, A/m  (D) 80.2$\mathbf{u}_z + 48.3\mathbf{u}_y + 178.9\mathbf{u}_z$, A/m

42. The plane separates air ($z > 0$, $\mu_r = 1$) from iron ($z < 0$, $\mu_r = 20$). In air magnetic field intensity is $\mathbf{H} = 10\mathbf{u}_z + 15\mathbf{u}_y - 3\mathbf{u}_x$ A/m. The magnetic flux density in iron is
(A) 5.02$\mathbf{u}_z + 7.5\mathbf{u}_y - 0.076\mathbf{u}_x$ mWb/m²  (B) 12.6$\mathbf{u}_z + 18.9\mathbf{u}_y - 75.4\mathbf{u}_x$ $\mu$Wb/m²
(C) 251$\mathbf{u}_z + 377\mathbf{u}_y - 377\mathbf{u}_x$ $\mu$Wb/m²  (D) 251$\mathbf{u}_z + 377\mathbf{u}_y + 1508\mathbf{u}_x$ $\mu$Wb/m²

43. The plane $2x + 3y - 4z = 1$ separates two regions. Let $\mu_{r1} = 2$ in region 1 defined by $2x + 3y - 4z > 1$, while $\mu_{r2} = 5$ in region 2 where $2x + 3y - 4z < 1$. In region $\mathbf{H}_1 = 50\mathbf{u}_z - 30\mathbf{u}_y + 20\mathbf{u}_x$, A/m. In region 2, $\mathbf{H}_2$ will be
(A) 63.4$\mathbf{u}_z + 43.18\mathbf{u}_y - 19.4\mathbf{u}_x$, A/m  (B) 529$\mathbf{u}_z - 25.66\mathbf{u}_y + 14.2\mathbf{u}_x$, A/m
(C) 48.6$\mathbf{u}_z - 16.4\mathbf{u}_y - 46.3\mathbf{u}_x$, A/m  (D) None of the above
1. (C) \( \mathbf{H} = \iint_{\Sigma} \frac{\mathbf{I} d\mathbf{L} \times \mathbf{u}_R}{4\pi R^2} \)
\[ = \frac{I}{4\pi} \int_{\Sigma} \mathbf{u}_R d\mathbf{L} \times \mathbf{u}_R \]
\[ = \frac{I}{4\pi} \int_{\Sigma} \frac{-\mathbf{u}_R d\mathbf{L} [2\mathbf{u}_R + (3 - y)\mathbf{u}_y]}{[2^2 + (3 - y)^2]^{3/2}} \]
\[ = \frac{I}{4\pi} \int_{\Sigma} \frac{2\mathbf{u}_y d\mathbf{L}}{[2^2 + (3 - y)^2]^{3/2}} \]
\[ = 3 - y = 2\tan \theta,\quad -dy = 2\sec^2 \theta, \]
\[ \theta_1 = 56.31^\circ, \quad \theta_2 = -90^\circ \]
\[ \mathbf{H} = \frac{I}{4\pi} \int_{\Sigma} \frac{2\mathbf{u}_y d\theta}{2\sec \theta} \]
\[ = \frac{I}{4\pi} \mathbf{u}_y [\sin \theta]^{56.31/2} = 145.8 \mathbf{u}_y \text{ mA/m} \]

2. (A) \( \mathbf{H} = \mathbf{H}_y + \mathbf{H}_z \)
\( \rho = \sqrt{(-3)^2 + (4)^2} = 5 \)
\( \mathbf{u}_s = \frac{-\mathbf{u}_x \times (-3\mathbf{u}_y + 4\mathbf{u}_z)}{5} = 3\mathbf{u}_x + 4\mathbf{u}_z \)
\( \mathbf{H}_y = \frac{24}{2\pi(5)} \frac{(4\mathbf{u}_z + 3\mathbf{u}_x)}{5} = 0.611\mathbf{u}_z + 0.458\mathbf{u}_y \text{ mA/m} \)
\( \mathbf{H}_z = \frac{I}{2\pi \rho} \mathbf{u}_s, \quad \rho = \sqrt{(-3)^2 + (5)^2} = \sqrt{34} \)
\( \mathbf{u}_s = \frac{\mathbf{u}_x \times (-3\mathbf{u}_y + 5\mathbf{u}_z)}{\sqrt{34}} = 3\mathbf{u}_x - 5\mathbf{u}_z \)
\( \mathbf{H}_y = \frac{12}{2\pi \sqrt{34}} \frac{(-5\mathbf{u}_x + 3\mathbf{u}_z)}{\sqrt{34}} \)
\[ = -0.281\mathbf{u}_x + 0.168\mathbf{u}_y \text{ mA/m} \]
\[ \mathbf{H} = \mathbf{H}_y + \mathbf{H}_z = 0.331\mathbf{u}_x + 0.458\mathbf{u}_y + 0.168\mathbf{u}_z \text{ mA/m} \]

3. (A) \( \mathbf{H} = \frac{I}{2\pi \rho} \mathbf{u}_x - \frac{I}{4\pi} \int_{\Sigma} \frac{\mathbf{d}z \times (\rho \mathbf{u}_s - z\mathbf{u}_y)}{4\pi(\rho^2 + z^2)^{3/2}} \)
\[ = \frac{I}{2\pi \rho} \mathbf{u}_x - \frac{I}{4\pi} \int_{\Sigma} \frac{\rho d\mathbf{z}}{4\pi(\rho^2 + z^2)^{3/2}} \mathbf{u}_s \]
\[ = \frac{I}{2\pi \rho} \mathbf{u}_x - \frac{I}{4\pi} \frac{8}{\rho \sqrt{\rho^2 + 16}} \mathbf{u}_s \]
\[ = \frac{I}{2\pi \rho} \left( 1 - \frac{4}{\sqrt{\rho^2 + 16}} \right) \mathbf{u}_s \]
At \( \rho = -3, \quad \phi = 60^\circ, \quad I = 3\pi, \)
\[ \mathbf{H} = 0.1\mathbf{u}_x \text{ A/m} \]

4. (B) \( \mathbf{H} = \int \frac{IdL \times \mathbf{u}_e}{4\pi R^2}, \quad IdL = 4dx \mathbf{u}_e \)

5. (B) \( \mathbf{H} = \int \frac{I}{2\pi \rho} \mathbf{u}_s - \int_{\Sigma} \frac{\mathbf{d}z \times (\rho \mathbf{u}_s - z\mathbf{u}_y)}{4\pi(\rho^2 + z^2)^{3/2}} \)
\[ = \frac{I}{2\pi \rho} \mathbf{u}_s - \frac{I}{4\pi} \frac{\rho d\mathbf{z}}{4\pi(\rho^2 + z^2)^{3/2}} \mathbf{u}_s \]
\[ = \frac{I}{2\pi \rho} \left( 1 - \frac{a}{\sqrt{\rho^2 + a^2}} \right) \mathbf{u}_s \text{ A/m} \]

At \( \rho = 1, \quad \mathbf{H} = \frac{I}{2\pi \rho} \)
\[ = 1 - \frac{a}{\sqrt{1 + a^2}} = \frac{1}{2} \]
\[ \Rightarrow \quad a = \frac{1}{\sqrt{3}} = 0.577 \text{ m} \]

6. (A) \( \mathbf{H} = \int \frac{IdL \times \mathbf{u}_e}{4\pi R^2} \)
\[ = \frac{2\pi}{4\pi} \int_{\Sigma} \frac{dx dy}{4\pi(\rho^2 + z^2)^{3/2}} \mathbf{u}_e \]
\[ = \frac{I}{2\pi} \mathbf{u}_e \text{ A/m} \]
\[ = 3 \mathbf{u}_y, \quad a = 0.5 \text{ m}, \quad \mathbf{H} = 3\mathbf{u}_y \text{ A/m} \]

7. (D) \( \mathbf{H} = \iiint \frac{K \times \mathbf{u}_e dx dy}{4\pi R^2} \)
\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx dy}{4\pi(\rho^2 + y^2)^{3/2}} \mathbf{u}_e \]
\[ = \frac{12}{4\pi} \frac{\mathbf{u}_e dx dy}{(x^2 + y^2 + 9)^{3/2}} \]
\[ = \frac{12}{4\pi} \frac{\mathbf{u}_e dx dy}{(x^2 + y^2 + 9)^{3/2}} \]
\[ = \frac{3}{\pi} \mathbf{u}_e \int_{-\infty}^{\infty} \frac{2dy}{y^2 + 9} \]
\[ = \frac{6}{\pi} \mathbf{u}_e \int_{-\infty}^{\infty} \tan^{-1} \left( \frac{y}{3} \right)^2 \]
\[ = -0.75 \mathbf{u}_e \text{ A/m} \]
8. (D) \( \mathbf{H} = \frac{1}{4\pi R^2} \int \mathbf{u}_a \times \mathbf{u}_b \times \mathbf{u}_c \) 
\[ = \frac{1}{4\pi} \int \mathbf{x} \mathbf{u}_a \times (-\mathbf{u}_a + \mathbf{u}_b) + \mathbf{x} \mathbf{u}_b \times (-\mathbf{u}_b + \mathbf{u}_c) \]
\[ = \frac{1}{4\pi} \int \mathbf{x} \mathbf{u}_a \times \mathbf{u}_c \]
\[ = \frac{I}{4\pi} \left[ \mathbf{z} \mathbf{u}_a + \mathbf{x} \mathbf{u}_c \right] = 0.8(\mathbf{u}_a + \mathbf{u}_c) \text{ mA/m} \]

9. (A) Using Ampere's circuital law
\[ \oint \mathbf{H} \cdot d\mathbf{L} = 2\pi \rho H_x = I_{\text{enclosed}} \]
At \( \rho = 0.5 \text{ cm} \), \( I_{\text{enclosed}} = 10 \text{ mA} \)

2\( \pi \times 10^{-5} \) \( H_x = 10 \), \( H_y = 0.32 \text{ A/m} \)

10. (B) At \( \rho = 15 \text{ cm} \) enclosed current
\[ I_{\text{enclosed}} = 10 + 2\pi(0.01)(400) = 35.13 \text{ mA} \]
\[ 2\pi(0.015) H_y = 35.13 \times 10^{-3} \Rightarrow H_y = 0.37 \text{ A/m} \]

11. (C) The enclosed current is
\[ I_{\text{enclosed}} = 10 + 2\pi(0.01)(400) - 2\pi(0.02)(250) + 2\pi(0.03)(300) \]
\[ = 60.3 \text{ mA/m} \]
\[ 2\pi(0.035) H_y = 60.3 \text{ M} \Rightarrow H_y = 0.27 \text{ A/m} \]

12. (A) \( \mathbf{H} = \frac{1}{2}(-30 - 40) \mathbf{u}_a \times (-\mathbf{u}_b) = -35 \mathbf{u}_a \text{ A/m} \)

13. (B) \( \mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{u}_n \)
\[ = \frac{1}{2} (30 - 40) \mathbf{u}_a \times (-\mathbf{u}_b) = -5 \mathbf{u}_a \text{ A/m} \]

14. (A) \( \mathbf{H} = \frac{1}{2}(-30 + 40) \mathbf{u}_a + (-\mathbf{u}_b) = 5 \mathbf{u}_a \text{ A/m} \)

15. (C) \( \mathbf{J} = \nabla \times \mathbf{H} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( 2 + \frac{\rho^2}{2} \right) \mathbf{u}_a = \frac{3}{\rho} \mathbf{u}_a \text{ A/m} \)

16. (D) \( \mathbf{J} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \frac{3}{\rho} \right) = 0 \)

17. (A) \( \mathbf{F} = \varepsilon (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \)
If \( \mathbf{F} = 0 \), \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} = \mathbf{B} \times \mathbf{v} \)
22. (A) This will be the vector sum of the forces on the four sides. By symmetry, the forces on sides AB and CD will be equal and opposite, and so will cancel. This leaves the sum of forces on side BC and DA
\[ F_{DA} = \int \left(-6 \times 10^{-3}\right) dx \mathbf{u}_x \times \frac{15 \mu_0}{2\pi(1)} \mathbf{u}_y = 54 \mathbf{u}_y \text{ nN} \]
\[ F_{total} = F_{DA} + F_{BC} = (54 - 18)\mathbf{u}_y = 36 \mathbf{u}_y \text{ nN} \]

23. (A) \[ F = \int \frac{I dL \times R}{\mu_0} \]
\[ = \frac{1}{2} \int dx \mathbf{u}_x \times B + \frac{1}{2} \int dy \mathbf{u}_y \times B + \frac{1}{2} \int dx \mathbf{u}_x \times B + \frac{1}{2} \int dy \mathbf{u}_y \times B \]
\[ \mathbf{u}_x \times B = \mathbf{u}_z \left[ 6 \mathbf{u}_x - 9 \mathbf{u}_y + 3 \mathbf{u}_z \right] = 3 \mathbf{u}_y - 6 \mathbf{u}_x \]
\[ \mathbf{u}_y \times B = \mathbf{u}_z \left[ 6 \mathbf{u}_x - 9 \mathbf{u}_y + 3 \mathbf{u}_z \right] = 3 \mathbf{u}_y - 6 \mathbf{u}_x \]
\[ z = 0 \text{ for all element} \]
\[ F = \left( \frac{1}{2} \int dx \left(-9 \mathbf{u}_y\right)_{x=0} + \frac{1}{2} \int dy \left(-6 \mathbf{u}_x\right)_{y=0} \right) \]
\[ = I \left(-18 + 36 + 6\right) \mathbf{u}_y = 5 \times 6 \mathbf{u}_y = 30 \mathbf{u}_y \text{ N} \]

24. (B) Within the region \(-1 < y < 1\), the magnetic fields from the two outer sheets (carrying \(-4 \mathbf{u}_y\)) cancel, leaving only the field from the center sheet. Therefore \(H = -4 \mathbf{u}_y \text{ A/m} \) (0 < y < 1) and \(H = 4 \mathbf{u}_y \text{ A/m} \) (-1 < y < 0). Outside (y > 1 or y < -1) the fields from all three sheet cancel, leaving \(H = 0 \text{ for } y > 1, y < -1\). So at \(x = 0, y = 0.5\)
\[ \frac{F}{m} = \int dx \mathbf{u}_y \times B = (7 \times 10^{-3}) \mathbf{u}_y - 4 \mu_0 \mathbf{u}_x = -352 \mathbf{u}_y \text{ nN/m} \]

25. (D) \[ \frac{F}{m} = \int dx \mathbf{u}_x \times (-4 \mu_0 \mathbf{u}_y) = 0 \]

26. (A) \[ F = \frac{1}{2} \int IdL \times B \]
\[ = \frac{1}{2} \int 100 dx \mathbf{u}_x \times \frac{-100 \mu_0}{2\pi(5 \times 10^{-3})} \mathbf{u}_y = 0.4 \mathbf{u}_y \text{ N/m} \]

27. (B) The field from the current strip at the filament location
\[ B = \frac{1}{2} \int \frac{6 \mu_0}{2\piy} dy = \frac{3 \mu_0}{2\pi} \ln \left( \frac{1.5}{0.5} \right) \mathbf{u}_z \]
\[ = 1.32 \times 10^{-6} \mathbf{u}_z \text{ Wb/m}^2 \]
\[ F = \int IdL \times B \]
\[ = \frac{1}{2} \int 5dz \mathbf{u}_z \times 1.32 \times 10^{-6} \mathbf{u}_z dz = 6.6 \mathbf{u}_z \mu\text{N/m} \]

28. (A) \( F_\text{total} = \iiint B \cdot dS = \frac{1}{2\pi} \mathbf{u}_y \times \frac{-5 \mu_0}{2\pi} \mathbf{u}_z \)
\[ = -\frac{15 \mu_0}{\pi} \ln \left( \frac{1.5}{0.5} \right) \mathbf{u}_y = -6.6 \mathbf{u}_y \mu\text{N/m} \]

29. (A) \( \chi_m + 1 = \mu_r, \chi_m + 1 = 65, \chi_m = 55 \)

30. (A) \( B = \mu H = \mu_0 \mu_r H \)
\[ = 4 \pi \times 10^{-7} \times 6.5(10 \mathbf{u}_x + 25 \mathbf{u}_y - 40 \mathbf{u}_z) \]
\[ = 82 \mathbf{u}_x + 204 \mathbf{u}_y - 327 \mathbf{u}_z \mu\text{Wb/m}^2 \]

31. (C) \( M = \chi_m H = 55 \mathbf{u}_x + 137.5 \mathbf{u}_y - 220 \mathbf{u}_z \text{ A/m} \)

32. (B) \( W = \frac{1}{2} \mathbf{H} \cdot \mathbf{B} = \frac{1}{2} \mu H^2 \)

33. (D) \( \mu_r = \chi_m + 1 = 31 + 1 = 41, \mu = \mu_0 \mu_r = 4.1 \mu_0 \)
\[ \mathbf{H} = \frac{4.4 \mathbf{u}_y}{4.1 \times 4 \pi \times 10^{-7}} = 77.6 \mathbf{u}_y \text{ kA/m} \]

34. (A) \( \mathbf{M} = \chi_m \mathbf{H} = (31)(77.6) \mathbf{u}_y = 241 \mathbf{u}_y \text{ kA/m} \)

35. (C) For case 1, \( \mu = \frac{B_1}{H_1} = \frac{2}{1200} \)
\[ \mu_{s1} = \frac{\mu}{\mu_0} = \frac{1}{600} = \frac{1}{4 \pi \times 10^{-7}} = 1326.3 \]
\[ \chi_m = \mu_r - 1 = 1325.3 \]
\[ M_1 = \chi_m H_1 = 1590 \times 10^6 \text{ A/m} \]
For case 2, \( \mu = \frac{B_2}{H_2} = \frac{1.4}{400} \)
\[ \mu_{s2} = \frac{1.4}{400} = \frac{1.4}{4 \pi \times 10^{-7}} = 27852 \]
\[ \chi_m = 27842 \]
\[ M_2 = \chi_m H_2 = 1.114 \times 10^6 \text{ A/m} \]
\[ \Delta M = (1590 - 1.114) \times 10^6 = 476 \text{ kA/m} \]

36. (B) \( \mathbf{M} = \mathbf{N} m = (2.7 \times 10^{20})(2.6 \times 10^{-30}) \mathbf{u}_y \)
\[ = 0.7 \mathbf{u}_y \text{ A/m} \]
\[ \mathbf{H} = \frac{\mathbf{M}}{\mu_r - 1} = \frac{0.7 \mathbf{u}_y}{42 - 1} = 0.022 \mathbf{u}_y \text{ A/m} \]

37. (A) \( \mathbf{M} = \frac{B}{\mu_0} \left( \frac{1}{\chi_m + 1} \right)^2 \)

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38. (A) \( W = \frac{1}{2} \mathbf{H} \cdot \mathbf{B} = \frac{1}{2} \mu_n H^2 \)
\[
= \frac{1}{2} (4\pi \times 10^{-7})(600)^2 = 0.226 \text{ J/m}^3
\]

39. (A) \( W = \int_0^\infty \mathbf{H} \cdot dB = \int_0^\infty \left( \frac{1}{3} + 2H \right) dH \)
\[
= \frac{H^2}{6} + \frac{2H^3}{3} = 6.2 \text{ MJ/m}^3
\]

40. (A) \( \mathbf{J}_n = \nabla \times \mathbf{M} = \frac{12}{a} \mathbf{u}_z \)

41. (B) \( \mathbf{B}_1 = 200\mu_n \mathbf{u}_x - 600\mu_n \mathbf{u}_y + 400\mu_n \mathbf{u}_z \)
Its normal component at the boundary is
\[
\mathbf{B}_{1N} = (\mathbf{B}_1 \cdot \mathbf{u}_{N12}) \mathbf{u}_{N12} = (52.8 \mathbf{u}_x + 42.24 \mathbf{u}_y + 56.32 \mathbf{u}_z) \mu_n = \mathbf{B}_{2N}
\]
\[
\Rightarrow \quad \mathbf{H}_{2N} = \frac{\mathbf{B}_{2N}}{\mu_n} = 6.60 \mathbf{u}_x + 5.28 \mathbf{u}_y + 7.04 \mathbf{u}_z
\]
\[
\begin{align*}
\mathbf{H}_{1N} &= \frac{\mathbf{B}_{1N}}{12\mu_n} = 26.40 \mathbf{u}_x + 21.12 \mathbf{u}_y + 28.16 \mathbf{u}_z \\
\mathbf{H}_{1T} &= \mathbf{H}_1 - \mathbf{H}_{1N} = (100 \mathbf{u}_x - 300 \mathbf{u}_y + 200 \mathbf{u}_z) - (26.40 \mathbf{u}_x + 21.12 \mathbf{u}_y + 28.16 \mathbf{u}_z) \\
&= 73.6 \mathbf{u}_x - 321.2 \mathbf{u}_y + 171.84 \mathbf{u}_z \\
\mathbf{H}_{1T} &= \mathbf{H}_{2T} \quad \mathbf{H}_2 = \mathbf{H}_{2N} + \mathbf{H}_{2T} = 80.2 \mathbf{u}_x - 315.8 \mathbf{u}_y + 178.9 \mathbf{u}_z \text{ A/m}
\end{align*}
\]

42. (C) \( \mathbf{H}_{N1} = -3 \mathbf{u}_x, \quad \mathbf{H}_{T1} = 10 \mathbf{u}_x + 15 \mathbf{u}_y \)
\[
\mathbf{H}_{T2} = \mathbf{H}_{T1} = 10 \mathbf{u}_x + 15 \mathbf{u}_y \\
\mathbf{H}_{N2} = \frac{\mu_1}{\mu_2} \mathbf{H}_{N1} = \frac{1}{20} (-3 \mathbf{u}_x) = 0.15 \mathbf{u}_x \\
\mathbf{H}_2 = \mathbf{H}_{N2} + \mathbf{H}_{T2} = 10 \mathbf{u}_x + 15 \mathbf{u}_y - 0.15 \mathbf{u}_x \\
\mathbf{B}_2 = \mu_0 \mathbf{H}_2 = 20 \times 4\pi \times 10^{-7} (10 \mathbf{u}_x + 15 \mathbf{u}_y - 0.15 \mathbf{u}_x) \\
&= 25 \mathbf{u}_x + 377 \mathbf{u}_y - 377 \mathbf{u}_x \mu \text{Wb/m}^2
\]

43. (B) At the boundary normal unit vector
\[
\mathbf{u}_n = \frac{\nabla (2x + 3y - 4z)}{|\nabla (2x + 3y - 4z)|} = \frac{2\mathbf{u}_x + 3\mathbf{u}_y - 4\mathbf{u}_z}{\sqrt{29}}
\]
\[
= 0.37 \mathbf{u}_x + 0.56 \mathbf{u}_y - 0.74 \mathbf{u}_z
\]
Since this vector is found through the gradient, it will point in the direction of increasing values of \(2x + 3y - 4z\), and so will be directed into region 1. Thus
\[
\mathbf{u}_n = \mathbf{u}_{n21} \cdot
\]

The normal component of \( \mathbf{H}_1 \) is
\[
\mathbf{H}_{N1} = (\mathbf{H}_1 \cdot \mathbf{u}_{N21}) \mathbf{u}_{N21} = (50 \mathbf{u}_x - 30 \mathbf{u}_y + 20 \mathbf{u}_z) \cdot (0.37 \mathbf{u}_x + 0.56 \mathbf{u}_y - 0.74 \mathbf{u}_z) \\
= 18.5 - 16.8 - 14.8 = -131
\]
\[
(\mathbf{H}_1 \cdot \mathbf{u}_{N21}) \mathbf{u}_{N21} = (-131)(0.37 \mathbf{u}_x + 0.56 \mathbf{u}_y - 0.74 \mathbf{u}_z) \\
= -4.83 \mathbf{u}_x - 7.24 \mathbf{u}_y + 9.66 \mathbf{u}_z \text{ A/m}
\]

Tangential component of \( \mathbf{H}_1 \) at the boundary
\[
\mathbf{H}_{T1} = \mathbf{H}_1 - \mathbf{H}_{N1} = (50 \mathbf{u}_x - 30 \mathbf{u}_y + 20 \mathbf{u}_z) - (-4.83 \mathbf{u}_x - 7.24 \mathbf{u}_y + 9.66 \mathbf{u}_z) \\
= 54.83 \mathbf{u}_x - 22.76 \mathbf{u}_y + 10.34 \mathbf{u}_z \text{ A/m}
\]
\[
\mathbf{H}_{T2} = \mathbf{H}_{T1} \\
\mathbf{H}_{N2} = \frac{\mu_1}{\mu_2} \mathbf{H}_{N1} = \frac{2}{5} (-4.83 \mathbf{u}_x - 7.24 \mathbf{u}_y + 9.66 \mathbf{u}_z) \\
= -193 \mathbf{u}_x - 290 \mathbf{u}_y + 386 \mathbf{u}_z \text{ A/m}
\]
\[
\mathbf{H}_2 = \mathbf{H}_{T2} + \mathbf{H}_{N2} = (54.83 \mathbf{u}_x - 22.76 \mathbf{u}_y + 10.34 \mathbf{u}_z) + (-193 \mathbf{u}_x - 29 \mathbf{u}_y + 386 \mathbf{u}_z) \\
= 529 \mathbf{u}_x - 25.66 \mathbf{u}_y + 142 \mathbf{u}_z
\]

**********
Statement for Q.8–9:

The location of the sliding bar in fig. P8.4.8–9 is given by \( x = 5t + 4t^2 \). The separation of the two rails is 30 cm. Let \( B = x^2 \mathbf{u}_z \) T.

\[ x = 5t + 4t^2 \]

3. The voltmeter reading at \( t = 0.5 \) s is
   (A) -21.6 V  
   (B) 21.6 V  
   (C) -6.3 V  
   (D) 6.3 V

4. The voltmeter reading at \( x = 0.6 \) m is
   (A) -1.68 V  
   (B) 1.68 V  
   (C) -0.933 V  
   (D) 0.933 V

Statement for Q.10–11:

A perfectly conducting filament containing a 250\( \Omega \) resistor is formed into a square as shown in fig. P8.4.10-11.

\[ I(t) \]

If \( B = 6 \cos (120\pi t - 30^\circ) \mathbf{u}_z \) T, then the value of \( I(t) \) is
   (A) \( 2.26 \sin (120\pi t - 30^\circ) \) A  
   (B) \( 2.26 \cos (120\pi t - 30^\circ) \) A  
   (C) \(-2.26 \sin (120\pi t - 30^\circ) \) A  
   (D) \(-2.26 \cos (120\pi t - 30^\circ) \) A

If \( B = 2 \cos \pi \nu (t - y) \mathbf{u}_z, \mu T \), where \( \nu \) is the velocity of light, then \( I(t) \) is
   (A) \( 1.2(\cos \pi \nu t - \sin \pi \nu t) \mu A \)  
   (B) \( 1.2(\cos \pi \nu t - \sin \pi \nu t) \mu A \)  
   (C) \( 1.2(\sin \pi \nu t - \sin \pi \nu t) \mu A \)  
   (D) \( 1.2(\sin \pi \nu t - \sin \pi \nu t) \mu A \)

Statement for Q.12–13:

Consider the fig. P8.4.12–13. The rails have a resistance of 2 \( \Omega /m \). The bar moves to the right at a constant speed of 9 m/s in a uniform magnetic field of 0.8 T. The bar is at \( x = 2 \) m at \( t = 0 \).

\[ B \]

If 6 \( \Omega \) resistor is present across the left-end with the right end open-circuited, then at \( t = 0.5 \) sec the current \( I \) is
   (A) -45 mA  
   (B) 45 mA  
   (C) -60 mA  
   (D) 60 mA

If 6 \( \Omega \) resistor is present across each end, then \( I \) at 0.5 sec is
   (A) -12.3 mA  
   (B) 12.3 mA  
   (C) -7.77 mA  
   (D) 77.7 mA

Statement for Q.14–15:

The internal dimension of a coaxial capacitor is \( a = 1.2 \) cm, \( b = 4 \) cm and \( c = 40 \) cm. The homogeneous material inside the capacitor has the parameter \( \varepsilon = 10^{-11} \) F/m, \( \mu = 10^{-5} \) H/m and \( \sigma = 10^{-5} \) S/m. The electric field intensity is \( E = \frac{\omega}{r} \cos (10^5 t) \mathbf{u}_r, V/m \).

\[ E = \frac{\omega}{r} \cos (10^5 t) \mathbf{u}_r \]

The current density \( J \) is
   (A) \( \frac{200}{\rho} \sin (10^5 t) \mathbf{u}_r, A/m^2 \)  
   (B) \( \frac{400}{\rho} \sin (10^5 t) \mathbf{u}_r, A/m^2 \)  
   (C) \( \frac{100}{\rho} \cos (10^5 t) \mathbf{u}_r, A/m^2 \)  
   (D) None of the above

The quality factor of the capacitor is
   (A) 0.1  
   (B) 10  
   (C) 0.2  
   (D) 20
16. The following fields exist in charge free regions

\[ P = 60 \sin (\omega t + 10x) \mathbf{u}_x \]
\[ Q = \frac{60}{\pi} \cos (\omega t - 2p) \mathbf{u}_y \]
\[ R = 3p^2 \cot \phi \mathbf{u}_r + \frac{1}{\pi} \cos \phi \mathbf{u}_\phi \]
\[ S = \frac{1}{\pi} \sin \theta \sin (\omega t - 6r) \mathbf{u}_\theta \]

The possible electromagnetic fields are (A) \( P, Q \) (B) \( R, S \) (C) \( P, R \) (D) \( Q, S \)

17. A parallel-plate capacitor with plate area of 5 \( \text{cm}^2 \) and plate separation of 3 mm has a voltage \( 50 \sin (10^5 t) \) \( \text{V} \) applied to its plates. If \( \varepsilon_r = 2 \), the displacement current is (A) \( 148 \cos (10^5 t) \) \( \text{nA} \) (B) \( 261 \cos (10^5 t) \) \( \mu \text{A} \) (C) \( 261 \cos (10^5 t) \) \( \text{nA} \) (D) \( 148 \cos (10^5 t) \) \( \mu \text{A} \)

18. In a coaxial transmission line \( \varepsilon_r = 1 \), the electric field intensity is given by

\[ E = \frac{100}{\rho} \cos (10^5 t - 6z) \mathbf{u}_\rho \text{ V/m} \]

The displacement current density is (A) \( -\frac{100}{\rho} \sin (10^5 t - 6z) \mathbf{u}_\rho \text{ A/m}^2 \) (B) \( \frac{116}{\rho} \sin (10^5 t - 6z) \mathbf{u}_\rho \text{ A/m}^2 \) (C) \( -\frac{9}{\rho} \sin (10^5 t - 6z) \mathbf{u}_\rho \text{ A/m}^2 \) (D) \( -\frac{216}{\rho} \cos (10^5 t - 6z) \mathbf{u}_\rho \text{ A/m}^2 \)

Statement for Q.19–21:

Consider the region defined by \( |x|, |y| \) and \( |z| < 1 \). Let \( \varepsilon = 5 \varepsilon_r, \mu = 4 \mu_s \), and \( \sigma = 0 \) the displacement current density \( J_d = 20 \cos (15 \times 10^3 t - ax) \mathbf{u}_x \mu \text{A/m}^2 \). Assume no DC fields are present.

19. The electric field intensity \( E \) is (A) \( 6 \sin (15 \times 10^3 t - ax) \mathbf{u}_x \text{ mV/m} \) (B) \( 6 \cos (15 \times 10^3 t - ax) \mathbf{u}_y \text{ mV/m} \) (C) \( 3 \cos (15 \times 10^3 t - ax) \mathbf{u}_x \text{ mV/m} \) (D) \( 3 \sin (15 \times 10^3 t - ax) \mathbf{u}_y \text{ mV/m} \)

20. The magnetic field intensity is (A) \( -4a \sin (15 \times 10^3 t - ax) \mathbf{u}_x \mu \text{A/m} \)

21. The value of \( a \) is (A) 4.3 (B) 2.25 (C) 5 (D) 6

Statement for Q.22–23:

Let \( H = 2 \cos (10^3 t - \beta x) \mathbf{u}_z \mu \text{A/m}, \mu = 3 \times 10^{-5} \text{ H/m}, \varepsilon = 1.2 \times 10^{-10} \text{ F/m} \) and \( \sigma = 0 \) everywhere.

22. The electric flux density \( D \) is (A) \( 120 \cos (10^3 t - \beta x) \text{ nC/m}^2 \) (B) \( -120 \cos (10^3 t - \beta x) \text{ nC/m}^2 \) (C) \( 120 \cos (10^3 t + \beta x) \text{ nC/m}^2 \) (D) None of the above

23. The magnetic flux density \( B \) is (A) \( 6.67 \times 10^4 \cos (10^3 t + \beta x) \text{ T} \) (B) \( 6.67 \times 10^4 \cos (10^3 t - \beta x) \text{ T} \) (C) \( 6 \times 10^{-5} \cos (10^3 t + \beta x) \text{ T} \) (D) \( 6 \times 10^{-5} \cos (10^3 t - \beta x) \text{ T} \)

Statement for Q.24–25:

A material has \( \sigma = 0 \) and \( \varepsilon_r = 1 \). The magnetic field intensity is \( H = 4 \cos (10^3 t - 0.01z) \mathbf{u}_y \text{ A/m} \).

24. The electric field intensity \( E \) is (A) \( 4.52 \sin (10^3 t - 0.01z) \text{ kV/m} \) (B) \( 4.52 \sin (10^3 t - 0.01z) \text{ V/m} \) (C) \( 4.52 \cos (10^3 t - 0.01z) \text{ V/m} \) (D) \( 4.52 \cos (10^3 t - 0.01z) \text{ kV/m} \)

25. The value of \( \mu_s \) is (A) 2 (B) 3 (C) 4 (D) 16

26. The surface \( \rho = 3 \) and 10 mm, and \( \varepsilon = 0 \) and 25 cm are perfect conductors. The region enclosed by these surface has \( \mu = 2.5 \times 10^{-6} \text{ H/m}, \varepsilon = 4 \times 10^{-11} \text{ F/m} \) and \( \sigma = 0 \). If \( H = \frac{1}{2} \cos 8 \pi \cos \omega t \mathbf{u}_y \text{ A/m} \), then the value of \( \omega \) is (A) \( 2\pi \times 10^6 \text{ rad/s} \) (B) \( 8\pi \times 10^6 \text{ rad/s} \) (C) \( 2 \pi \times 10^6 \text{ rad/s} \) (D) \( 8 \pi \times 10^6 \text{ rad/s} \)
27. For distilled water $\mu = \mu_\varepsilon = 81 \varepsilon_\varepsilon$, and $\sigma = 2 \times 10^{-3}$ S/m, the ratio of conduction current density to displacement current density at 1 GHz is

(A) $1.11 \times 10^{-3}$  
(B) $4.44 \times 10^{-4}$  
(C) $2.68 \times 10^{-6}$  
(D) $1.68 \times 10^{-7}$

28. A conductor with cross-sectional area of 10 cm$^2$ carries a conductor current $2 \sin(10^8 t)$ mA. If $\sigma = 2.5 \times 10^6$ S/m and $\varepsilon_r = 4.6$, the magnitude of the displacement current density is

(A) $48.4 \mu$A/m$^2$  
(B) $8.11$ nA/m$^2$  
(C) $32.6$ nA/m$^2$  
(D) $16.4$ $\mu$A/m$^2$

29. In a certain region

$$J = (4y\mathbf{u}_x + 2xz\mathbf{u}_y + z^2\mathbf{u}_z) \sin(10^4 t) \, \text{A/m}$$

If volume charge density $\rho_v$ in $z=0$ plane is zero, then $\rho_v$ is

(A) $3z^2 \cos(10^4 t)$ mC/m$^3$  
(B) $0.3z^2 \cos(10^4 t)$ mC/m$^3$  
(C) $-3z^2 \cos(10^4 t)$ mC/m$^3$  
(D) $-0.3z^2 \cos(10^4 t)$ mC/m$^3$

30. In a charge-free region ($\sigma = 0$, $\varepsilon = \varepsilon_\varepsilon$, $\mu = \mu_\mu$) magnetic field intensity is $\mathbf{H} = 10 \cos(10^8t - 4y) \mathbf{u}_z$ A/m. The displacement current density is

(A) $-40 \sin(10^8t - 4y) \mathbf{u}_y$ A/m  
(B) $40 \sin(10^8t - 4y) \mathbf{u}_y$ A/m  
(C) $-40 \sin(10^8t - 4y) \mathbf{u}_x$ A/m  
(D) $40 \sin(10^8t - 4y) \mathbf{u}_x$ A/m

31. In a nonmagnetic medium ($\varepsilon_r = 6.25$) the magnetic field of an EM wave is $\mathbf{H} = 6 \cos \left[ x \cos(10^8t) \right] \mathbf{u}_y$ A/m. The corresponding electric field is

(A) $903 \sin(0.83x) \sin(10^8t)$ V/m  
(B) $903 \sin(12x) \sin(10^8t)$ V/m  
(C) $903 \sin(0.83x) \cos(10^8t)$ V/m  
(D) $903 \sin(12x) \cos(10^8t)$ V/m

32. In a nonmagnetic medium

$$\mathbf{E} = 5 \cos(10^5t - 8x) \mathbf{u}_x + 4 \sin(10^5t - 8x) \mathbf{u}_y \, \text{V/m}.$$  

The dielectric constant of the medium is

(A) 3.39  
(B) 1.84  
(C) 5.76  
(D) 2.4

*************

**SOLUTIONS**

1. (B) $\text{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{S}$

$$= 2\pi(0.2)^2(20)(377) \sin 377t \, \text{mV} = 0.95 \cos 377t \, \text{V}$$

2. (A) $\text{emf} = \frac{1}{2} B \cdot \alpha l^2 = \frac{1}{2} \left( \frac{4}{2} \right) (2)^2 = 16 \, \text{V}$

3. (C) Since B is constant over the loop area, the flux is

$$\Phi = \pi(0.1)^2B = 0.31 \cos(120\pi t)$$

$$\text{emf} = V_{ab}(t) = -\frac{d\Phi}{dt}$$

$$= 0.31(120\pi \sin (120\pi t)) = 118.43 \sin(120\pi t)$$

$$V_{ab} = -118.43 \sin (120\pi t)$$

4. (D) $I = \frac{V_{ab}}{R} = \frac{118.43 \sin (120\pi t)}{250} = 0.47 \sin (120\pi t)$

5. (A) $\text{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int \mathbf{B} \cdot \mathbf{u} \, dz$

$$= \frac{d}{dt} (3)(4)(6) \cos 5000t = -360000 \cos 5000t$$

$$I = \frac{\text{emf}}{R} = \frac{-360000 \sin 5000t}{900 \times 10^2} = -0.4 \sin 5000t \, \text{A}$$

6. (C) $\Phi = \int_0^1 \int_0^{1.2} 20 \mu_\mu \cos(3 \times 10^8 t - y) \, dx \, dy$

$$= [20 \mu_\mu \sin(3 \times 10^8 t - y)]_0^1$$

$$= 20 \mu_\mu [\sin(3 \times 10^8 t - 1) - \sin(3 \times 10^8 t)] \, \text{Wb}$$

$$\text{Emf} = -\frac{d\Phi}{dt}$$

$$= -20 \times (4 \pi \times 10^{-7}) (3 \times 10^8) \times [\cos(3 \times 10^8 t - 1) - \cos(3 \times 10^8 t)]$$

$$= 7540[\cos(3 \times 10^8 t) - \cos(3 \times 10^8 t - 1)] \, \text{V}$$

7. (D) In this case $\Phi = [20 \mu_\mu (2\pi \sin(3 \times 10^8 t - y))]_0^{2\pi} = 0$

8. (A) $\Phi = \oint \mathbf{B} \cdot d\mathbf{S} = \int_0^1 \int_0^{0.3} r^2 \, dx \, dy$

$$= 0.1x^3 = 0.1(5t + 4t^3)^3 \, \text{Wb}$$

$$\text{emf} = -\frac{d\Phi}{dt} = -0.1(3)(5t + 4t^3)^2(5 + 12 t^2)$$

At $t = 0.5$ s, $\text{emf} = -0.1(3)(2.5 + 0.5)^2(5 + 12 \times 0.5^2) = -41.4 \, \text{V}$

9. (C) At $x = 0.6$ m, $0.6 - 5t + 4 \times 10^3 \Rightarrow t = 0.119 \, \text{s}$

At $t = 0.119 \, \text{s}$, $\text{emf} = -0.933 \, \text{V}$
10. (A) \( \Phi = \oint B \cdot dS = 6(0.5)^2 \cos (120\pi t - 30^\circ) \) Wb
\[ \text{emf} = -\frac{d\Phi}{dt} = 6(0.5)^2 (120\pi) \sin (120\pi t - 30^\circ) \]
The current is \( \text{emf} = \frac{(6(0.5)^2 (120\pi)}{250} \sin (120\pi t - 30^\circ) \) A
\[ = 226 \sin (120\pi t - 30^\circ) \) A

11. (D) \( \Phi = \oint B \cdot dS = (0.5)(2) \frac{0.5}{0} \cos (\pi t - \pi ct) \) dy
\[ = \frac{1}{\pi} \left[ \sin \left( \frac{\pi t - \pi ct}{2} \right) - \sin \pi ct \right] = \frac{1}{\pi} \left[ -\cos \pi ct - \sin \pi ct \right] \mu \text{Wb} \]
\[ \text{emf} = -\frac{d\Phi}{dt} = c \cos \pi ct - \sin \pi ct \] \mu V
\[ I(t) = \text{emf} = \frac{3 \times 10^8}{250} \left[ \cos \pi ct - \sin \pi ct \right] \mu A \]
\[ = 12 \cos (\pi ct - \sin \pi ct) \] A

12. (A) The flux in the left-hand closed loop is
\[ \Phi_l = B \times \text{area} = (0.8)(0.2)(2 + 9t) \]
\[ \text{emf}_l = -\frac{d\Phi_l}{dt} = -(0.16)(9) = -1.44 \text{ V} \]
While the bar in motion, the loop resistance is increasing with time,
\[ R_t = 6 + 2[2(2 + 9t)] \Omega, \text{ At } t = 0.5, \ R_t = 32 \Omega \]
\[ I_i = -\frac{1.44}{32} = -45 \text{ mA} \]

13. (C) In this case, there will be contribution to the current from the right loop, which is now closed. The flux in the right loop, whose area decreases with time, is \( \Phi_r = (0.8)(0.2)(16 - 2 - 9t) \)
\[ \text{emf}_r = -\frac{d\Phi_r}{dt} = 1.44 \text{ V} \]
\[ R_r = 6 + 2(2(14 + 9t)), \text{ At } 0.5 s, \ R_r = 44 \Omega \]
The contribution to the current from the right loop
\[ I_r = -\frac{144}{44} = 3.27 \text{ mA} \]
The total current is \(-32.7 - 45 = 77.7 \text{ mA}\)

14. (C) \( \textbf{J} = \sigma \textbf{E} = \frac{100}{\rho} \cos (10^5 t) \) \textbf{u}_x, \text{ A/m}^2

15. (A) Total conduction current
\[ I_c = \oint J \cdot dS = 2\pi \rho J = 2\pi \rho \frac{100}{\rho} \cos (10^5 t) \] \textbf{u}_x, \text{ A/m}^2
\[ = 80\pi \cos (10^5 t) \] A
\[ J_d = \frac{\partial \textbf{D}}{\partial t} = \frac{\partial \textbf{E}}{\partial t} = -\frac{10}{\rho} \sin (10^5 t) \] A/m^2
\[ I_d = 2\pi \rho J_d = -2\pi \rho (10) \sin (10^5 t) = -8\pi \sin (5^5 t) \] A
Quality factor \( \frac{I_d}{|I|} = \frac{8}{80} = 0.1 \)

16. (A) \( \nabla \cdot \textbf{P} = 0, \nabla \times \textbf{P} = -\frac{\partial}{\partial x} \textbf{u}_y \neq 0 \)
\( \textbf{P} \) is a possible EM field
\[ \nabla \cdot \textbf{Q} = 0, \nabla \times \textbf{Q} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (10 \cos (\omega t - 2\rho)) \] \textbf{u}_z \neq 0
\( \textbf{Q} \) is a possible EM field
\[ \nabla \cdot \textbf{R} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \sin \phi \neq 0, \textbf{R} \text{ is not an EM field}. \]
\[ \nabla \cdot \textbf{S} = \frac{1}{r^2} \sin (\omega t - 6r) \frac{\partial}{\partial r} (\sin^2 \phi) \neq 0 \]
\( \textbf{S} \) is not an EM field. Hence (A) is correct.

17. (A) \( D = \varepsilon_0 \varepsilon \frac{V}{d} \Rightarrow J = \frac{dD}{dt} = \frac{\varepsilon}{d} \frac{dV}{dt} \)
\[ I_d = \textbf{J} \cdot \textbf{S} = \frac{\varepsilon S dV}{d} \frac{dt}{dt} = \frac{2\varepsilon S 5 \times 10^{-4}}{3 \times 10^{-3}} 10^5 \cos (10^5 t) \]
\[ = 148 \cos (10^5 t) \text{ nA} \]

18. (C)
\[ J_d = \frac{\partial \textbf{D}}{\partial t} = \frac{\partial \textbf{E}}{\partial t} = \frac{\varepsilon_0}{\varepsilon} \frac{100}{\rho} \left[ -\sin (10^5 t - 6x) \right] 10^5 \textbf{u}_x, \text{ A/m}^2 \]
\[ = \frac{0.9}{\rho} \sin (10^5 t - 6x) \] \textbf{u}_x, \text{ A/m}^2

19. (D) \( \textbf{D} = \oint \textbf{J} dt + C_1 = \frac{20 \times 10^{-6}}{15 \times 10^5} \sin (15 \times 10^5 - ax) \) \textbf{u}_y \]
\[ = 1.33 \times 10^{-13} \sin (15 \times 10^5 t - ax) \] \textbf{u}_y, \text{ C/m}^3 \]
\( C_1 \) is set to zero since no DC fields are present.
\[ \textbf{E} = \frac{\textbf{D}}{\varepsilon} = \frac{133 \times 10^{-13}}{5 \varepsilon_0} \sin (15 \times 10^5 t - ax) \] \textbf{u}_y \]
\[ = 3 \times 10^{-3} \sin (15 \times 10^5 t - ax) \] V/m

20. (D) \( \nabla \times \textbf{E} = -\frac{\partial}{\partial x} \textbf{u}_y \neq -\frac{\partial}{\partial t} \textbf{B} \]
\[ = -a(3 \times 10^{-3}) \cos (15 \times 10^5 t - ax) \] \textbf{u}_z \]
\[ \textbf{B} = \frac{a(3 \times 10^{-3})}{15 \times 10^8} \sin (15 \times 10^5 t - ax) \] \textbf{u}_z \]
\[ \textbf{H} = \frac{\textbf{B}}{\mu} = \frac{2 \times 10^{-11}}{4 \pi \times 10^2 \sin (15 \times 10^5 t - ax) \textbf{u}_x, \text{ A/m}} \]
\[ = 4 \times 10^{-8} \text{a sin (15 \times 10^5 t - ax) u}_x \text{ mA/m} \]

21. (B) \( \nabla \times \textbf{H} = -\frac{\partial}{\partial x} \textbf{u}_y = \textbf{J}_d \)
= a^2(4 \times 10^{-6}) \cos(1.5 \times 10^6 t - ax) = J_d

Comparing the result

\[ a^2 \times 4 \times 10^{-6} = 20 \times 10^{-6}, \quad a = \sqrt{5} = 2.25 \]

22. (B) \( \nabla \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t} \), \( \mathbf{u}_y = \frac{\partial \mathbf{D}}{\partial t} \)

\[ \frac{\partial \mathbf{D}}{\partial t} = 2\beta \sin(10^6 t - \beta x) \mathbf{u}_y \]

\[ \mathbf{D} = -2\beta \frac{10^6}{10^6} \cos(10^6 t - \beta x) \mathbf{u}_y, \quad \text{C/m}^2 \]

\[ \beta = \frac{\omega}{v} = 10^6 \sqrt{\mu_e} = 10^6 \sqrt{3 \times 10^{-5} \times 12 \times 10^{-10}} = 600 \]

\[ \mathbf{D} = -120 \cos(10^6 t - \beta x) \mathbf{u}_y, \quad \text{nC/m}^2 \]

23. (D) \( \mathbf{B} = \mu \mathbf{H} = 6 \times 10^{-5} \cos(10^6 t - \beta x) \mathbf{u}_y \)

24. (A) \( \nabla \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t} \), \( \mathbf{u}_y = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \)

\[ \Rightarrow \nabla \times \mathbf{H} = 0.04 \cos(10^6 t - 0.01z) \mathbf{u}_y = \frac{\epsilon_0}{10^6} \frac{\partial \mathbf{E}}{\partial t} \]

\[ \mathbf{E} = 0.04 \sin(10^6 t - 0.01z) \mathbf{u}_y \quad \text{kV/m} \]

25. (B) \( \nabla \times \mathbf{E} = \frac{\partial \mathbf{H}}{\partial t} \), \( \mathbf{u}_y = -\mu \frac{\partial \mathbf{H}}{\partial t} \)

\[ \frac{-0.04(0.01)}{10^6 \epsilon_0} \cos(10^6 t - 0.01z) \mathbf{u}_y = -\mu \frac{\partial \mathbf{H}}{\partial t} \]

\[ \mathbf{H} = \frac{0.04(0.01)}{10^6(10^6) \mu \mu_0 \epsilon_0} \sin(10^6 t - 0.013) \mathbf{u}_y \]

\[ = \frac{0.04(0.01)}{10^6(10^6) \mu \mu_0 \epsilon_0} = 4 \Rightarrow \mu_r = \frac{0.04(0.01)}{4(10^6)} \times (3 \times 10^4)^2 = 9 \]

26. (D) \( \nabla \times \mathbf{H} = -\frac{\partial \mathbf{E}}{\partial t} \), \( \mathbf{u}_y = \epsilon \frac{\partial \mathbf{E}}{\partial t} \)

\[ \frac{16 \pi \epsilon_0}{\rho} \sin(8 \pi t) \cos(\omega t) \mathbf{u}_y = \epsilon \frac{\partial \mathbf{E}}{\partial t} \]

\[ \mathbf{E} = \frac{16 \pi \epsilon_0}{\rho \mu_0 \omega} \sin(8 \pi t) \mathbf{u}_y \]

\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{H}}{\partial t} \], \( \mathbf{u}_y = -\frac{\partial \mathbf{H}}{\partial t} \)

\[ \mathbf{H} = \frac{128 \pi^2}{\rho \epsilon_0^2} \cos(8 \pi t) \cos(\omega t) \mathbf{u}_y \]

This result must be equal to the given \( \mathbf{H} \) field. Thus

\[ \frac{128 \pi^2}{\rho \epsilon_0^2} = \frac{2}{\rho} \Rightarrow \omega = \frac{8 \pi}{\sqrt{\mu e}} = \frac{8 \pi}{\sqrt{4 \times 10^{-11} \times 2.5 \times 10^{-2}}} \]

\[ = 8 \pi \times 10^8 \, \text{rad/s} \]

27. (B) At high frequency \( \frac{J_d}{J_d} = \frac{\sigma E}{\omega \epsilon_0} = \frac{\sigma}{\omega \epsilon_0} \)

\[ = \frac{2 \times 10^{-3}}{2 \pi \times 10^8 \times 81 \times \epsilon_0} = 4.44 \times 10^{-4} \]

28. (C) \( J_c = \frac{I}{S} = \frac{\alpha E}{\epsilon_0} \Rightarrow E = \frac{I}{\alpha S} \)

\[ \Rightarrow J_d = \frac{\epsilon I}{\epsilon_0 \alpha S} \]

\[ \Rightarrow J_d = \frac{4.6 \epsilon I_0 (10^3)}{2.5 \times 10^8 \times 10^{-4}} \cos(10^4 t) \]

\[ |J_d| = 32.6 \, \text{nA/m}^2 \]

29. (B) \( \nabla \cdot \mathbf{J} = (0 + 0 + 3z^2) \sin(10^4 t) = -\frac{8 \rho}{\partial t} \)

\[ \rho_c = \frac{3z^2 \cos(10^4 t)}{10^4} + C_1 \]

At \( z = 0, \rho_c = 0, C_1 = 0 \)

\[ \rho_c = 0.3z^2 \cos(10^4 t) \, \text{mC/m}^3 \]

30. (D) \( \mathbf{J} = \nabla \times \mathbf{H} = \frac{\partial \mathbf{H}}{\partial t} \), \( \mathbf{u}_x = \frac{\partial \mathbf{E}}{\partial t} \)

\[ = 4 \sin(10^6 t - 4 \gamma) \mathbf{u}_x \]

31. (A) \( \nabla \times \mathbf{E} = \frac{\partial \mathbf{D}}{\partial t} \), \( \mathbf{u}_x = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \)

\[ = 6 \beta \sin(\beta x) \cos(10^6 t) \mathbf{u}_x \]

\[ \mathbf{E} = \frac{1}{\epsilon} \int 6 \beta \sin(\beta x) \cos(10^6 t) \mathbf{u}_x \, dt \]

\[ = \frac{6 \beta}{\epsilon} \, \sin(\beta x) \sin(10^6 t) \mathbf{u}_y \]

\[ \epsilon = 6.25 \epsilon_0, \quad \beta = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{\epsilon_r} = \frac{10^8}{3 \times 10^{-4}} \sqrt{625} = 0.833 \]

\[ \mathbf{E} = \frac{6(0.833)}{6.25 \epsilon_0, \times 10^8} \sin(\beta x) \sin(10^6 t) \mathbf{u}_y, \quad \text{V/m} \]

\[ = 903 \sin(0.83x) \sin(10^6 t) \mathbf{u}_y, \quad \text{V/m} \]

32. (C) For nonmagnetic medium \( \mu_r = 1 \)

\[ \beta = \frac{\omega}{v} = \frac{\omega}{c} \sqrt{\epsilon_r}, \quad \omega = 10^8, \beta = 8, \]

\[ = \frac{10^8}{3} \times 8 \sqrt{\epsilon_r} \Rightarrow \epsilon_r = 5.76 \]

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CHAPTER 8.5

ELECTROMAGNETIC WAVE PROPAGATION

Statement for Q.1–3:
A y-polarized uniform plane wave with a frequency of 100 MHz propagates in air in the +x direction and impinges normally on a perfectly conducting plane at x = 0. The amplitude of incident E-field is 6 mV/m.

1. The phasor \( \mathbf{H} \) of the incident wave in air is
(A) \( 16e^{j2\pi x} \mathbf{u}_x \) \( \mu \) A/m
(B) \( -16e^{j2\pi x} \mathbf{u}_y \) \( \mu \) A/m
(C) \( 16e^{j2\pi x} \mathbf{u}_x \) \( \mu \) A/m
(D) \( -16e^{j2\pi x} \mathbf{u}_y \) \( \mu \) A/m

2. The E-field of total wave in air is
(A) \( j12 \sin \left( \frac{2\pi}{3} x \right) \mathbf{u}_y \) mV/m
(B) \( -j12 \sin \left( \frac{2\pi}{3} x \right) \mathbf{u}_x \) mV/m
(C) \( 12\cos \left( \frac{2\pi}{3} x \right) \mathbf{u}_x \) mV/m
(D) \( -12\cos \left( \frac{2\pi}{3} x \right) \mathbf{u}_y \) mV/m

3. The location in air nearest to the conducting plane, where total E-field is zero, is
(A) \( x = 15 \) m
(B) \( x = -15 \) m
(C) \( x = 3 \) m
(D) \( x = -3 \) m

4. The phasor magnetic field intensity for a 400 MHz uniform plane wave propagating in a certain lossless material is \( (6\mathbf{u}_y - j5\mathbf{u}_x) e^{-j\beta x} \) A/m. The phase velocity \( v_p \) is
(A) \( 6.43 \times 10^6 \) m/s
(B) \( 2.2 \times 10^7 \) m/s
(C) \( 1.4 \times 10^8 \) m/s
(D) None of the above

Statement for Q.5–6:
A uniform plane wave in free space has electric field \( \mathbf{E} = (2\mathbf{u}_x + 3\mathbf{u}_y) e^{-j\beta x} \) V/m.

5. The magnetic field phasor \( \mathbf{H} \) is
(A) \( (-5.3\mathbf{u}_y - 8\mathbf{u}_x) e^{-j\beta x} \) mA/m
(B) \( (5.3\mathbf{u}_y - 8\mathbf{u}_x) e^{-j\beta x} \) mA/m
(C) \( (-5.3\mathbf{u}_y + 8\mathbf{u}_x) e^{-j\beta x} \) mA/m
(D) \( (5.3\mathbf{u}_y + 8\mathbf{u}_x) e^{-j\beta x} \) mA/m

6. The average power density in the wave is
(A) \( 34 \) mW/m²
(B) \( 17 \) mW/m²
(C) \( 22 \) mW/m²
(D) \( 44 \) mW/m²

7. The electric field of a uniform plane wave in free space is given by \( \mathbf{E} = 12\pi \mathbf{(u}_y + j\mathbf{u}_x) e^{-j\beta x} \). The magnetic field phasor \( \mathbf{H} \) is
(A) \( \frac{12}{\eta_0} (\mathbf{u}_x + j\mathbf{u}_y) e^{-j\beta x} \)
(B) \( \frac{12}{\eta_0} (\mathbf{u}_x - j\mathbf{u}_y) e^{-j\beta x} \)
(C) \( \frac{12}{\eta_0} (\mathbf{u}_y - j\mathbf{u}_x) e^{-j\beta x} \)
(D) \( \frac{12}{\eta_0} (\mathbf{u}_y + j\mathbf{u}_x) e^{-j\beta x} \)

Statement for Q.8–9:
A lossy material has \( \mu = 5\mu_0 \), \( \epsilon = 2\epsilon_0 \). The phase constant is 10 rad/m at 5 MHz.

8. The loss tangent is
(A) 2913
(B) 1823
(C) 2468
(D) 1374
3. The attenuation constant $\alpha$ is
   (A) 4.43  (B) 9.99
   (C) 5.57  (D) None of the above

**Statement for Q.10–11:**
At 50 MHz a lossy dielectric material is characterized by $\mu = 2.1\mu$, $\varepsilon = 3.6\varepsilon$, and $\sigma = 0.08$ S/m. The electric field is $E = 6e^{-j\beta}$ V/m.

10. The propagation constant $\gamma$ is
   (A) 7.43 + j2.46 per meter
   (B) 2.46 + j7.43 per meter
   (C) 6.13 + j5.41 per meter
   (D) 5.41 + j6.13 per meter

11. The impedance $\eta$ is
   (A) 101.4 $\Omega$  (B) 167.4 $\Omega$
   (C) 98.3 $\Omega$  (D) 67.3 $\Omega$

**Statement for Q.12–13:**
A non magnetic medium has an intrinsic impedance $360 \angle 30^\circ$ $\Omega$.

12. The loss tangent is
   (A) 0.866  (B) 0.5
   (C) 1.732  (D) 0.577

13. The Dielectric constant is
   (A) 1.634  (B) 1.234
   (C) 0.936  (D) 0.548

**Statement for Q.14–15:**
The amplitude of a wave traveling through a lossy nonmagnetic medium reduces by 18% every meter. The wave operates at 10 MHz and the electric field leads the magnetic field by $24^\circ$.

14. The propagation constant is
   (A) 0.198 + j0.448 per meter
   (B) 0.346 + j0.713 per meter
   (C) 0.448 + j0.198 per meter
   (D) 0.713 + j0.346 per meter

15. The skin depth is
   (A) 2.52 m  (B) 5.05 m
   (C) 8.46 m  (D) 4.23 m

16. A 60 m long aluminium ($\sigma = 3.5 \times 10^7$ S/m, $\mu = 1$, $\varepsilon = 1$) pipe with inner and outer radii 9 mm and 12 mm carries a total current of $16\sin (10^6 \pi t)$ A. The effective resistance of the pipe is
   (A) 0.19 $\Omega$  (B) 3.48 $\Omega$
   (C) 1.46 $\Omega$  (D) 2.43 $\Omega$

17. Silver platted brass wave guide is operating at 12 GHz. If at least the thickness of silver ($\sigma = 6.1 \times 10^7$ S/m, $\mu = \varepsilon = 1$) is $5\delta$, the minimum thickness required for wave-guide is
   (A) 6.41 $\mu$m  (B) 3.86 $\mu$m
   (C) 5.21 $\mu$m  (D) 2.94 $\mu$m

**Statement for Q.18–19:**
A uniform plane wave in a lossy nonmagnetic media has
   $E = (5u_x + 12u_y)e^{-j\phi}$, $\gamma = 0.2 + j3.4$ m$^{-1}$

18. The magnitude of the wave at $z = 4$ m and $t = T/8$ is
   (A) 10.34  (B) 5.66
   (C) 4.36  (D) 12.60

19. The loss suffered by the wave in the interval $0 < z < 3$ m is
   (A) 4.12 dB  (B) 8.24 dB
   (C) 10.42 dB  (D) 5.21 dB

**Statement for Q.20–22:**
The plane wave $E = 42\cos (\omega t - z) u_x$ V/m in air normally hits a lossless medium ($\mu = 1$, $\varepsilon = 4$) at $z = 0$.

20. The SWR $s$ is
   (A) 2  (B) 1
   (C) $\frac{1}{2}$  (D) None of the above

21. The transmission coefficient $\tau$ is
   (A) $\frac{2}{3}$  (B) $\frac{4}{3}$
   (C) $\frac{1}{3}$  (D) 3

22. The reflected electric field is
   (A) $-14\cos (\omega t - z) u_x$ V/m
   (B) $-14\cos (\omega t + z) u_x$ V/m
33. The region \( z < 0 \) is characterized by \( \varepsilon_r = \mu_r = 1 \) and \( \sigma = 0 \). The total electric field here is given as \( \mathbf{E}_z = 150 e^{-j\omega t} \mathbf{u}_z + 50 Z \omega e^{j\omega t} \mathbf{u}_z \) V/m. The intrinsic impedance of the region \( z > 0 \) is

(A) \( 692 + j176 \Omega \)  
(B) \( 193 - j49 \Omega \)  
(C) \( 176 + j692 \Omega \)  
(D) \( 49 - j193 \Omega \)

**Statement for Q.34–35:**

Region 1, \( z < 0 \) and region 2, \( z > 0 \), are both perfect dielectrics. A uniform plane wave traveling in the \( \mathbf{u}_z \) direction has a frequency of \( 3 \times 10^10 \) rad/s. Its wavelength in the two region are \( \lambda_1 = 5 \) cm and \( \lambda_2 = 3 \) cm.

34. On the boundary the reflected energy is

(A) 6.25%  
(B) 12.5%  
(C) 25%  
(D) 50%

35. The SWR is

(A) 1.67  
(B) 0.6  
(C) 2  
(D) 1.16

36. A uniform plane wave is incident from region 1 \( (\mu_r = 1, \ \sigma = 0) \) to free space. If the amplitude of incident wave is one-half that of reflected wave in region, then the value of \( \varepsilon_r \) is

(A) 4  
(B) 3  
(C) 16  
(D) 9

37. A 150 MHz uniform plane wave is normally incident from air onto a material. Measurements yield a SWR of 3 and the appearance of an electric field minimum at 0.3\( \lambda_1 \) in front of the interface. The impedance of material is

(A) \( 502 - j641 \Omega \)  
(B) \( 641 - j502 \Omega \)  
(C) \( 641 + j502 \Omega \)  
(D) \( 502 + j641 \Omega \)

38. A plane wave is normally incident from air onto a semi-infinite slab of perfect dielectric \( (\varepsilon_r = 3.45) \). The fraction of transmitted power is

(A) 0.91  
(B) 0.3  
(C) 0.7  
(D) 0.49

**Statement for Q.39–40:**

Consider three lossless region:

Region 2 (\( 0 < z < 6 \) cm): \( \mu_2 = 2 \mu \) H/m, \( \varepsilon_2 = 25 \) pF/m  
Region 3 (\( z > 6 \) cm): \( \mu_3 = 4 \mu \) H/m, \( \varepsilon_3 = 10 \) pF/m

39. The lowest frequency, at which a uniform plane wave incident from region 1 onto the boundary at \( z = 0 \) will have no reflection, is

(A) 2.96 GHz  
(B) 4.38 GHz  
(C) 1.18 GHz  
(D) 590 MHz

40. If frequency is 50 MHz, the SWR in region 1 is

(A) 0.64  
(B) 1.27  
(C) 2.38  
(D) 4.16

41. A uniform plane wave in air is normally incident onto a lossless dielectric plate of thickness \( \lambda/8 \), and of intrinsic impedance \( \eta = 260 \) \( \Omega \). The SWR in front of the plate is

(A) 1.12  
(B) 1.34  
(C) 1.70  
(D) 1.93

42. The E-field of a uniform plane wave propagating in a dielectric medium is given by

\[
\mathbf{E} = 2\cos \left(10^8 t - \frac{z}{\sqrt{3}}\right) \mathbf{u}_z - \sin \left(10^8 t - \frac{z}{\sqrt{3}}\right) \mathbf{u}_y \text{ V/m}
\]

The dielectric constant of medium is

(A) 3  
(B) 9  
(C) 6  
(D) \( \sqrt{6} \)

43. An electromagnetic wave from an under water source with perpendicular polarization is incident on a water-air interface at angle 20° with normal to surface. For water assume \( \varepsilon_r = 81, \mu_r = 1 \). The critical angle \( \theta_c \) is

(A) 83.62°  
(B) 6.38°  
(C) 42.6°  
(D) None of the above

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SOLUTIONS

1. (A) \( \omega = 2\pi \times 10^8 \) rad/s
   \[ \beta = \frac{\omega}{c} = \frac{2\pi \times 10^8 - \frac{2\pi}{3}}{3 \times 10^8} \text{ rad/m} \]
   \[ E_x = 6e^{-\frac{2\pi}{3} r} u_y \text{ mV/m} \]
   \[ u_y \times u_H = u_x, \quad u_y \times u_H = u_x, \quad u_H = u_y \]
   \[ H_y = \frac{6}{120\pi} e^{-\frac{2\pi}{3} r} u_x = 16e^{-\frac{2\pi}{3} r} u_x \mu \text{A/m} \]

2. (B) For conducting plane \( \Gamma = -1 \),
   \[ E_z = -6e^{-\frac{2\pi}{3} r} u_y \text{ mV/m}, \]
   \[ E_E = E_y = \left(6e^{-\frac{2\pi}{3} r} u_y - 6e^{-\frac{2\pi}{3} r} u_y\right) \text{ mV/m} \]
   \[ = -j12 \sin \left(\frac{2\pi}{3} x\right) u_y \text{ mV/m} \]

3. (B) The electric field vanishes at the surface of the conducting plane at \( x = 0 \). In air the first null occur at \( x = -\frac{\lambda}{\beta} = -\frac{\pi}{2} = -\frac{3}{2} \) m

4. (C) \( v_p = \frac{\omega}{\beta} = \frac{2\pi \times 400 \times 10^6}{18} = 1.4 \times 10^8 \text{ m/s} \)

5. (C) The wave is propagating in forward \( x \) direction.
   Therefore \( u_y \times u_H = u_x, \)
   For \( u_y = u_y, \quad u_y \times u_H = u_x \quad \Rightarrow \quad u_H = -u_y \)
   For \( u_y = u_x, \quad u_y \times u_H = u_x \quad \Rightarrow \quad u_H = u_x \)
   \[ H_y = \frac{1}{120\pi}(2u_y + 3u_x)e^{-\beta x} = (-5.3u_y + 8u_x)e^{-\beta x} \mu \text{A/m} \]

6. (B) \( p_{avg} = \frac{1}{2} \Re (E_x \times H_y) \)
   \[ \frac{1}{2}(5.3u_x + 8u_x) \times 10^{-3} = 17.3u_x \text{ mW/m}^2 \]

7. (D) Since Pointing vector is in the positive \( x \) direction, therefore \( u_y \times u_H = u_x, \)
   For \( u_y = u_y, \quad u_y \times u_H = u_x \quad \Rightarrow \quad u_H = u_x \)
   For \( u_y = u_x, \quad u_y \times u_H = u_x \quad \Rightarrow \quad u_H = -u_y \)
   \[ H_y = \frac{12}{\eta_0}(u_x - ju_y)e^{-j15x} \]

8. (B) Loss tangent \( \frac{\sigma}{\omega \epsilon_0} = x \)
   \[ \beta = \frac{\omega}{\sqrt{2}} \left[ 1 + \left(\frac{\sigma}{\omega \epsilon_0}\right)^2 \right]^{1/2} \]
   \[ \Rightarrow 10 = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8} \sqrt{\frac{5 \times 2}{2}\left[1 + x^2 + 1\right]} \]
   \[ \Rightarrow x = \frac{\sigma}{\omega \epsilon_0} = 1823 \]

9. (B) \[ \frac{\alpha}{\beta} = \frac{\sqrt{1 + x^2 - 1}}{\sqrt{1 + x^2 + 1}} \]
   \[ \alpha = \frac{\sqrt{1822}}{\sqrt{1824}} \]
   \[ \alpha = 10 \times 0.999 = 9.99 \]

10. (D) \[ \frac{\alpha}{\beta} = \frac{\sqrt{1 + x^2 - 1}}{\sqrt{1 + x^2 + 1}} \]
    \[ \frac{\sigma}{\omega \epsilon_0} = \frac{0.08}{3.6 \times 10^8 \times 2 \pi \epsilon_0} = 8 \]
    \[ \alpha = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} \sqrt{\frac{(2.1)(3.6)}{2} \left[\frac{1}{1825} + 1\right]} = 5.41 \]
    \[ \beta = \frac{\omega}{\sqrt{2}} \left[ 1 + \left(\frac{\sigma}{\omega \epsilon_0}\right)^2 \right]^{1/2} \]
    \[ = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} \sqrt{\frac{(2.1)(3.6)}{2} \left[\frac{1}{65 + 1}\right]} = 6.13 \]
    \[ \gamma = \alpha + j\beta = 5.41 + j6.13 \text{ per meter} \]

11. (A) \[ |\eta| = \frac{\mu}{\sqrt{\epsilon_0}} = \frac{120\pi \sqrt{21}}{36} = 101.4 \]
    \[ \frac{1}{\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_0}\right)^2}} \]

12. (C) \[ \frac{\sigma}{\omega \epsilon_0} = \tan 2\theta_n, \quad \tan 60^\circ = 1.732 \]

13. (D) \[ |\eta| = \frac{\sqrt{\frac{\mu}{\epsilon_0}}}{\left(1 + \left(\frac{\sigma}{\omega \epsilon_0}\right)^2\right)^{1/2}} \]
    \[ = \frac{120\pi}{36} \frac{e}{e_x} \Rightarrow e_x = 0.548 \]
14. (A) $|\mathbf{E}| = E_0 e^{-\omega t}$

$E_0 e^{-\omega t} = (1 - 0.18) E_0$

$e^{-\omega t} = 0.82 \Rightarrow \alpha = \ln \frac{1}{0.82} = 0.198$

$\theta_n = 24^\circ \Rightarrow \tan 2\theta_n = \frac{\sigma}{\omega \varepsilon} = 1.111$

$\alpha = \frac{1}{\beta} = \frac{1}{\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}}$

$0.198 = \frac{1}{\sqrt{234 + 1}} \Rightarrow \beta = 0.448$

$\gamma = \alpha + j\beta = 0.198 + j0.448$

15. (B) $\delta = \frac{1}{\alpha} = \frac{24}{0.198} = 505$

16. (A) $\omega = 10^6 \Rightarrow f = 5 \times 10^5$ Hz,

$\delta = \frac{1}{\sqrt{\pi \sigma \varepsilon}} = \frac{1}{\sqrt{\pi \times 5 \times 10^5 \times 35 \times 10^{-7} \times \mu_o}} = 120 \mu m$

$R_{ac} = \frac{L}{\alpha \delta w}$

Since $\delta$ is very small, $w = 2\pi \rho_{outer}$

$R_{ac} = \frac{60}{35 \times 10^7 \times 120 \times 10^{-6} \times 2 \pi \times 12 \times 10^{-3}} = 0.19 \Omega$

17. (D) $t = 5\delta = \frac{5}{\sqrt{\pi \sigma \mu \varepsilon}}$

$= \frac{5}{\sqrt{\pi \times 12 \times 10^{-9} \times \mu_o \times 6.1 \times 10^{-7}}} = 294 \mu m$

18. (B) $\mathbf{E} = \text{Re}(\mathbf{E} e^{i\omega t}) = (5\mathbf{u}_x + 12\mathbf{u}_y) e^{-a_2x} \cos(\omega t - 3.4z)$

At $z = 4$ m, $t = \frac{T}{8}$

$\mathbf{E} = (5\mathbf{u}_x + 12\mathbf{u}_y) e^{-0.8} \cos \left(\frac{\pi}{4} - 13.6\right)$

$|\mathbf{E}| = 13 e^{-0.8} \cos \left(\frac{\pi}{4} - 13.6\right) = 5.66$

19. (D) Loss = $\alpha \Delta z = 0.2 \times 3 = 0.6 Np$

$1 Np = 8.686$ DB, $0.6 Np = 5.21$ dB.

20. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\mu_r \varepsilon_r} = \frac{\eta_o}{2}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o - \eta_o}{\eta_o + \eta_o} = -\frac{1}{3}$

$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2$

21. (A) $\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \eta_o}{2 \eta_o + 2} = \frac{2}{3}$

22. (A) $E_{ds} = \Gamma E_{ds} = \frac{1}{3} (42) = 14$

$E_x = -14 \cos (\omega t - z) \mathbf{u}_x \text{ V/m}$

23. (C) $\eta_1 = \eta_o$, $\eta_2 = \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_o - \eta_o}{\eta_o + \eta_o} = -\frac{1}{3}$

24. (D) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\mu_r \varepsilon_r} = \frac{\eta_o}{2}$

$E_{ds} = \Gamma \eta_2 - \eta_1 = \frac{\eta_o - \eta_o}{\eta_o + \eta_o}$

But $E_{ds} = \eta_1 H_{ds}$

$\eta_1 H_{ds} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_{ds} \Rightarrow \eta_1 = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \frac{18}{3 \times 10^{-7}}$

$\eta_1 = \eta_o \Rightarrow \eta_o = \left(\frac{\eta_o - \eta_o}{\eta_2 + \eta_o}\right) 3000$

$\frac{377}{3000} \Rightarrow \eta_2 = 485.37 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}}$

$\Rightarrow \varepsilon_r = 12.5$, $\mu_r = 20.75$

25. (A) $\eta_1 = \eta_o$, $\eta_2 = \eta_o \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_o}{2}$

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -\frac{1}{3}$

$E_{ds} = \frac{1}{3} (10) = -\frac{10}{3}$

$H_{ds} = \frac{E_{ds}}{\eta_o} = \frac{10}{3 \times 377} = 8.8 \times 10^{-3}$

$\mathbf{u}_x \times \mathbf{u}_x = \mathbf{u}_y$, $-\mathbf{u}_x \times \mathbf{u}_x = -\mathbf{u}_z \Rightarrow \mathbf{u}_H = -\mathbf{u}_x$

$\mathbf{H}_x = -8.8 \cos (\omega t - z) \mathbf{u}_x \text{ mA/m}$

$\beta = \frac{c}{\nu} = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$
\[
\omega = \frac{3 \times 10^8}{\sqrt{12 \times 3}} = 0.5 \times 10^8 \text{ rad/s}.
\]

26. (D) \( \eta_1 = \eta_0, \eta_2 = \eta_0 \sqrt{\frac{\mu_2}{\varepsilon_2}} = \eta_0 = 0.58\eta_0 \)

\[
\begin{align*}
\Gamma &= \frac{\eta_2 - \eta_0}{\eta_2 + \eta_1}, \quad \eta_2 = 0.58\eta_0 - \eta_0 \\quad \eta_2 + \eta_1 = 0.58\eta_0 + \eta_0 = -0.266 \\
\tau &= 1 + \Gamma = 0.734, \quad E_{\text{refl}} = tE_{\text{inc}} = 7.34
\end{align*}
\]

\[E_i = 7.34\cos(\omega t - z) \mathbf{u}_y, \quad \text{V/m} \]

27. (B) \( E_{\text{total}} = E_i + E_r, \quad E_{\text{refl}} = \Gamma E_{\text{inc}} = -2.66 \)

\[E_{\text{total}} = 10\cos(\omega t - z) \mathbf{u}_y - 2.66\cos(\omega t + z) \mathbf{u}_y, \quad \text{V/m} \]

28. (B) \( \mu_o = \mu_1 = \mu_2 \)

\[
sin \theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sin \theta_i \\Rightarrow \sin \theta_i = \sqrt{\frac{1}{45}} \sin 45^\circ = 0.333
\]

\[\theta_i = 19.47^\circ \]

29. (B) \( \sin \theta_r = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sin \theta_i = \sqrt{\frac{45}{2.25}} (0.333) = 0.47 \)

\[\theta_r = \sin^{-1} 0.47 = 28^\circ \]

30. (A) Since both media are non-magnetic

\[
tan \theta_b = \frac{\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_2}} = \frac{2.6\varepsilon_0}{\varepsilon_o} = \sqrt{2.6}
\]

But \( \cos \theta_i = \frac{\eta_1}{\eta_2} \cos \theta_b = \frac{\eta_0}{\eta_2} \cos 58.2^\circ = \sqrt{2.6} \cos 58.2^\circ \)

\[\theta_i = 31.8^\circ \]

31. (A) \( \eta_1 = \eta_0, \eta_2 = \eta_0 \sqrt{\frac{\mu_2}{\varepsilon_2}} = \eta_0 = 0.447\eta_0 \)

\[\Gamma = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_1} = -0.38, \quad \tau = 1 + \Gamma = 0.62 \]

\[E_i = tE_i = 92.7\cos(\omega t - 8\gamma) \mathbf{u}_y, \quad \text{V/m} \]

32. (B) \( |\Gamma|^2 = 0.02, \quad \Gamma = \pm 0.447 \)

\[
\begin{align*}
\Gamma &= \frac{\eta_2 - \eta_0}{\eta_2 + \eta_1}, \quad \eta_2 = \eta_0 \frac{\mu_2}{\varepsilon_2} = \eta_0 = 0.447\eta_0 \\
\tau &= 1 + \Gamma = 0.62 \\
\Gamma &= \frac{\mu_2}{\mu_1}, \quad \mu_2 = 1.047, 2.62 \\
\end{align*}
\]

\[\frac{\varepsilon_2}{\varepsilon_1} = \left(\frac{\mu_2}{\mu_1}\right)^3 = 0.056, 179 \]

33. (A) \( \Gamma = \frac{E_r}{E_i} = -0.266 \quad e^{2\gamma} \)

\[\Gamma = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_1}, \quad \eta_1 = \eta_0, \quad \eta_2 = \eta_0 \left(1 + \frac{1}{\Gamma}\right) = \frac{377}{1 - \Gamma} \quad E_{\text{refl}} = 692 + j176 \Omega \]

34. (A) \( \varepsilon_1 = \left(\frac{2\pi}{\lambda_1}\right)^2, \quad \varepsilon_2 = \left(\frac{2\pi}{\lambda_2}\right)^2 \quad \Rightarrow \quad \frac{\varepsilon_1}{\varepsilon_2} = \left(\frac{\lambda_1}{\lambda_2}\right)^2 \)

\[\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0}{\sqrt{\varepsilon_2}} - \frac{\eta_0}{\sqrt{\varepsilon_1}} = \frac{\varepsilon_1 - 1}{\sqrt{\varepsilon_1}} = \frac{\lambda_2 - 1}{\lambda_1} \]

\[\Rightarrow \quad \Gamma = \frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1} = \frac{3 - 5}{3 + 5} = \frac{1}{4} \]

The fraction of the incident energy that is reflected is \( \Gamma^2 = \frac{1}{16} = 0.625\% \).

35. (A) \( s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{5}{3} \)

36. (D) \( \eta_2 = \eta_o, \quad \eta_1 = \eta_o \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{\sqrt{\varepsilon_2}} \)

\[\Gamma = \frac{|E_r|}{|E_i|} = \frac{1}{2} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0}{\sqrt{\varepsilon_2}} - \frac{\eta_0}{\sqrt{\varepsilon_1}} = \frac{\eta_0}{\sqrt{\varepsilon_2}} - \frac{1}{2} \Rightarrow \varepsilon_2 = 9 \]

37. (C) At minimum \( \frac{\phi + \pi}{2\beta} = 0.3\lambda, \)

\[\beta = \frac{2\pi}{\lambda} \Rightarrow \phi = 0.2\pi \]

\[|\Gamma| = \frac{s - 1}{s + 1} = \frac{3 - 1}{3 + 1} = \frac{1}{2} \]

\[\Gamma = 0.5e^{j2\pi} = \frac{\eta_2 - \eta_0}{\eta_2 + \eta_0} \Rightarrow \eta_2 = \eta_0 \left(\frac{1 + 0.5e^{j2\pi}}{1 - 0.5e^{j2\pi}}\right) = 641 + j502 \Omega \]

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38. (A) \( \eta_1 = \eta_o \), \( \eta_2 = \eta_o \sqrt{\frac{\mu_2}{\varepsilon_r}} = \frac{\eta_o}{\sqrt{3.45}} \)

\[
\gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{3.45} - \eta_o}{\sqrt{3.45} + \eta_o} = -0.3
\]

The transmitted fraction is \( 1 - |\gamma|^2 = 1 - 0.09 = 0.91 \).

39. (C) This frequency gives the condition \( \beta_z d = \pi \)

Where \( d = 6 \text{ cm} \), \( \beta_z = \omega \sqrt{\mu_2 \varepsilon_2} \)

\[
\Rightarrow \ \omega \sqrt{\mu_2 \varepsilon_2} = \frac{\pi}{0.06} \Rightarrow f = \frac{1}{2 \times 0.06 \sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}}} = 118 \text{ GHz}
\]

40. (B) At 50 MHz,

\[
\beta_z = \omega \sqrt{\mu_2 \varepsilon_2} = 2\pi \times 50 \times 10^6 \sqrt{2 \times 10^{-6} \times 25 \times 10^{-12}} = 2.2
\]

\[
\beta_z d = 2.22(0.06) = 0.133
\]

\[
\eta_i = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{4 \times 10^{-6}}{10^{-11}}} = 632 \Omega
\]

\[
\eta_o = 632 \Omega
\]

\[
\eta_3 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{2 \times 10^{-6}}{25 \times 10^{-12}}} = 283 \Omega
\]

The input impedance at the first interface is

\[
\eta_{in} = \eta_2 \left( \eta_3 + j\eta_1 \tan (\beta_z d) \right) / \eta_2 + j\eta_1 \tan (\beta_z d) = 283 \left( \frac{632 + j283(0.134)}{283 + j632(0.134)} \right)
\]

\[
= 590 - j138
\]

\[
\eta_{in} - \eta_o = 590 - j138 - 632 / 590 - j138 + 632 = 0.12 \angle -100.5^\circ
\]

\[
\frac{1 + |\gamma|}{1 - |\gamma|} = \frac{1 + 0.12}{1 - 0.12} = 1.27
\]

41. (C) \( \beta d = \frac{2\pi}{\lambda} = \frac{\pi}{8} \), \( \tan \frac{\pi}{4} = 1 \)

\[
\eta_2 = 260, \ \eta_1 = \eta_3 = \eta_o
\]

\[
\eta_{in} = \eta_2 \left( \eta_3 + j\eta_1 \tan (\beta_z d) \right) / \eta_2 + j\eta_1 \tan (\beta_z d) = 260 \left( \frac{377 + j260}{260 + j377} \right)
\]

\[
= 243 - j92 \Omega
\]

\[
\eta_{in} - \eta_o = 243 - j92 - 377 / 243 - j92 + 377 = 0.26 \angle -137^\circ
\]

\[
\frac{1 + |\gamma|}{1 - |\gamma|} = \frac{126}{0.74} = 170
\]

42. (A) \( \omega = 10^8 \text{ rad/s}, \ \beta = \frac{1}{\sqrt{3}} \ \text{ rad/m}, \ \nu = \frac{c}{\sqrt{\varepsilon_r}} = \frac{\omega}{\beta} \)
Statement for Q.1–3:

A 2 cm by 3 cm rectangular waveguide is filled with a dielectric material with \( \varepsilon_r = 6 \). The waveguide is operating at 20 GHz with \( TM_{11} \) mode.

1. The cutoff frequency is
   (A) 3.68 GHz  
   (B) 22.09 GHz  
   (C) 9.02 GHz  
   (D) 16.04 GHz

2. The phase constant is
   (A) 816 rad/m  
   (B) 412 rad/m  
   (C) 1009 rad/m  
   (D) 168 rad/m

3. The phase velocity is
   (A) \( 1.24 \times 10^8 \) m/s  
   (B) \( 1.54 \times 10^8 \) m/s  
   (C) \( 3.05 \times 10^8 \) m/s  
   (D) \( 7.48 \times 10^8 \) m/s

4. In an an-filled rectangular wave guide, the cutoff frequency of a \( TE_{10} \) mode is 5 GHz where as that of \( TE_{01} \) mode is 12 GHz. The dimensions of the guide is
   (A) 3 cm by 1.25 cm  
   (B) 1.25 cm by 3 cm  
   (C) 6 cm by 2.5 cm  
   (D) 2.5 cm by 6 cm

5. Consider a 150 m long air-filled hollow rectangular waveguide with cutoff frequency 6.5 GHz. If a short pulse of 7.2 GHz is introduced into the input end of the guide, the time taken by the pulse to return the input end is
   (A) 920 ns  
   (B) 460 ns  
   (C) 230 ns  
   (D) 430 ns

Statement for Q.6–7:

In an air-filled rectangular waveguide the cutoff frequencies for \( TM_{11} \) and \( TE_{03} \) modes are both equal to 12 GHz.

6. The dominant mode is
   (A) \( TM_{10} \)  
   (B) \( TM_{01} \)  
   (C) \( TE_{01} \)  
   (D) \( TE_{10} \)

7. At dominant mode the cutoff frequency is
   (A) 11.4 GHz  
   (B) 4 GHz  
   (C) 5 GHz  
   (D) 8 GHz

8. For an air-filled rectangular waveguide given that
   \[ E_x = 10 \sin \left( \frac{2\pi x}{a} \right) \sin \left( \frac{3\pi y}{b} \right) \cos (10^2 t - \beta z) \text{ V/m} \]

   If the waveguide has cross-sectional dimension \( a = 6 \) cm and \( b = 3 \) cm, then the intrinsic impedance of this mode is
   (A) 373.2 \( \Omega \)  
   (B) 378.9 \( \Omega \)  
   (C) 375.1 \( \Omega \)  
   (D) 380.0 \( \Omega \)

Statement for Q.9–10:

In an air-filled waveguide, a TE mode operating at 6 GHz has
   \[ E_x = 15 \sin \left( \frac{2\pi x}{a} \right) \cos \left( \frac{\pi y}{b} \right) \sin (\omega t - 12z) \text{ V/m} \]
9. The cutoff frequency is
   (A) 4.189 GHz  
   (B) 5.973 GHz  
   (C) 8.438 GHz  
   (D) 7.946 GHz

10. The intrinsic impedance is
    (A) 35.72 \Omega  
    (B) 3978 \Omega  
    (C) 1989 \Omega  
    (D) 7144 \Omega  

Statement for Q.11–12.

Consider an air-filled rectangular waveguide with \( a = 2.286 \text{ cm} \) and \( b = 1.016 \text{ cm} \). The \( y \)-component of the \( TE \) mode is

\[
E_y = 12 \sin \left( \frac{2\pi x}{a} \right) \cos \left( \frac{3\pi y}{b} \right) \sin (10\pi \times 10^9 t - \beta z) \text{ V/m}
\]

11. The propagation constant \( \gamma \) is
    (A) \( j4094.2 \)  
    (B) \( j400.7 \)  
    (C) \( j2733.3 \)  
    (D) \( j276.4 \)  

12. The intrinsic impedance is
    (A) 743 \Omega  
    (B) 168 \Omega  
    (C) 986 \Omega  
    (D) 144 \Omega  

Statement for Q.13–14:

Consider a air-filled waveguide operating in the \( TE_{10} \) mode at a frequency 20% higher than the cutoff frequency.

13. The phase velocity is
    (A) \( 1.66 \times 10^8 \text{ m/s} \)  
    (B) \( 5.42 \times 10^8 \text{ m/s} \)  
    (C) \( 2.46 \times 10^8 \text{ m/s} \)  
    (D) \( 9.43 \times 10^8 \text{ m/s} \)  

14. The group velocity is
    (A) \( 1.66 \times 10^8 \text{ m/s} \)  
    (B) \( 4.42 \times 10^8 \text{ m/s} \)  
    (C) \( 2.46 \times 10^8 \text{ m/s} \)  
    (D) \( 9.43 \times 10^8 \text{ m/s} \)  

15. A rectangular waveguide is filled with a polyethylene (\( \varepsilon_r = 2.25 \)) and operates at 24 GHz. The cutoff frequency of a certain mode is 16 GHz. The intrinsic impedance of this mode is
    (A) 2248 \Omega  
    (B) 337.2 \Omega  
    (C) 421.4 \Omega  
    (D) 632.2 \Omega  

16. The cross section of a waveguide is shown in fig. P8.7.16. It has dielectric discontinuity as shown in fig. P8.7.16. If the guide operate at 8 GHz in the dominant mode, the standing wave ratio is
    \[
    \frac{\mu_0 \varepsilon_0}{\mu_{0,\text{ref}} \varepsilon_{0,\text{ref}}} = 2.25 \varepsilon_0
    \]
    (A) -3.911  
    (B) 2.468  
    (C) 1.564  
    (D) 4.389  

Statement for Q.17–19:

Consider the rectangular cavity as shown in fig. P8.7.17–19.

17. If \( a < b < c \), the dominant mode is
    (A) \( TE_{011} \)  
    (B) \( TM_{110} \)  
    (C) \( TE_{101} \)  
    (D) \( TM_{101} \)  

18. If \( a > b > c \), then the dominant mode is
    (A) \( TE_{011} \)  
    (B) \( TM_{110} \)  
    (C) \( TE_{101} \)  
    (D) \( TM_{101} \)  

19. If \( a = c > b \), then the dominant mode is
    (A) \( TE_{011} \)  
    (B) \( TM_{110} \)  
    (C) \( TE_{101} \)  
    (D) \( TM_{101} \)  

20. The air filled cavity resonator has dimension \( a = 3 \text{ cm} \), \( b = 2 \text{ cm} \), \( c = 4 \text{ cm} \). The resonant frequency for the \( TM_{110} \) mode is
    (A) 5 GHz  
    (B) 6.4 GHz  
    (C) 16.2 GHz  
    (D) 9 GHz
frequency, the \( TM_1 \) mode propagates through the guide without suffering any reflective loss at the dielectric interface. This frequency is

\[
\frac{f_c}{2} = \frac{\omega}{v} = \sqrt{\frac{m^2}{a} + \frac{n^2}{b}} = \frac{3 \times 10^8}{2} \sqrt{\frac{1}{2} + \frac{1}{3}} = 3.68 \text{ GHz}
\]

Statement for Q.35–36:

A 6 cm \( \times \) 4 cm rectangular wave guide is filled with dielectric of refractive index 1.25.

35. The range of frequencies over which single mode operation will occur is
(A) 2.24 GHz < \( f \) < 3.33 GHz
(B) 2 GHz < \( f \) < 3 GHz
(C) 4.48 GHz < \( f \) < 7.70 GHz
(D) 4 GHz < \( f \) < 6 GHz

36. The range of frequencies, over which guide support both \( TE_{10} \) and \( TE_{01} \) modes and no other, is
(A) 3.35 GHz < \( f \) < GHz
(B) 2.5 GHz < \( f \) < 3.6 GHz
(C) 3 GHz < \( f \) < 3.6 GHz
(D) 2.5 GHz < \( f \) < 4.02 GHz

37. Two identical rectangular waveguide are joined end to end where \( a = 2b \). One guide is air filled and other is filled with a lossless dielectric of \( \varepsilon_r \). it is found that up to a certain frequency single mode operation can be simultaneously ensured in both guide. For this frequency range, the maximum allowable value of \( \varepsilon_r \) is
(A) 4
(B) 2
(C) 1
(D) 6

38. A parallel-plate guide operates in the \( TEM \) mode only over the frequency range \( 0 < f < 3 \text{ GHz} \). The dielectric between the plates is teflon (\( \varepsilon_r = 2.1 \)). The maximum allowable plate separation \( b \) is
(A) 3.4 cm
(B) 6.8 cm
(C) 4.3 cm
(D) 8.6 cm

SOLUTIONS

1. (A) \( f_c = \frac{v}{2} \sqrt{\frac{m^2}{a} + \frac{n^2}{b}} \)
\[
= \frac{3 \times 10^8}{2} \sqrt{\frac{1}{2} + \frac{1}{3}} = 3.68 \text{ GHz}
\]

2. (C) \( \beta_p = \beta = \sqrt{\frac{1}{f} - \frac{1}{f_c^2}} = \frac{\omega}{v} \sqrt{\frac{1}{f} - \frac{1}{f_c^2}} \)
\[
\Rightarrow \beta_p = \frac{2\pi \times 20 \times 10^3 \sqrt{6}}{3 \times 10^8} \left[ 1 - \left( \frac{3.68}{20} \right)^2 \right] = 1009 \text{ rad/m}
\]

3. (A) \( v_p = \frac{\omega}{\beta_p} = \frac{2 \times 20 \times 10^3}{1009} = 1.24 \times 10^8 \text{ m/s} \)

4. (A) For \( TE_{10} \) mode \( f_c = \frac{v}{2a} \),
\[
a = \frac{v}{2f_c} = \frac{3 \times 10^8}{2 \times 5 \times 10^9} = 3 \text{ cm}
\]

For \( TE_{01} \) mode \( f_c = \frac{v}{2b} \),
\[
b = \frac{v}{2f_c} = \frac{3 \times 10^8}{2 \times 12 \times 10^9} = 1.25 \text{ cm}
\]

5. (D) \( v = \frac{c}{\sqrt{1 - \left( \frac{f}{f_c} \right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left( \frac{6.5}{72} \right)^2}} = 6.975 \times 10^8 \text{ ms} \)
\[
t = \frac{2l}{v} = \frac{2 \times 150}{6.975 \times 10^8} = 430 \text{ ns}
\]

6. (C) \( 12 \times 10^9 = \frac{3 \times 10^8}{2} \sqrt{0 + \left( \frac{3}{b} \right)^2} \Rightarrow b = 3.75 \text{ cm} \)
\[
12 \times 10^9 = \frac{3 \times 10^8}{2} \left[ \frac{1}{a} + \left( \frac{1}{375 \times 10^{-2}} \right)^2 \right] \Rightarrow a = 1.32 \text{ cm}
\]

Since \( a < b \), the dominant mode is \( TE_{01} \).

7. (B) \( f_{c_{01}} = \frac{v}{2b} = \frac{3 \times 10^8}{2 \times 375 \times 10^{-2}} = 4 \text{ GHz} \)

8. (C) \( E \neq 0 \), this must be \( TM_{23} \) mode \( (m = 2, n = 3) \)
\[
f_c = \frac{3 \times 10^8}{2} \left[ \frac{2}{6} + \left( \frac{3}{3} \right)^2 \right] = 15.81 \text{ GHz}
\]
\[
f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.2 \text{ GHz}
\]
\[ \eta_{TM} = 377 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 377 \sqrt{1 - \left(\frac{15.81}{1592}\right)^2} = 375.1 \Omega \]

9. (B) \( m = 2, \ n = 1, \ \beta_p = 12, \ f = 6 \ GHz \)

\[ \beta_p = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \Rightarrow 12 = \frac{2\pi \times 6 \times 10^9}{3 \times 10^8} \sqrt{1 - \left(\frac{f_c}{6}\right)^2} \]

\[ f_c = 5.973 \ GHz \]

10. (B) \[ \eta_{RE} = \frac{377}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{5973}{6}\right)^2}} = 3978 \ \Omega \]

11. (B) \( m = 2, \ n = 3, \)

\[ f_c = \frac{c}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} - \frac{3 \times 10^8}{2 \times 10^{-2}} \sqrt{\left(\frac{2}{2.286}\right)^2 + \left(\frac{3}{1.016}\right)^2}} = 46.2 \ GHz \]

\[ f = \frac{10\pi \times 10^{10}}{2\pi} = 50 \ GHz \]

\[ \beta_p = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{2\pi \times 50 \times 10^9}{3 \times 10^8} \sqrt{1 - \left(\frac{46.2}{50}\right)^2} = 400.7 \ m^{-1}, \ \gamma = j\beta_p = j \times 400.7 \]

12. (C) \[ \eta_{RE} = \frac{377}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{462}{50}\right)^2}} = 986 \ \Omega \]

13. (A) \( v = c, \ f = 1.2f_c \)

\[ v_p = \frac{v}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{f_c}{1.2f_c}\right)^2}} = 5.42 \times 10^8 \ m/s \]

14. (A) \( v = v \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = c \sqrt{1 - \left(\frac{f_c}{12f_c}\right)^2} = 1.66 \times 10^8 \ m/s \]

15. (B) \[ \eta = \frac{377}{\sqrt{\varepsilon_r}} = \frac{377}{15} = 251.33 \ \Omega \]

\[ \eta_{RE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{251.33}{\sqrt{1 - \left(\frac{16}{24}\right)^2}} = 337.2 \ \Omega \]

16. (C) Since \( a > b, \) the dominant mode is \( TE_{10}. \)

In free space \( f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.05} = 3 \ GHz \)

\[ \eta_1 = \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \sqrt{1 - \left(\frac{3}{8}\right)^2} = 0.65 \]

In dielectric medium

\[ f_c = \frac{c}{2a\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{2 \times 0.05 \times 2.25} = 2 \ GHz \]

\[ \eta = \frac{\eta_1}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.25}} = 251.33 \ \Omega, \ \eta_2 = \frac{251.33}{\sqrt{1 - \left(\frac{3}{8}\right)^2}} = 259.23 \ \Omega \]

\[ \Gamma = \eta_2 - \eta_1 = 259.23 - 406.7 = -0.22 \]

\[ s = 1 + |\Gamma| = 1 + 0.22 = 1.22 \]

\[ 1 - |\Gamma| = 1 - 0.22 = 0.78 \]

17. (A) \[ f_c = \frac{v}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{c^2}} \]

where for TM mode to \( z \)

\( m = 1, 2, 3,..., \)

\( n = 1, 2, 3,..., \)

\( p = 0, 1, 2 ... \)

For TE mode to \( z \)

\( m = 1, 2, 3,..., \)

\( n = 1, 2, 3,..., \)

\( p = 1, 2, 3,..., \)

if \( a < b < c, \) then \( \frac{1}{a} > \frac{1}{b} > \frac{1}{c} \)

The lowest TM mode is \( TM_{110} \) with

\[ f_{r1} = \frac{v}{2} \left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 \]

The lowest TE mode is \( TE_{011} \) with

\[ f_{r2} = \frac{v}{2} \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2 \]

\( f_{r2} > f_{r1}, \) Hence the dominant mode is \( TE_{011} \)

18. (B) If \( a > b > c \) then \( \frac{1}{a} < \frac{1}{b} < \frac{1}{c} \)

The lowest TM mode is \( TM_{110} \) with

\[ f_{r1} = \frac{v}{2} \left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 \]

The lowest TE mode is \( TE_{101} \) with

\[ f_{r2} = \frac{v}{2} \left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2 \]

\( f_{r2} > f_{r1} \) Hence the dominant mode is \( TM_{110}. \)
19. (C) If \( a = c > b \), then \( \frac{1}{a} = \frac{1}{a} < \frac{1}{b} \)

The lowest TM mode is \( TM_{10} \) with

\[
f_{n1} = \frac{\nu}{2} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{1}{b} \right)^2}
\]

The lowest TE mode is \( TE_{10} \) with

\[
f_{e2} = \frac{\nu}{2} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{1}{c} \right)^2}
\]

\( f_{n1} < f_{e2} \) Hence the dominant mode is \( TE_{10} \).

20. (D) \( f_e = \frac{\nu}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{P}{c} \right)^2}
\]

\[
= \frac{3 \times 10^8}{2 \times 0.001} \sqrt{\left( \frac{1}{3} \right)^2 + \left( \frac{1}{2} \right)^2} = 9 \text{ GHz}
\]

21. (A) \( m = n = 1, \ p = 0, a = b = c, f_r = 2 \text{ GHz},
\]

\[
f_r = \frac{\nu}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{P}{c} \right)^2}
\]

\[
= 2 \times 10^9 = \frac{3 \times 10^8 \sqrt{2}}{2} = 9 \text{ GHz}
\]

22. (A) \( f_e = \frac{mc}{2b \sqrt{\varepsilon_r}} = \frac{2 \times 3 \times 10^6}{2 \times 0.01 \sqrt{\varepsilon_r}} = 10 \times 10^6 \Rightarrow \varepsilon_r = 9
\]

23. (A) For a propagating mode \( f > f_{cm},
\]

\[
f_{cm} = \frac{mc}{2b \sqrt{\varepsilon_r}}, \ f > \frac{mc}{2b \sqrt{\varepsilon_r}} \Rightarrow m < \frac{2fb \sqrt{\varepsilon_r}}{c}
\]

\[
m < \frac{2 \times 30 \times 10^9 \times 0.01 \sqrt{2.5}}{3 \times 10^8} \Rightarrow m < 3.16
\]

The maximum allowed \( m \) is 3. The propagating mode will be \( TM_1, TE_1, TM_2, TE_2, TM_3, TE_3 \) and TEM Thus total 7 modes.

24. (B) \( f_{cm} = \frac{mc}{2b \sqrt{\varepsilon_r}}, \ f_{e2} = 2f_{e1} = 15 \text{ GHz}
\]

\[
f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.015} = 20 \text{ GHz}
\]

\[
v_{s2} = \frac{c}{\sqrt{1 - \left( \frac{f}{f_e} \right)^2}} = 3 \times 10^8 \sqrt{1 - \left( \frac{15}{20} \right)^2} = 2 \times 10^8 \text{ m/s}
\]

25. (A) \( f_e = \frac{c}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2}, \ \lambda_r = \frac{2}{\sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2}}
\]

<table>
<thead>
<tr>
<th>Mode</th>
<th>( TE_{10} )</th>
<th>( TE_{01} )</th>
<th>( TE_{11} )</th>
<th>( TE_{20} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_r \text{ (cm)} )</td>
<td>14.4</td>
<td>6.8</td>
<td>6.15</td>
<td>7.21</td>
</tr>
</tbody>
</table>

\( \lambda > \lambda_r \). Hence \( TE_{10} \) mode can be used.

26. (C) Let \( a = kb, \ 1 < k < 2 \)

\[
f_{cm} = \frac{c}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} = \frac{15 \times 10^8}{a} \sqrt{m^2 + k^2 n^2}
\]

Dominant mode is \( TE_{10}, \ f_{110} = \frac{15 \times 10^8}{a} \)

\[3 \text{ GHz} > 12 f_e \Rightarrow 3 \times 10^8 > \frac{12 \times 15 \times 10^8}{a}\]

\[\Rightarrow a > 6 \text{ cm}\]

The next higher mode is \( TE_{01}, f_{011} = \frac{15 \times 10^8}{b}, \)

\[3 \text{ GHz} < \frac{0.8 f_{10}}{0.8} \Rightarrow 3 \times 10^8 < \frac{0.8 \times 15 \times 10^8}{b} \]

\[\Rightarrow b < 4 \text{ cm, Thus (C) is correct option.}\]

27. (C) \( f_e = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.065} = 2.3 \text{ GHz}
\]

\[
v_p = \sqrt{\frac{1 - \left( \frac{f}{f_e} \right)^2}{1 - \left( \frac{2.3}{2} \right)^2}} = 4.7 \times 10^8 \text{ m/s}
\]

28. (B) For \( TE_{10} \) mode

\[
R = \frac{1}{\sigma_\delta} = \sqrt{\frac{\pi f \mu}{\sigma_e}} = \sqrt{\frac{\pi \times 9 \times 10^7 \times 2 \times 10^9}{1.2 \times 10^5}} = 0.0568
\]

\[
R \left[ 1 + \frac{2b f_e}{a f_e} \right] = 0.0568 \left[ 1 + \frac{2 \times 15 \times \left( \frac{3.876}{9} \right)^2}{2.4} \right] = 0.022
\]

29. (B) \( \sigma_d = \frac{10^{-15}}{2 \pi \times 9 \times 10^7 \times 2.6 \times 8.85 \times 10^{-12}} = 10^{-15}
\]

\[
\sigma_d < \frac{10}{10}, \ \text{hence} \ v \approx \frac{c}{\sqrt{2.6}}, \ \eta = \frac{377}{\sqrt{2.6}} = 233.8
\]

\[
f_e = \frac{3 \times 10^8}{2 \times 2.4 \times 10^{-2} \sqrt{2.6}} = 3.876 \text{ GHz}
\]

\[
\sigma_d = \frac{\sigma e \eta}{\sqrt{1 - \left( \frac{f}{f_e} \right)^2}} = \frac{10^{-15} \times 233.8}{2 \sqrt{1 - \left( \frac{3.876}{9} \right)^2}} = 1.3 \times 10^{-12} \text{ Np/m}
\]
30. (D) Dominant mode is TE\textsubscript{10} mode

\[ f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.072} = 2.08 \text{ GHz} \]

\[ R_c = \sqrt{\frac{n_0^2}{\sigma_0}} = \sqrt{\frac{\pi \times 3 \times 10^8 \times 4 \times 10^{-7}}{5.8 \times 10^7}} = 1.429 \times 10^{-2} \Omega \]

For TE\textsubscript{10} mode

\[ R = \frac{1 + 2b}{a} \left( \frac{f_c}{f} \right)^2 = \frac{1.429 \times 10^{-2}}{1 - \left( \frac{208}{3} \right)^2} \]

\[ = 2.25 \times 10^{-3} \text{ Np/m} \]

\[ e^{-\alpha_x} = \frac{1}{2} \Rightarrow z = \frac{1}{a_c} \ln 2 = 308 \text{ m} \]

31. (B)

\[ f_{\text{cm}} = \frac{c}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} = \frac{3 \times 10^8}{2 \times 0.01} \sqrt{\left( \frac{m}{8} \right)^2 + \left( \frac{n}{10} \right)^2} \]

\[ = 15 \sqrt{\left( \frac{m}{8} \right)^2 + \left( \frac{n}{10} \right)^2} \text{ GHz} \]

\[ f_{\text{c01}} = 1.875 \text{ GHz} \]

\[ f_{\text{c02}} = 15 \text{ GHz}, \quad f_{\text{c11}} = 2.4 \text{ GHz} \]

\[ f_{\text{c03}} = 3.75 \text{ GHz}, \quad f_{\text{c02}} = 3 \text{ GHz}, \]

\[ f_{\text{c11}} = 4.04 \text{ GHz}, \quad f_{\text{c12}} = 354 \text{ GHz}, \]

\[ f_{\text{c03}} = 5.625 \text{ GHz}, \quad f_{\text{c03}} = 4.5 \text{ GHz} \]

If \( f_c < f \), then mode will be transmit. Hence six mode will be transmitted.

32. (C) For dominant mode \((m = 1, n = 0)\)

\[ f_c = \frac{c}{2} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} = \frac{3 \times 10^8}{2 \times 0.04} = 375 \text{ GHz} \]

Since given frequency is below the cutoff frequency, 3 GHz will not be propagated and get attenuated

\[ \gamma = \alpha + j\beta = \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2} - \left( \frac{\omega}{c} \right)^2 \]

\[ \beta = 0, \text{ Since wave is attenuated,} \]

\[ \alpha = \sqrt{\left( \frac{m}{a} \right)^2 - \left( \frac{\omega}{c} \right)^2} = \sqrt{\left( \frac{\pi}{0.04} \right)^2 - \left( \frac{2 \pi \times 3 \times 10^8}{3 \times 10^8} \right)^2} = 47.1 \]

33. (B) \( f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.08} = 1.875 \text{ GHz} \)

34. (B) The ray angle is such that the wave is interface at Brewster’s angle \( \theta = \tan^{-1} \sqrt{\frac{21}{4}} = 35.9^\circ \).

The ray angle \( \theta = 90^\circ - 35.9^\circ = 54.1^\circ \)

\[ f_{c1} = \frac{c}{2b\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{2 \times 1 \times 2} = 7.5 \text{ GHz} \]

\[ f = \frac{f_{c1}}{\cos \theta} = \frac{7.5}{\cos 54.1^\circ} = 12.8 \text{ GHz} \]

35. (A) \( f_c = \frac{c}{2\sqrt{\varepsilon_r}} \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right] \)

\[ f_{c01} = \frac{c}{a2\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{2 \times 125 \times 0.06} = 2 \text{ GHz} \]

\[ f_{c02} = \frac{c}{b2\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{2 \times 125 \times 0.04} = 3 \text{ GHz} \]

2 GHz < \( f < 3 \text{ GHz} \)

36. (C) \( f_{c11} = \frac{3 \times 10^8}{2 \times 125 \times 10^{-2}} \left[ \left( \frac{1}{6} \right)^2 + \left( \frac{1}{4} \right)^2 \right] = 3.6 \text{ GHz} \)

3 GHz < \( f < 3.6 \text{ GHz} \)

37. (A) \( f_c = \frac{c}{2\sqrt{\varepsilon_r}} \left[ \left( \frac{m}{2b} \right)^2 + \left( \frac{n}{b} \right)^2 \right], \) In guide \( \varepsilon_r = 1 \)

lowest cutoff frequency \( f_{c10} = \frac{c}{2(2b)} \)

Next lowest cutoff frequency \( f_{c20} = \frac{c}{2b} \)

In guide 2 lowest cutoff frequency \( f'_{c10} = \frac{c}{2\sqrt{\varepsilon_r} (2b)} \)

Next lowest cutoff frequency \( f'_{c20} = \frac{c}{2\sqrt{\varepsilon_r} (2b)} \)

For single mode \( f'_{c10} < f < f'_{c20} \)

\[ \Rightarrow \frac{c}{2(2b)} < f < \frac{c}{2\sqrt{\varepsilon_r} (2b)} \Rightarrow \varepsilon_r < 4 \]

38. (A) \( f < f_c \Rightarrow \frac{f}{2b} = \frac{3 \times 10^8}{2 \times b \times \sqrt{2.1}} \Rightarrow 3 \times 10^9 < \frac{3 \times 10^8}{2 \times b \times \sqrt{2.1}} \Rightarrow b < 3.4 \text{ cm} \)

***************
1. A Hertizian dipole at the origin in free space has \( dl = 10 \, \text{cm} \) and \( I = 20 \cos(2\pi \times 10^7 t) \, \text{A} \). The \( |E| \) at the distant point \((100, 0, 0)\) is
   (A) 0.252 V/m \hspace{1cm} (B) 0.126 V/m \hspace{1cm} (C) 0.04 V/m \hspace{1cm} (D) 0.08 V/m

**Statement for Q.2–3:**

A 25 A source operating at 300 MHz feeds a Hertizian dipole of length 4 mm situated at the origin. Consider the point \( P(10, 30^\circ, 90^\circ) \).

2. The \( H \) at point \( P \) is
   (A) \( j0.25 \, \text{mA/m} \) \hspace{1cm} (B) 94.25 \( \, \text{mA/m} \) \hspace{1cm} (C) \( j0.5 \, \text{mA/m} \) \hspace{1cm} (D) 188.5 \( \, \text{mA/m} \)

3. The \( E \) at point \( P \) is
   (A) \( j0.25 \, \text{mV/m} \) \hspace{1cm} (B) \( j0.5 \, \text{mV/m} \) \hspace{1cm} (C) \( j94.25 \, \text{mV/m} \) \hspace{1cm} (D) \( j188.5 \, \text{mV/m} \)

4. An antenna can be modeled as an electric dipole of length 4 m at 3 MHz. If current is uniform over its length, then radiation resistance of the antenna is
   (A) 1.974 \( \, \Omega \) \hspace{1cm} (B) 1.263 \( \, \Omega \) \hspace{1cm} (C) 2.186 \( \, \Omega \) \hspace{1cm} (D) 2.693 \( \, \Omega \)

**Statement for Q.5–6:**

5. The time-average poynting vector at 50 km is
   (A) 6.36 \( \mu \text{W/m}^2 \) \hspace{1cm} (B) 4.78 \( \mu \text{W/m}^2 \) \hspace{1cm} (C) 9.55 \( \mu \text{W/m}^2 \) \hspace{1cm} (D) 12.73 \( \mu \text{W/m}^2 \)

6. The maximum electric field at that location is
   (A) 24 \( \, \text{mV/m} \) \hspace{1cm} (B) 85 \( \, \text{mV/m} \) \hspace{1cm} (C) 109 \( \, \text{mV/m} \) \hspace{1cm} (D) 12 \( \, \text{mV/m} \)

7. In free space, an antenna has a far-zone field given by \( E = \frac{1}{r} 10 \sin 20 e^{-ir} \, u_s \, \text{V/m} \). The radiated power is
   (A) 0.23 W \hspace{1cm} (B) 0.89 W \hspace{1cm} (C) 1.68 W \hspace{1cm} (D) 1.23 W

8. At the far field, an antenna produces \( P_{\text{ave}} = \frac{1}{r} \cos \theta \cos \phi \, u_s \, \text{W/m}^2 \), where \( 0 < \theta < \pi \) and \( 0 < \phi < \frac{\pi}{2} \). The directive gain of the antenna is
   (A) \( \cos \theta \cos \phi \) \hspace{1cm} (B) \( 2 \sin \theta \cos \phi \) \hspace{1cm} (C) \( 8 \cos \theta \sin \phi \) \hspace{1cm} (D) \( 8 \sin \theta \cos \phi \)

**Statement for Q.9–10:**

The radiation intensity of antennas has been given. Determine the directivity of antenna.

9. \( U(\theta, \phi) = \sin^2 \theta \, \cos \phi \, u_s \, \text{W/m}^2\), where \( 0 < \theta < \pi \) and \( 0 < \phi < 2\pi \)
   (A) 1.875 \hspace{1cm} (B) 2.468 \hspace{1cm} (C) 3.943 \hspace{1cm} (D) 6.743

10. \( U(\theta, \phi) = 4 \sin^2 \theta \sin^2 \phi \, \text{W/m}^2\), where \( 0 < \theta < \pi \) and \( 0 < \phi < \pi \)
    (A) 15 \hspace{1cm} (B) 12 \hspace{1cm} (C) 3 \hspace{1cm} (D) 6
11. The radiation intensity of an antenna is given by
\[ U(\theta, \phi) = 8 \sin^2 \theta \cos^2 \phi \], where \( 0 < \theta < \pi \) and \( 0 < \phi < \pi \). The directive gain is
(A) \( 6 \sin^2 \theta \cos^2 \phi \)  
(B) \( 3\sin^2 \theta \cos^2 \phi \)  
(C) \( 3\sin^2 \phi \cos^2 \theta \)  
(D) \( 6 \sin^2 \phi \cos^2 \theta \)

Statement for Q.12–13:
At the far field, an antenna radiates a field
\[ E_r = \frac{0.4 \cos^2 \theta}{4\pi r} e^{-jkr} \text{ kV/m} \]

12. The total radiated power is
(A) 1.36 W  
(B) 2.14 W  
(C) 0.844 W  
(D) 3.38 W

13. The directive gain at \( \theta = \pi/3 \) is
(A) 0.3125  
(B) 0.625  
(C) 1.963  
(D) 3.927

14. An antenna has directivity of 100 and operates at 150 MHz. The maximum effective aperture is
(A) 31.8 m²  
(B) 62.4 m²  
(C) 26.4 m²  
(D) 13.2 m²

15. Two half wave dipole antenna are operated at 100 MHz and separated by 1 km. If 100 W is transmitted by one, the power received by the other is \( D = 1.68 \)
(A) 12 μW  
(B) 10 mW  
(C) 18 mW  
(D) 16 μW

16. The electric field strength impressed on a half wave dipole is 6 mV/m at 60 MHz. The maximum power received by the antenna is \( D = 1.68 \)
(A) 159 nW  
(B) 230 nW  
(C) 196 μW  
(D) 318 μW

17. The power transmitted by a synchronous orbit satellite antenna is 480 W. The antenna has a gain of 40 dB at 15 GHz. The earth station is located at distance of 24,567 km. If the antenna of earth station has a gain of 32 dB, the power received is
(A) 32 pW  
(B) 3.2 fW  
(C) 10.2 pW  
(D) 1.3 fW

18. The directive gain of an antenna is 36 dB. If the antenna radiates 15 kW at a distance of 60 km, the time average power density at that distance is
(A) 9.42 μW/m²  
(B) 6.83 mW/m²  
(C) 1.32 mW/m²  
(D) 10.46 mW/m²

19. Two identical antenna separated by 12 m are oriented for maximum directive gain. At a frequency of 5 GHz, the power received by one is 30 dB down from the transmitted by the other. The gain of antenna is
(A) 22 dB  
(B) 16 dB  
(C) 19 dB  
(D) 13 dB

Statement for Q.20–21:
An L-band pulse radar has common transmitting and receiving antenna. The antenna having directive gain of 36 dB operates at 1.5 GHz and transmits 200 kW. The object is 120 km from the radar and its scattering cross section is 8 m².

20. The magnitude of the incident electric field intensity of the object is
(A) 1.82 V/m  
(B) 2.46 V/m  
(C) 0.34 V/m  
(D) 0.17 V/m

21. The magnitude of the scattered electric field at the radar is
(A) 18 μW  
(B) 12 μW  
(C) 17 mW  
(D) 126 mW

22. A transmitting antenna with a 300 MHz carrier frequency produces 2 kW of power. If both antennas has unity power gain, the power received by another antenna at a distance of 1 km is
(A) 11.8 mW  
(B) 18.4 mW  
(C) 18.4 μW  
(D) 12.7 μW

23. A bistatic radar system shown in fig. P8.7.23 has following parameters: \( f = 5 \text{ GHz} \), \( G_{dr} = 34 \text{ dB} \), \( G_{dr} = 22 \text{ dB} \). To obtain a return power of 8 pW the minimum necessary radiated power is
(A) 1.394 kW  
(B) 2.046 kW  
(C) 1.038 kW  
(D) 3.46 kW

24. The radiation resistance of an antenna is 63 Ω and loss resistance 7 Ω. If antenna has power gain of 16, then directivity is
(A) 48.26 dB  
(B) 12.5 dB  
(C) 38.96 dB  
(D) 24.7 dB
25. An antenna is desired to operate on a frequency of 40 MHz whose quality factor is 50. The bandwidth of antenna is
(A) 5.03 MHz  
(B) 800 kHz  
(C) 127 kHz  
(D) None of the above

26. A thin dipole antenna is \( \lambda/15 \) long. If its loss resistance is 1.2 \( \Omega \), the efficiency is
(A) 41.1%  
(B) 59%  
(C) 74.5%  
(D) 25.5%

**Statement for Q.27–29:**

An array comprises of two dipoles that are separated by the wavelength. The dipoles are fed by currents of the same magnitude and phase.

**27. The array factor is**

(A) \( 2 \cos (\pi \cos \theta + 45^\circ) \)  
(B) \( 2 \cos (\pi \sin \theta) \)  
(C) \( 2 \cos (\pi \sin \theta + 45^\circ) \)  
(D) \( 2 \cos (\pi \cos \theta) \)

**28. The nulls of the pattern occur when \( \theta \) is**

(A) 30°, 150°  
(B) 60°, 120°  
(C) 45°, 135°  
(D) 0°, 180°

**29. The maximum of the pattern occur at**

(A) \( \theta = 45^\circ, 135^\circ \)  
(B) \( \theta = 0°, 90°, 180° \)  
(C) \( \theta = 30°, 150° \)  
(D) \( \theta = 60°, 150° \)

**30. An array comprises two dipoles that are separated by half wavelength. If the dipoles are fed by currents, that are 180° out of phase with each other, then array factor is**

(A) \( \sin \left(\frac{\pi}{2} \cos \theta + \frac{\pi}{2}\right) \)  
(B) \( \cos \left(\frac{\pi}{2} \cos \theta + \frac{\pi}{2}\right) \)  
(C) \( \cos \left(\frac{\pi}{2} \cos \theta + \frac{\pi}{2}\right) \)  
(D) \( \sin \left(\frac{\pi}{2} \cos \theta + \frac{\pi}{2}\right) \)

**31. An antenna consists of 4 identical Hertizian dipoles uniformly located along the z-axis and polarized in the z-direction. The spacing between the dipole is \( \lambda/4 \). The group pattern function is**

(A) \( 4 \cos \left(\frac{\pi}{2} \cos \theta \right) \cos \left(\frac{\pi}{2} \cos \theta \right) \)  
(B) \( 4 \cos \left(\frac{\pi}{2} \cos \theta \right) \cos \left(\frac{\pi}{2} \cos \theta \right) \)  
(C) \( 4 \cos \left(\frac{\pi}{2} \cos \theta \right) \sin \left(\frac{\pi}{2} \cos \theta \right) \)  
(D) \( 4 \cos \left(\frac{\pi}{2} \cos \theta \right) \sin \left(\frac{\pi}{2} \cos \theta \right) \)

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**SOLUTIONS**

1. (B) \( \beta = \frac{\omega}{c} = \frac{2\pi \times 10^7}{3 \times 10^8} = \frac{2\pi}{30} \)

At far field \( |E_o| = \frac{\eta \beta dl}{4\pi r} \sin \theta \)

\( \eta = 120\pi = 377 \), \( I_o = 20 \), \( dl = 10 \) cm

At \( (100 \) cm, 0, 0), \( \theta = \frac{\pi}{2} \)

\( |E_o| = \frac{120\pi \times 20 \times 0.1}{4\pi \times 100} \times \frac{2\pi}{30} = 0.126 \) V/m

2. (A) \( \beta = \frac{\omega}{c} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \)

\( r = 10 \) m, \( \theta = 30^\circ \), \( \phi = 90^\circ \)

At far field \( H = H_\theta = \frac{jI \beta dl}{4\pi r} \sin \theta e^{-j\mu r} \)

\( \Rightarrow H_\theta = \frac{j(2.5)(2\pi)(4 \times 10^{-3})}{4\pi(10)} e^{-j2\pi0} = j0.25 \) mA/m

3. (C) \( E = E_o = \eta H_\theta = 377 H_\theta = j94.25 \) mV/m

4. (B) \( \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^6} = 100 \)

\( dl = \frac{4}{100} = \frac{1}{25} = 0.04 \) m

\( R_{rad} = 80\pi \left( \frac{dl}{\lambda} \right)^2 = \frac{80\pi^2}{625} = 1263 \) \( \Omega \)

5. (C) \( P_{rad} = \int P_{ave} \cdot dS = P_{ave} \cdot 2\pi r^2 \)

\( P_{ave} = \frac{P_{rad}}{2\pi r^2} = \frac{150 \times 10^3}{2\pi(50 \times 10^3)^2} = 9.55 \) \( \mu W/m^2 \)

\( P = 9.55uW, \) \( \mu W/m^2 \)

6. (B) \( P_{ave} = \frac{(E_{max})^2}{2\eta} \Rightarrow E_{max} = \sqrt{2\eta P_{ave}} \)

\( \Rightarrow E_{max} = \sqrt{2 \times 377 \times 9.55 \times 10^{-6}} = 85 \) mV/m

7. (B) \( P_{ave} = \frac{|E|^2}{2\eta} \)

\( P_{rad} = \int \frac{100 \sin^2 2\theta}{2\eta} \sin \theta d\theta d\phi \)

\( = \frac{100}{2 \times 120\pi} \int (2\sin^2 \theta \cos^2 \theta) \sin \theta d\theta \)

\( = \frac{10}{3} \int \sin^3 \theta \cos \theta \cdot 0 \) d\theta = 0.89 \) W
20. (A) \( G_d = 36 \text{ dB} = 3981 \)

\[
G_d = \frac{4\pi^2P_i}{P_{rad}} \Rightarrow P_i = \frac{G_dP_{rad}}{4\pi^2} = \frac{|E|^2}{2\eta}
\]

\[
|E| = \sqrt{\frac{(240\pi)G_dP_{rad}}{4\pi^2}} = \sqrt{\frac{(60)(3981)(200 \times 10^3)}{(120 \times 10^3)^2}} = 1.82 \text{ V/m}
\]

21. (B) \( |E_x| = \frac{|E|^2\sigma}{4\pi r} = \frac{|E|}{r} \sqrt{\frac{\sigma}{4\pi}} = \frac{1.82}{120 \times 10^3} \sqrt{\frac{8}{4\pi}} = 12 \mu \text{W} \)

22. (D) \( \lambda = c \omega = \frac{3 \times 10^8}{300 \times 10^6} = 1 \text{ m} \)

\[
P_r = G_dG_{dr} \left( \frac{\lambda}{4\pi} \right)^2 P = (1)(1) \left( \frac{1}{4\pi^2} \right)^2 2000 = 12.7 \mu \text{W}
\]

23. (C) \( P_r = G_dG_{dr} \left( \frac{\lambda}{4\pi} \right)^2 \sigma P_{rad} \)

\[
G_{dr} = 34 \text{ dB} = 2512, \quad G_{dr} = 22 \text{ dB} = 1585 \]

\( r_1 = 3 \text{ km}, \quad r_2 = \sqrt{3^2 + 4^2} = 5 \text{ km} \)

\( \lambda = \frac{3 \times 10^8}{5 \times 10^3} = 0.06 \text{ m}, \quad P_r = 8 \text{ pW} \)

\[
8 \times 10^{-12} = \left( \frac{2512)(1585)}{4\pi} \right) \left( \frac{0.06}{4\pi(3k)(5k)} \right)^2 (2.4)P_{rad}
\]

\[\Rightarrow P_{rad} = 1.038 \text{ kW}\]

24. (B) Efficiency = \( \frac{63}{63 + 7} = 0.9 \)

\[D = \frac{\text{Gain}}{\text{Efficiency}} = \frac{16}{0.9} = 17.78 = 125 \text{ dB}\]

25. (B) \( BW = \frac{f}{Q} = \frac{40 \times 10^6}{50} = 800 \text{ kHz}\)

26. (C) Radiation resistance \( R_r = 80\pi^2 \left( \frac{dl}{l} \right)^2 \)

\[
= 80 \times \pi^2 \left( \frac{1}{15} \right)^2 = 351 \Omega
\]

Efficiency = \( \frac{R_r}{R_r + R_L} = \frac{351}{351 + 12} = 74.5% \)

27. (D) \( \beta d = \frac{2\pi}{\lambda} = 2\pi, \quad \alpha = 0 \)

\[
AF = 2\cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) = 2 \cos (\pi \cos \theta) = 2 \cos (\pi \cos \theta)
\]

28. (B) \( \cos (\pi \cos \theta) = 0 \)

\[
\Rightarrow \pi \cos \theta = 120^\circ, \quad 120^\circ
\]

29. (B) Maxima occur when \( \frac{d(AF)}{d\theta} = 0 \)

\( \sin (\pi \cos \theta) \pi \cos \theta = 0 \Rightarrow \theta = 0, 90^\circ, 180^\circ \)

30. (B) \( \beta d = \frac{2\pi}{\lambda} = \frac{\pi}{2}, \quad \alpha = \pi \)

\[
AF = 2\cos \left( \frac{\beta d \cos \theta + \alpha}{2} \right) = \cos \left( \frac{\pi}{4} \cos \theta \right)
\]

31. (A) \( (AF)_N = \frac{\sin \left( \frac{N\theta}{2} \right)}{\sin \left( \frac{\psi}{2} \right)} \)

\( \psi = \beta d \cos \theta + \alpha, \quad N = 4 \)

\[
\frac{\sin 4x}{\sin x} = \frac{2 \sin 2x \cos 2x}{\sin x} = 4 \cos x \cos 2x
\]

\( \beta d = \frac{2\pi}{\lambda} = \frac{\pi}{4}, \quad \alpha = 0, \quad \psi = \frac{\pi}{4} \cos \theta \)

\[
AF = 4 \cos \left( \frac{\pi}{4} \cos \theta \right) \cos \left( \frac{\pi}{2} \cos \theta \right)
\]

**************
1. If \( A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -2 & \lambda \end{bmatrix} \) is a singular matrix, then \( \lambda \) is
   (A) 0  
   (B) 2  
   (C) 2  
   (D) 1

2. If \( A \) and \( B \) are square matrices of order \( 4 \times 4 \) such that \( A = 5B \) and \( |A| = \alpha \cdot |B| \), then \( \alpha \) is
   (A) 5  
   (B) 25  
   (C) 625  
   (D) None of these

3. If \( A \) and \( B \) are square matrices of the same order such that \( AB = A \) and \( BA = A \), then \( A \) and \( B \) are both
   (A) Singular  
   (B) Idempotent  
   (C) Involutory  
   (D) None of these

4. The matrix, \( A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix} \) is
   (A) Idempotent  
   (B) Involutory  
   (C) Singular  
   (D) None of these

5. Every diagonal elements of a skew–symmetric matrix is
   (A) 1  
   (B) 0  
   (C) Purely real  
   (D) None of these

6. The matrix, \( A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \) is
   (A) Orthogonal  
   (B) Idempotent  
   (C) Unitary  
   (D) None of these

7. Every diagonal elements of a Hermitian matrix is
   (A) Purely real  
   (B) 0  
   (C) Purely imaginary  
   (D) 1

8. Every diagonal element of a Skew–Hermitian matrix is
   (A) Purely real  
   (B) 0  
   (C) Purely imaginary  
   (D) 1

9. If \( A \) is Hermitian, then \( iA \) is
   (A) Symmetric  
   (B) Skew–symmetric  
   (C) Hermitian  
   (D) Skew–Hermitian

10. If \( A \) is Skew–Hermitian, then \( iA \) is
    (A) Symmetric  
    (B) Skew–symmetric  
    (C) Hermitian  
    (D) Skew–Hermitian

11. If \( A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 3 & -2 & 1 \end{bmatrix} \), then adj. \( A \) is equal to
    (A) \( A \)  
    (B) \( A' \)  
    (C) \( 3A' \)  
    (D) \( 3A \)

12. The inverse of the matrix \( \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix} \) is
    (A) \( \begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix} \)  
    (B) \( \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} \)  
    (C) \( \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix} \)  
    (D) None of these
13. Let \( \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 3 & 1 & 2 \end{bmatrix} \), then \( \mathbf{A}^{-1} \) is equal to
(A) \( \begin{bmatrix} 2 & 0 & 0 \\ 10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \) 
(B) \( \begin{bmatrix} -5 & 1 & 0 \\ -1 & -1 & 2 \end{bmatrix} \) 
(C) \( \begin{bmatrix} 1 & 0 & 0 \\ -10 & 2 & 0 \\ -1 & -1 & 2 \end{bmatrix} \) 
(D) None of these

14. If the rank of the matrix, \( \mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 7 & \lambda \\ 1 & 4 & 5 \end{bmatrix} \) is 2, then the value of \( \lambda \) is
(A) -13 
(B) 13 
(C) 3 
(D) None of these

15. Let \( \mathbf{A} \) and \( \mathbf{B} \) be non–singular square matrices of the same order. Consider the following statements.
(I) \( (\mathbf{AB})^T = \mathbf{A}^T \mathbf{B}^T \) 
(II) \( (\mathbf{AB})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1} \) 
(III) \( \text{adj}(\mathbf{AB}) = (\text{adj} \mathbf{A})(\text{adj} \mathbf{B}) \) 
(IV) \( \rho(\mathbf{AB}) = \rho(\mathbf{A})\rho(\mathbf{B}) \) 
(V) \( |\mathbf{AB}| = |\mathbf{A}| \cdot |\mathbf{B}| \)
Which of the above statements are false ?
(A) I, III & IV 
(B) IV & V 
(C) I & II 
(D) All the above

16. The rank of the matrix \( \mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix} \) is
(A) 3 
(B) 2 
(C) 1 
(D) None of these

17. The system of equations \( 3x - y + z = 0, \) \( 15x - 6y + 5z = 0, \) \( \lambda x - 2y + 2z = 0 \) has a non–zero solution, if \( \lambda \) is
(A) 6 
(B) -6 
(C) 2 
(D) -2

18. The system of equation \( x - 2y + z = 0, \) \( 2x - y + 3z = 0, \) \( \lambda x + y - z = 0 \) has the trivial solution as the only solution, if \( \lambda \) is
(A) \( \lambda \neq -\frac{4}{3} \) 
(B) \( \lambda = \frac{4}{3} \) 
(C) \( \lambda = 2 \) 
(D) None of these

19. The system equations \( x + y + z = 6, \) \( x + 2y + 3z = 10, \) \( x + 2y + \lambda z = 12 \) is inconsistent, if \( \lambda \) is
(A) 3 
(B) -3 
(C) 0 
(D) None of these

20. The system of equations \( 5x + 3y + 7z = 4, \) \( 3x + 26y + 2z = 9, \) \( 7x + 2y + 10z = 5 \) has
(A) a unique solution 
(B) no solution 
(C) an infinite number of solutions 
(D) none of these

21. If \( \mathbf{A} \) is an \( n \)–row square matrix of rank \((n - 1)\), then
(A) \( \text{adj} \mathbf{A} = 0 \) 
(B) \( \text{adj} \mathbf{A} \neq 0 \) 
(C) \( \text{adj} \mathbf{A} = \mathbf{I}_n \) 
(D) None of these

22. The system of equations \( x - 4y + 7z = 14, \) \( 3x + 8y - 2z = 13, \) \( 7x - 8y + 26z = 5 \) has
(A) a unique solution 
(B) no solution 
(C) an infinite number of solutions 
(D) none of these

23. The eigen values of \( \mathbf{A} = \begin{bmatrix} 3 & 4 \\ 9 & -5 \end{bmatrix} \) are
(A) \pm 1 
(B) 1, 1 
(C) -1, -1 
(D) None of these

24. The eigen values of \( \mathbf{A} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \) are
(A) 0, 3, -15 
(B) 0, -3, -15 
(C) 0, 3, 15 
(D) 0, -3, 15

25. If the eigen values of a square matrix be 1, -2 and 3, then the eigen values of the matrix \( 2\mathbf{A} \) are
(A) \( \frac{1}{2}, -1, \frac{3}{2} \) 
(B) 2, -4, 6 
(C) 1, -2, 3 
(D) None of these

26. If \( \mathbf{A} \) is a non–singular matrix and the eigen values of \( \mathbf{A} \) are \( 2, 3, -3 \) then the eigen values of \( \mathbf{A}^{-1} \) are
(A) \( 2, 3, -3 \) 
(B) \( \frac{1}{2}, \frac{1}{3}, -\frac{1}{3} \) 
(C) \( 2|\mathbf{A}|, 3|\mathbf{A}| - 3|\mathbf{A}| \) 
(D) None of these
27. If $-1, 2, 3$ are the eigen values of a square matrix $A$ then the eigen values of $A^2$ are
(A) $-1, 2, 3$  
(B) $1, 4, 9$  
(C) $1, 2, 3$  
(D) None of these

28. If $2, -4$ are the eigen values of a non-singular matrix $A$ and $|A| = 4$, then the eigen values of $\text{adj} A$ are
(A) $\frac{1}{2}, -1$  
(B) $2, -1$  
(C) $2, -4$  
(D) $8, -16$

29. If $2$ and $4$ are the eigen values of $A$ then the eigenvalues of $A^r$ are
(A) $\frac{1}{2}, \frac{1}{4}$  
(B) $2, 4$  
(C) $4, 16$  
(D) None of these

30. If $1$ and $3$ are the eigenvalues of a square matrix $A$ then $A^3$ is equal to
(A) $13A - I_3$  
(B) $13A - 12I_3$  
(C) $12(A - I_3)$  
(D) None of these

31. If $A$ is a square matrix of order $3$ and $|A| = 2$ then $A (\text{adj} A)$ is equal to
(A) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$  
(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  
(C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$  
(D) None of these

32. The sum of the eigenvalues of $A = \begin{bmatrix} 8 & 2 & 3 \\ 4 & 5 & 9 \\ 2 & 0 & 5 \end{bmatrix}$ is equal to
(A) $18$  
(B) $15$  
(C) $10$  
(D) None of these

33. If $1, 2$ and $5$ are the eigenvalues of the matrix $A$ then $|A|$ is equal to
(A) $8$  
(B) $10$  
(C) $9$  
(D) None of these

34. If the product of matrices
$$A = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}$$

is a null matrix, then $\theta$ and $\phi$ differ by
(A) an odd multiple of $\pi$  
(B) an even multiple of $\pi$  
(C) an odd multiple of $\frac{\pi}{2}$  
(D) an even multiple $\frac{\pi}{2}$

35. If $A$ and $B$ are two matrices such that $A + B$ and $AB$ are both defined, then $A$ and $B$ are
(A) both null matrices  
(B) both identity matrices  
(C) both square matrices of the same order  
(D) None of these

36. If $A = \begin{bmatrix} 0 & -\tan \frac{n \pi}{2} \\ \tan \frac{n \pi}{2} & 0 \end{bmatrix}$

then $(I - A) \begin{bmatrix} \cos \alpha & -\sin \frac{n \pi}{2} \\ \sin \alpha & \cos \alpha \end{bmatrix}$ is equal to
(A) $I + A$  
(B) $I - A$  
(C) $I + 2A$  
(D) $I - 2A$

37. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then for every positive integer $n$, $A^n$ is equal to
(A) $\begin{bmatrix} 1 + 2n & 4n \\ n & 1 + 2n \end{bmatrix}$  
(B) $\begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$  
(C) $\begin{bmatrix} 1 - 2n & 4n \\ n & 1 + 2n \end{bmatrix}$  
(D) None of these

38. If $A_\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ then consider the following statements:
I. $A_\alpha A_\beta = A_{\alpha \beta}  
II. A_\alpha A_\beta = A_{(\alpha + \beta)}$
III. $(A_\alpha)^n = \begin{bmatrix} \cos^n \alpha & \sin^n \alpha \\ -\sin^n \alpha & \cos^n \alpha \end{bmatrix}$
IV. $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$

Which of the above statements are true?
(A) I and II  
(B) I and IV  
(C) II and III  
(D) II and IV
39. If $A$ is a 3-rowed square matrix such that $|A| = 3$, then $\text{adj}(\text{adj} A)$ is equal to:
(A) $3A$  
(B) $9A$  
(C) $27A$  
(D) none of these

40. If $A$ is a 3-rowed square matrix, then $|\text{adj}(\text{adj} A)|$ is equal to:
(A) $|A|^6$  
(B) $|A|^3$  
(C) $|A|^2$  
(D) $|A|^1$

41. If $A$ is a 3-rowed square matrix such that $|A| = 2$, then $|\text{adj}(\text{adj} A^2)|$ is equal to:
(A) $2^4$  
(B) $2^8$  
(C) $2^{16}$  
(D) None of these

42. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of $x$ is:
(A) 1  
(B) 2  
(C) $\frac{1}{2}$  
(D) None of these

43. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ then $A^{-1}$ is:
(A) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$  
(B) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \\ 1 & 2 \end{bmatrix}$  
(C) $\begin{bmatrix} 2 & 3 \\ 3 & 1 \\ 2 & 7 \end{bmatrix}$  
(D) Undefined

44. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix}$ then $AB$ is:
(A) $\begin{bmatrix} -1 & -8 \\ -10 & -10 \end{bmatrix}$  
(B) $\begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix}$  
(C) $\begin{bmatrix} 1 & -8 \\ 9 & 22 \\ 15 \end{bmatrix}$  
(D) $\begin{bmatrix} 0 & -8 \\ -10 & -10 \end{bmatrix}$

45. If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$, then $AA^T$ is:
(A) $\begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$  
(B) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  
(C) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  
(D) Undefined

46. The matrix, that has an inverse is:
(A) $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$  
(B) $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$  
(C) $\begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix}$  
(D) $\begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix}$

47. The skew symmetric matrix is:
\[ \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & 6 \\ -5 & -6 & 0 \end{bmatrix} \]

48. If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, the product of $A$ and $B$ is:
(A) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  
(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  
(C) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$  
(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

49. Matrix $D$ is an orthogonal matrix $D = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ The value of $|B|$ is:
(A) $\frac{1}{2}$  
(B) $\frac{1}{\sqrt{2}}$  
(C) 1  
(D) 0

50. If $A_{n \times n}$ is a triangular matrix then $\text{det} A$ is:
(A) $\prod_{i=1}^{n}(-1)^{i}a_{ii}$  
(B) $\prod_{i=1}^{n}a_{ii}$  
(C) $\sum_{i=1}^{n}(-1)^{i}a_{ii}$  
(D) $\sum_{i=1}^{n}a_{ii}$
51. If \( A = \begin{bmatrix} t^2 & \cos t \\ e^t & \sin t \end{bmatrix} \), then \( \frac{dA}{dt} \) will be

(A) \( \begin{bmatrix} t^2 & \sin t \\ e^t & \cos t \end{bmatrix} \)  
(B) \( \begin{bmatrix} 2t & \cos t \\ e^t & \sin t \end{bmatrix} \)  
(C) \( \begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix} \)  
(D) Undefined

52. If \( A \in \mathbb{R}_{n \times n} \), \( \det A \neq 0 \), then

(A) \( A \) is non singular and the rows and columns of \( A \) are linearly independent.
(B) \( A \) is non singular and the rows \( A \) are linearly dependent.
(C) \( A \) is non singular and the \( A \) has one zero rows.
(D) \( A \) is singular.

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**SOLUTIONS**

1. (B) \( A \) is singular if \( |A|=0 \)

\[
\begin{bmatrix}
0 & 1 & -2 \\
-1 & 0 & 3 \\
2 & -2 & \lambda \\
\end{bmatrix} = 0
\]

\[
\begin{align*}
\Rightarrow & \quad (-1)^1 1^2 \lambda + 2^1 \lambda + 3^0 \lambda + (-2)^2 \lambda + 0^3 \lambda = 0 \\
\Rightarrow & \quad \lambda - 4 + 2(3) = 0 \\
\Rightarrow & \quad \lambda - 4 + 6 = 0 \\
\Rightarrow & \quad \lambda = -2
\end{align*}
\]

2. (C) If \( k \) is a constant and \( A \) is a square matrix of order \( n \times n \) then \( \det kA = k^n \det A \)

\( A = 5B \Rightarrow |A| = |5B| = 5^n |B| = 625 |B| \)

\( \Rightarrow \alpha = 625 \)

3. (B) \( A \) is singular, if \( |A|=0 \),

\( A \) is Idempotent, if \( A^2 = A \)
\( A \) is Involutory, if \( A^2 = I \)

Now, \( A^2 = AA = (AB) A = A(BA) = AB = A \)

and \( B^2 = BB = (BA)B = B(AB) = BA = B \)

\( \Rightarrow \quad A^2 = A \) and \( B^2 = B \),

Thus \( A \) & \( B \) both are Idempotent.

4. (B) Since, \( A^2 = \begin{bmatrix}
-5 & -8 & 0 \\
3 & 5 & 0 \\
1 & 2 & -1 \\
\end{bmatrix} \)

\[ \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} = I \]

\( \Rightarrow \quad A^2 = I \)

5. (B) Let \( A = [a_{ij}] \) be a skew-symmetric matrix, then

\( A^T = -A \), \( \Rightarrow \quad a_{ij} = -a_{ji} \),

if \( i = j \) then \( a_{ii} = -a_{ii} \) \( \Rightarrow \quad 2a_{ii} = 0 \) \( \Rightarrow \quad a_{ii} = 0 \)

Thus diagonal elements are zero.

6. (C) \( A \) is orthogonal if \( AA^T = I \)

\( A \) is unitary if \( AA^q = I \), where \( A^q \) is the conjugate transpose of \( A \) i.e., \( A^q = \overline{(A^T)} \).

Here,

\[
AA^q = \begin{bmatrix}
1 & i \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
i & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
i & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} = I_2
\]

Thus \( A \) is unitary.
7. (A) A square matrix $A$ is said to be Hermitian if $A^H = A$. So $a_{ji} = \bar{a}_{ij}$. If $i = j$ then $a_{ii} = \bar{a}_{ii}$ i.e. conjugate of an element is the element itself and $a_{ii}$ is purely real.

8. (C) A square matrix $A$ is said to be Skew-Hermitian if $A^H = -A$. If $A$ is Skew-Hermitian then $A^H = -A$ $\Rightarrow$ $\bar{a}_{ji} = -a_{ij}$, $i$ $j$, if $i = j$ then $\bar{a}_{ii} = -a_{ii} \Rightarrow a_{ii} + \bar{a}_{ii} = 0$

it is only possible when $a_{ii}$ is purely imaginary.

9. (D) $A$ is Hermitian then $A^H = A$

Now, $(iA)^H = iA^H = -iA^H = -iA$, $\Rightarrow$ $(iA)^H = -(iA)$

Thus $iA$ is Skew-Hermitian.

10. (C) $A$ is Skew–Hermitian then $A^H = -A$

Now, $(iA)^H = iA^H = -(A) = iA$ then $iA$ is Hermitian.

11. (C) If $A = [a_{ij}]_{n \times n}$ then $\det A = \left[ c_{ij} \right]_{n \times n}$

Where $c_{ij}$ is the cofactor of $a_{ij}$

Also $c_{ij} = (-1)^{i+j}M_{ij}$, where $M_{ij}$ is the minor of $a_{ij}$, obtained by leaving the row and the column corresponding to $a_{ij}$ and then take the determinant of the remaining matrix.

Now, $M_{11}$ minor of $a_{11}$ i.e. $-1 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} = -3$

Similarly

$M_{12} = \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = 6$ ; $M_{13} = \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} = -6$

$M_{21} = \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = 6$ ; $M_{22} = \begin{vmatrix} 2 & -2 \\ 2 & 1 \end{vmatrix} = 3$

$M_{31} = \begin{vmatrix} 2 & -2 \\ 2 & -2 \end{vmatrix} = 6$ ; $M_{32} = \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = 1$

$C_{11} = (-1)^{1+1}M_{11} = -3$ ; $C_{12} = (-1)^{1+2}M_{12} = -6$ ; $C_{13} = (-1)^{1+3}M_{13} = -6$ ; $C_{21} = (-1)^{2+1}M_{21} = 6$ ; $C_{22} = (-1)^{2+2}M_{22} = 3$ ; $C_{23} = (-1)^{2+3}M_{23} = -6$ ; $C_{32} = (-1)^{3+2}M_{32} = -6$ ; $C_{33} = (-1)^{3+3}M_{33} = 3$

$\det A = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$

$= \begin{vmatrix} -3 & 6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{vmatrix}$

$= \begin{vmatrix} 1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

$= 3A^T$

12. (A) Since $A^{-1} = \frac{1}{|A|} \text{adj} A$

Now, Here $|A| = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -1$

Also, $\text{adj} A = \begin{array}{cc} -5 & -3 \\ -2 & -1 \end{array}$ $\Rightarrow$ $\text{adj} A = \begin{array}{cc} -5 & 2 \\ -3 & -1 \end{array}$

$A^{-1} = \frac{1}{-1} \begin{array}{cc} -5 & 2 \\ -3 & -1 \end{array} = \begin{array}{cc} 5 & 2 \\ 3 & 1 \end{array}$

13. (A) Since, $A^{-1} = \frac{1}{|A|} \text{adj} A$

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 1 & 2 \end{vmatrix} = 4 \neq 0,$

$\text{adj} A = \begin{vmatrix} 4 & 10 & -10 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 10 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{vmatrix}$

$A^{-1} = \frac{1}{4} \begin{vmatrix} 4 & 10 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 10 & 2 & 0 \\ -1 & -1 & 2 \end{vmatrix}$

14. (B) A matrix $A_{(m \times n)}$ is said to be of rank $r$ if

(i) it has at least one non–zero minor of order $r$, and
(ii) all other minors of order greater than $r$, if any; are zero.

The rank of $A$ is denoted by $r(A)$. Now, given that $r(A) = 2 \Rightarrow$ minor of order greater than 2 i.e., 3 is zero.

$|A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & 5 \end{vmatrix}$

$\Rightarrow 2(35 - 4\lambda) + 1(20 - \lambda) + 3(16 - 7) = 0,

\Rightarrow 10 - 8\lambda + 20 - \lambda + 27 = 0,

\Rightarrow 9\lambda = 117 \Rightarrow \lambda = 13$

15. (A) The correct statements are

$(AB)^T = B^T A^T$, $(AB)^{-1} = B^{-1} A^{-1}$

$\text{adj} (AB) = \text{adj} (B) \text{adj} (A)$

$\rho(AB) = \rho(A) \rho(B)$, $|AB| = |A| |B|$

Thus statements I, II, and IV are wrong.

16. (B) Since

$|A| = 2(-9 + 8) + 2(-2 + 3) = -2 + 2 = 0$

$\Rightarrow \rho(A) < 3$

Again, one minor of order 2 is $\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = 6 \neq 0$

$\Rightarrow \rho(A) = 2$
24. (C) Characteristic equation is \(|A - \lambda I| = 0\)
\[
\begin{vmatrix}
8 - \lambda & -6 & 2 \\
-6 & 7 - \lambda & -4 \\
2 & -4 & 3 - \lambda
\end{vmatrix} = 0
\]
\[\Rightarrow 5 - 18\lambda + 45\lambda = 0\]
\[\Rightarrow \lambda^2 - 3(\lambda - 15) = 0 \Rightarrow \lambda = 0, 3, 15\]
Thus eigen values are \(-1, -1\).

25. (B) If eigen values of \(A\) are \(\lambda_1, \lambda_2, \lambda_3\) then the eigen values of \(kA\) are \(k\lambda_1, k\lambda_2, k\lambda_3\). So the eigen values of \(2A\) are \(2, -4\) and \(6\).

26. (B) If \(\lambda_1, \lambda_2, \ldots, \lambda_n\) are the eigen values of a non-singular matrix \(A\), then \(A^{-1}\) has the eigen values \(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots, \frac{1}{\lambda_n}\). Thus eigen values of \(A^{-1}\) are \(\frac{1}{2}, \frac{1}{3}, \frac{1}{3}\).

27. (B) If \(\lambda_1, \lambda_2, \ldots, \lambda_n\) are the eigen values of a matrix \(A\), then \(A^2\) has the eigen values \(\lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2\). So, eigen values of \(A^2\) are \(1, 4, 9\).

28. (B) If \(\lambda_1, \lambda_2, \ldots, \lambda_n\) are the eigen values of \(A\) then the eigen values of \(\text{adj} A\) are \(\frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2}, \ldots, \frac{|A|}{\lambda_n}\). Thus eigen values of \(\text{adj} A\) are \(\frac{4}{2}, \frac{-4}{4}\). i.e. \(2\) and \(-1\).

29. (B) Since, the eigen values of \(A\) and \(A^T\) are square so the eigen values of \(A^T\) are \(2\) and \(4\).

30. (B) Since \(1\) and \(3\) are the eigen values of \(A\) so the characteristic equation of \(A\) is
\[(\lambda - 1)(\lambda - 3) = 0 \Rightarrow \lambda^2 - 4\lambda + 3 = 0\]
Also, by Cayley–Hamilton theorem, every square matrix satisfies its own characteristic equation so
\[A^2 - 4A + 3I = 0\]
\[\Rightarrow A^2 = 4A - 3I\]
\[\Rightarrow A^3 = 4A^2 - 3A = 4(4A - 3I) - 3A\]
\[\Rightarrow A^3 = 13A - 12I\]

31. (A) Since \(A(\text{adj} A) = |A|I\)
\[
\begin{bmatrix}
1 & 0 & 0 \\
2 & 0 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]
\[\Rightarrow A(\text{adj} A) = 2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2 \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}
\]

32. (A) Since the sum of the eigen values of an \(n\)-square matrix is equal to the trace of the matrix (i.e. sum of the diagonal elements)
so, required sum = \(8 + 5 + 5 = 18\)

33. (B) Since the product of the eigen values is equal to the determinant of the matrix so \(|A| = 1 \times 2 \times 5 = 10\)

34. (C)
\[AB = \begin{bmatrix}
\cos \theta & \cos \phi \\
\sin \theta & \sin \phi
\end{bmatrix} \begin{bmatrix}
\cos \theta & \cos \phi \\
\sin \theta & \sin \phi
\end{bmatrix} = A\]
null matrix when \(\cos (\theta - \phi) = 0\)
This happens when \(\theta - \phi\) is an odd multiple of \(\frac{\pi}{2}\).

35. (C) Since \(A + B\) is defined, \(A\) and \(B\) are matrices of the same type, say \(m \times n\). Also, \(AB\) is defined. So, the number of columns in \(A\) must be equal to the number of rows in \(B\) i.e. \(n = m\). Hence, \(A\) and \(B\) are square matrices of the same order.

36. (A) Let \(\tan \frac{\alpha}{2} = t\), then, \(\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{1 - t^2}{t + t^2}\)
and \(\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} = \frac{2t}{1 + t^2}\)
\[\begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix} \cdot \begin{bmatrix}
\tan \frac{\alpha}{2} \\
-\tan \frac{\alpha}{2}
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
\cos \alpha & -\sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix} \begin{bmatrix}
\tan \frac{\alpha}{2} \\
-\tan \frac{\alpha}{2} + t
\end{bmatrix} \cdot \begin{bmatrix}
1 \\
0
\end{bmatrix}
\]
\[\begin{bmatrix}
1 & t \\
t & 1
\end{bmatrix} \cdot \begin{bmatrix}
\tan \frac{\alpha}{2} - \frac{1 - t^2}{1 + t^2} \\
\frac{2t}{(1 + t^2)} - \frac{1 - t^2}{1 + t^2}
\end{bmatrix} = \begin{bmatrix}
1 & -\tan \frac{\alpha}{2} \\
\tan \frac{\alpha}{2} & 1
\end{bmatrix} = (I + A)\]
37. (B) \[ \mathbf{A}^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} \]
\[ = \left[ 1 + 2n - 4n \right] \begin{bmatrix} 1 & -2n \\ n & 1-2n \end{bmatrix}, \text{ where } n = 2. \]

38. (D) \[ \mathbf{A}_a \cdot \mathbf{A}_b = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \]
\[ = \begin{bmatrix} \cos (\alpha + \beta) & \sin (\alpha + \beta) \\ -\sin (\alpha + \beta) & \cos (\alpha + \beta) \end{bmatrix} = \mathbf{A}_{a+b} . \]

Also, it is easy to prove by induction that
\[ (\mathbf{A}_a)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix} \]

39. (A) We know that \( \text{adj}(\text{adj} \mathbf{A}) = |\mathbf{A}|^{-2} \cdot \mathbf{A} \).
Here \( n = 3 \) and \( |\mathbf{A}| = 3 \).
So, \( \text{adj}(\text{adj} \mathbf{A}) = 3^{3-3} \cdot \mathbf{A} = 3 \mathbf{A} \).

40. (C) We have \( |\text{adj} (\text{adj} \mathbf{A})| = |\mathbf{A}|^{(n-1)2} \).
Putting \( n = 3 \), we get \( |\text{adj} (\text{adj} \mathbf{A})| = |\mathbf{A}|^4 \).

41. (C) Let \( \mathbf{B} = \text{adj} (\text{adj} \mathbf{A}^2) \).
Then, \( \mathbf{B} \) is also a \( 3 \times 3 \) matrix.
\[ |\text{adj} (\text{adj} (\mathbf{A}^2))| = |\text{adj} \mathbf{B}| = |\mathbf{B}^{[n-1]}| = |\mathbf{B}| \]
\[ = |\text{adj} (\text{adj} \mathbf{A}^2)|^2 = |\mathbf{A}^2|^{(n-1)2} = |\mathbf{A}|^{4n} = 2^n \]
\[ \Rightarrow |\mathbf{A}|^2 = |\mathbf{A}|^2 \]

42. (C) \[ \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
\[ \Rightarrow \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]
So, \( 2x = 1 \Rightarrow x = \frac{1}{2} \).

43. (D) Inverse matrix is defined for square matrix only.

44. (C) \[ \mathbf{A} \mathbf{B} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & -5 \end{bmatrix} \]
\[ = \begin{bmatrix} (2)(1) + (-1)(3) & (2)(-2) + (-1)(4) & (2)(-5) + (-1)(0) \\ (1)(1) + (0)(3) & (1)(-2) + (0)(4) & (1)(-5) + (0)(0) \\ (-3)(1) + (4)(3) & (-3)(-2) + (4)(4) & (-3)(-5) + (4)(0) \end{bmatrix} \]
\[ = \begin{bmatrix} 1 & -8 \\ -2 & -10 \\ 9 & -22 \end{bmatrix} \]

45. (C) \[ \mathbf{A} \mathbf{A}^T = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \]
\[ = \begin{bmatrix} (1)(1) + (2)(2) + (0)(0) & (1)(3) + (2)(-1) + (0)(4) \\ (3)(1) + (1)(2) + (4)(0) & (3)(3) + (1)(-1) + (4)(4) \end{bmatrix} \]
\[ = \begin{bmatrix} 5 & 1 \\ 1 & 26 \end{bmatrix} \]

46. (B) If \( |\mathbf{A}| \) is zero, \( \mathbf{A}^{-1} \) does not exist and the matrix \( \mathbf{A} \) is said to be singular. Only (B) satisfy this condition.
\[ |\mathbf{A}| = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = (5)(1) - (2)(2) = 1 \]

47. (A) A skew symmetric matrix \( \mathbf{A}_{n \times n} \) is a matrix with \( \mathbf{A}^T = -\mathbf{A} \). The matrices of (A) satisfy this condition.

48. (C) \[ \mathbf{AB} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1)(1) + (0)(0) + (0)(1) \\ (1)(0) + (0)(0) + (1)(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \]

49. (C) For orthogonal matrix \( \det \mathbf{M} = 1 \) and \( \mathbf{M}^{-1} = \mathbf{M}^T \), therefore \( \mathbf{D}^{-1} = \mathbf{D}^T \)
\[ \mathbf{D}^T = \begin{bmatrix} A & C \\ B & 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -C \end{bmatrix} \begin{bmatrix} B & -C \\ 0 & A \end{bmatrix} \]
This implies \( B = \frac{-C}{-BC} \Rightarrow B = \frac{1}{B} \Rightarrow B = \pm 1 \)
Hence \( B = 1 \)

50. (B) From linear algebra for \( \mathbf{A}_{n \times n} \) triangular matrix \( \det \mathbf{A} = \prod_{i=1}^{n} a_{ii} \), the product of the diagonal entries of \( \mathbf{A} \)

51. (C) \[ \frac{d\mathbf{A}}{dt} = \begin{bmatrix} \frac{d(t^2)}{dt} & \frac{d(\cos t)}{dt} \\ \frac{d(e^t)}{dt} & \frac{d(\sin t)}{dt} \end{bmatrix} = \begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix} \]

52. (A) If \( \det \mathbf{A} \neq 0 \), then \( \mathbf{A}_{n \times n} \) is non-singular, but if \( \mathbf{A}_{n \times n} \) is non-singular, then no row can be expressed as a linear combination of any other. Otherwise \( \det \mathbf{A} = 0 \)

**************
If \( f(x) = x^3 - 6x^2 + 11x - 6 \) is on \([1, 3]\), then the point \( c \in ]1, 3[ \) such that \( f'(c) = 0 \) is given by

(A) \( c = 2 + \frac{1}{\sqrt{2}} \)  
(B) \( c = 2 + \frac{1}{\sqrt{3}} \)  
(C) \( c = 2 + \frac{1}{2} \)  
(D) None of these

Let \( f(x) = \sin 2x, 0 \leq x \leq \frac{\pi}{4} \) and \( f'(c) = 0 \) for \( c \in ]0, \frac{\pi}{4}[ \).
Then, \( c \) is equal to

(A) \( \pi \)  
(B) \( \frac{\pi}{3} \)  
(C) \( \frac{\pi}{6} \)  
(D) None

Let \( f(x) = x(x - 1)(x - 2) \) be defined in \([0, \frac{1}{2}]\). Then, the value of \( c \) of the mean value theorem is

(A) 0.16  
(B) 0.20  
(C) 0.24  
(D) None

A point on the curve \( y = \sqrt{x - 2} \) on \([2, 3]\), where the tangent is parallel to the chord joining the end points of the curve is

(A) \( \left( \frac{9}{4}, \frac{1}{2} \right) \)  
(B) \( \left( \frac{7}{2}, \frac{1}{4} \right) \)  
(C) \( \left( \frac{7}{4}, \frac{1}{2} \right) \)  
(D) \( \left( \frac{9}{2}, \frac{1}{4} \right) \)
11. $\log \sin (x + h)$ when expanded in Taylor’s series, is equal to
(A) $\log x + h \cot x - \frac{1}{2} h^2 \cosec^2 x + \ldots$
(B) $\log x + h \cot x + \frac{1}{2} h^2 \sec^2 x + \ldots$
(C) $\log x - h \cot x + \frac{1}{2} h^2 \cosec^2 x + \ldots$
(D) None of these

12. $\sin x$ when expanded in powers of $(x - \frac{\pi}{2})$ is
(A) $1 + \frac{(x - \frac{\pi}{2})^2}{2!} + \frac{(x - \frac{\pi}{2})^3}{3!} + \frac{(x - \frac{\pi}{2})^4}{4!} + \ldots$
(B) $1 - \frac{(x - \frac{\pi}{2})^2}{2!} + \frac{(x - \frac{\pi}{2})^3}{4!} + \ldots$
(C) $\left( x - \frac{\pi}{2} \right)^2 + \frac{(x - \frac{\pi}{2})^3}{3!} + \frac{(x - \frac{\pi}{2})^5}{5!} + \ldots$
(D) None of these

13. $\tan \left( \frac{\pi}{4} + x \right)$ when expanded in Taylor’s series, gives
(A) $1 + x + x^2 + \frac{4}{3} x^3 + \ldots$
(B) $1 + 2x + 2x^2 + \frac{8}{3} x^3 + \ldots$
(C) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$
(D) None of these

14. If $u = e^{xy}$, then $\frac{\partial^3 u}{\partial x \partial y \partial z}$ is equal to
(A) $e^{xy}[1 + xyz + 3x^2y^2z^2]$
(B) $e^{xy} [1 + xyz + x^3y^3z^3]$
(C) $e^{xy} [1 + 3xyz + x^2y^2z^2]$
(D) $e^{xy} [1 + 3xyz + x^3y^3z^3]$

15. If $z = f(x + ay) + \phi(x - ay)$, then
(A) $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$
(B) $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$
(C) $\frac{\partial^2 z}{\partial y^2} = -\frac{1}{a^2} \frac{\partial^2 z}{\partial x^2}$
(D) $\frac{\partial^2 z}{\partial x^2} = -a^2 \frac{\partial^2 z}{\partial y^2}$

16. If $u = \tan^{-1}\left( \frac{x + y}{\sqrt{x + y}} \right)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ equals
(A) $2 \cos 2u$
(B) $\frac{1}{4} \sin 2u$
(C) $\frac{1}{4} \tan u$
(D) $2 \tan 2u$

17. If $u = \tan^{-1}\left( \frac{x^3 + y^3 + x^2y - xy^2}{x^2 - xy + y^2} \right)$, then the value of
\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \] is
(A) $\frac{1}{2} \sin 2u$
(B) $\sin 2u$
(C) $\sin u$
(D) 0

18. If $u = \phi\left( \frac{y}{x} \right) + yv\left( \frac{y}{x} \right)$, then the value of
\[ x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \] is
(A) 0
(B) $u$
(C) $2u$
(D) $-u$

19. If $z = e^x \sin y$, $x = \log_t t$ and $y = t^2$, then $\frac{dz}{dt}$ is given by the expression
(A) $\frac{e^x}{x} (\sin y - 2t^2 \cos y)$
(B) $\frac{e^x}{x} (\sin y + 2t^2 \cos y)$
(C) $\frac{e^x}{x} (\cos y + 2t^2 \sin y)$
(D) $\frac{e^x}{x} (\cos y - 2t^2 \sin y)$

20. If $z = z(u, v)$, $u = x^2 + 2xy - y^2$, $v = a$, then
\[ (x + y) \frac{\partial z}{\partial x} = (x - y) \frac{\partial z}{\partial y} \]
\[ (x - y) \frac{\partial z}{\partial x} = (x + y) \frac{\partial z}{\partial y} \]
\[ (x + y) \frac{\partial z}{\partial x} = (x - y) \frac{\partial z}{\partial y} \]
\[ (y - x) \frac{\partial z}{\partial x} = (x + y) \frac{\partial z}{\partial y} \]

21. If $f(x, y) = 0$, $\phi(x, z) = 0$, then
\[ \frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} = \frac{\partial f}{\partial z} \frac{\partial \phi}{\partial y} \]
\[ \frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} = \frac{\partial f}{\partial z} \frac{\partial \phi}{\partial y} \]
\[ \frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} = \frac{\partial f}{\partial z} \frac{\partial \phi}{\partial y} \]
\[ \frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} = \frac{\partial f}{\partial z} \frac{\partial \phi}{\partial y} \]

22. If $z = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, then at $x = a$, $y = a$, $\frac{dz}{dx}$ is equal to
(A) $2a$
(B) 0
(C) $2a^2$
(D) $a^3$
23. If \( x = r \cos \theta, y = r \sin \theta \) where \( r \) and \( \theta \) are the functions of \( x \), then \( \frac{dx}{dt} \) is equal to

(A) \( r \cos \theta \frac{dr}{dt} - r \sin \theta \frac{d\theta}{dt} \) 
(B) \( \cos \theta \frac{dr}{dt} - \sin \theta \frac{d\theta}{dt} \) 
(C) \( r \cos \theta \frac{dr}{dt} + \sin \theta \frac{d\theta}{dt} \) 
(D) \( r \cos \theta \frac{dr}{dt} - \sin \theta \frac{d\theta}{dt} \)

24. If \( r^2 = x^2 + y^2 \), then \( \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} \) is equal to

(A) \( r^2 \left[ \left( \frac{\partial r}{\partial x} \right)^2 + \left( \frac{\partial r}{\partial y} \right)^2 \right] \) 
(B) \( 2r^2 \left[ \left( \frac{\partial r}{\partial x} \right)^2 + \left( \frac{\partial r}{\partial y} \right)^2 \right] \) 
(C) \( \frac{1}{r} \left[ \left( \frac{\partial r}{\partial x} \right)^2 + \left( \frac{\partial r}{\partial y} \right)^2 \right] \) 
(D) None of these

25. If \( x = r \cos \theta, y = r \sin \theta \), then the value of \( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \) is

(A) 0 
(B) 1 
(C) \( \frac{\partial r}{\partial x} \) 
(D) \( \frac{\partial r}{\partial y} \)

26. If \( u = x^m y^n \), then

(A) \( du = mx^{m-1} y^n + nx^m y^{n-1} \) 
(B) \( du = m dx + n dy \) 
(C) \( u du = mx dx + ny dy \) 
(D) \( \frac{du}{u} = m \frac{dx}{x} + n \frac{dy}{y} \)

27. If \( y^3 - 3ax^2 + x^3 = 0 \), then the value of \( \frac{d^2 y}{dx^2} \) is equal to

(A) \( -\frac{a^2 x^2}{y^3} \) 
(B) \( \frac{2 a^2 x^2}{y^3} \) 
(C) \( \frac{2 a^2 x^4}{y^5} \) 
(D) \( -\frac{2 a^2 x^2}{y^3} \)

28. \( z = \tan^{-1} \frac{y}{x} \), then

(A) \( dz = \frac{xdy - ydx}{x^2 + y^2} \) 
(B) \( dz = \frac{xdy + ydx}{x^2 + y^2} \) 
(C) \( dz = \frac{xdy - ydx}{x^2 + y^2} \) 
(D) \( dz = \frac{xdy - ydx}{x^2 + y^2} \)

29. If \( u = \log \frac{x^2 + y^2}{x + y} \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \) is equal to

(A) 0 
(B) 1 
(C) \( u \) 
(D) \( eu \)

30. If \( u = x^{n-1} y \left( \frac{y}{x} \right) \), then \( x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 y}{\partial y \partial x} \) is equal to

(A) \( nu \) 
(B) \( n(n-1)u \) 
(C) \( (n-1) \frac{\partial u}{\partial x} \) 
(D) \( (n-1) \frac{\partial u}{\partial y} \)

31. Match the List–I with List–II.

**List–I**

(i) If \( u = \frac{x^2 y}{x + y} \) then \( \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y \partial x} \)

(ii) If \( u = \frac{x^2 - y^2}{x^2 + y^2} \) then \( x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \)

(iii) If \( u = x^2 + y^2 \) then \( x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \)

(iv) If \( u = f \left( \frac{y}{x} \right) \) then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \)

**List–II**

(1) \( -\frac{3}{16} u \) 
(2) \( \frac{\partial u}{\partial x} \) 
(3) 0 
(4) \( -\frac{1}{4} u \)

Correct match is—

<table>
<thead>
<tr>
<th>(I)</th>
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<th>(III)</th>
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<tbody>
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<td>(A)</td>
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<td>(D)</td>
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</tbody>
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32. If an error of 1% is made in measuring the major and minor axes of an ellipse, then the percentage error in the area is approximately equal to

(A) 1% 
(B) 2% 
(C) π% 
(D) 4%

33. Consider the Assertion (A) and Reason (R) given below:

**Assertion (A):** If \( u = xyf \left( \frac{y}{x} \right) \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \)

**Reason (R):** Given function \( u \) is homogeneous of degree 2 in \( x \) and \( y \).

Of these statements

(A) Both A and R are true and R is the correct explanation of A
34. If \( u = x \log xy \), where \( x^2 + y^3 + 3xy = 1 \), then \( \frac{du}{dx} \) is equal to
(A) \( (1 + \log xy) - \frac{x}{y} \left( x^2 + y \right) \)
(B) \( (1 + \log xy) - \frac{x}{y} \left( y^2 + x \right) \)
(C) \( (1 - \log xy) - \frac{x}{y} \left( x^3 + y \right) \)
(D) \( (1 - \log xy) - \frac{x}{y} \left( y^3 + x \right) \)

35. If \( z = xyf \left( \frac{y}{x} \right) \), then \( x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} \) is equal to
(A) \( z \)
(B) \( 2z \)
(C) \( xz \)
(D) \( yz \)

36. \( f(x) = 2x^3 - 15x^2 + 36x + 1 \) is increasing in the interval
(A) \( ] 2, 3 [ \)
(B) \( ] -\infty, 3 [ \)
(C) \( ] -\infty, 2 [ \cup ] 3, \infty \)
(D) None of these

37. \( f(x) = \frac{x}{(x^2 + 1)} \) is increasing in the interval
(A) \( ] -\infty, -1 [ \cup ] 1, \infty [ \)
(B) \( ] -1, 1 [ \)
(C) \( ] -1, \infty [ \)
(D) None of these

38. \( f(x) = x^4 - 2x^2 \) is decreasing in the interval
(A) \( ] -\infty, -1 [ \cup ] 0, 1 [ \)
(B) \( ] -1, 1 [ \)
(C) \( ] -\infty, -1 [ \cup ] 1, \infty [ \)
(D) None of these

39. \( f(x) = x^2 + 3x^3 + 6 \) is increasing for
(A) all positive real values of \( x \)
(B) all negative real values of \( x \)
(C) all non-zero real values of \( x \)
(D) None of these

40. If \( f(x) = kx^3 - 9x^2 + 9x + 3 \) is increasing in each interval, then
(A) \( k < 3 \)
(B) \( k \leq 3 \)
(C) \( k > 3 \)
(D) \( k \geq 3 \)

41. If \( a < 0 \), then \( f(x) = e^{ax} + e^{-ax} \) is decreasing for
(A) \( x > 0 \)
(B) \( x < 0 \)
(C) \( x > 1 \)
(D) \( x < 1 \)

42. \( f(x) = x^2 e^{-x} \) is increasing in the interval
(A) \( ] -\infty, \infty [ \)
(B) \( ] -2, 0 [ \)
(C) \( ] 2, \infty [ \)
(D) \( ] 0, 2 [ \)

43. The least value of \( a \) for which \( f(x) = x^2 + ax + 1 \) is increasing on \( ] 1, 2, \] is
(A) \( 2 \)
(B) \( -2 \)
(C) \( 1 \)
(D) \( -1 \)

44. The minimum distance from the point \( (4, 2) \) to the parabola \( y^2 = 8x \), is
(A) \( \sqrt{2} \)
(B) \( 2\sqrt{2} \)
(C) \( 2 \)
(D) \( 3\sqrt{2} \)

45. The co-ordinates of the point on the parabola \( y = x^2 + 7x + 2 \) which is closest to the straight line \( y = 3x - 3 \), are
(A) \( (-2, -8) \)
(B) \( (2, -8) \)
(C) \( (-2, 0) \)
(D) None of these

46. The shortest distance of the point \( (0, c) \), where \( 0 \leq c < 5 \), from the parabola \( y = x^2 \) is
(A) \( \sqrt{4c + 1} \)
(B) \( \frac{\sqrt{4c + 1}}{2} \)
(C) \( \sqrt{4c - 1} \)
(D) None of these

47. The maximum value of \( \left( \frac{1}{x} \right)^x \) is
(A) \( e \)
(B) \( e^{-\frac{1}{x}} \)
(C) \( \left( \frac{1}{e} \right)^x \)
(D) None of these

48. The minimum value of \( \left( x^2 + \frac{250}{x} \right) \) is
(A) \( 75 \)
(B) \( 50 \)
(C) \( 25 \)
(D) \( 0 \)

49. The maximum value of \( f(x) = (1 + \cos x) \sin x \) is
(A) \( 3 \)
(B) \( 3\sqrt{3} \)
(C) \( 4 \)
(D) \( \frac{3\sqrt{3}}{4} \)
50. The greatest value of
\[ f(x) = \frac{\sin 2x}{\sin \left( x + \frac{\pi}{4} \right)} \]
on the interval \([0, \frac{\pi}{2}]\) is
(A) \(\frac{1}{\sqrt{2}}\) (B) \(\sqrt{2}\)
(C) 1 (D) \(-\sqrt{2}\)

51. If \(y = a \log x + bx^2 + x\) has its extremum values at \(x = -1\) and \(x = 2\), then
(A) \(a = -\frac{1}{2}, b = 2\) (B) \(a = 2, b = -1\)
(C) \(a = 2, b = -\frac{1}{2}\) (D) None of these

52. The co-ordinates of the point on the curve
\[ 4x^2 + 5y^2 = 20 \]
that is farthest from the point \((0, -2)\) are
(A) \((\sqrt{5}, 0)\) (B) \((\sqrt{6}, 0)\)
(C) \((0, 2)\) (D) None of these

53. For what value of \(x \left( 0 \leq x \leq \frac{\pi}{2} \right)\) the function
\[ y = \frac{x}{1 + \tan x} \]
has a maxima?
(A) \(\tan x\) (B) 0
(C) \(\cot x\) (D) \(\cos x\)

**************

SOLUTIONS

1. (B) A polynomial function is continuous as well as differentiable. So, the given function is continuous and differentiable.
\[ f(1) = 0 \text{ and } f(3) = 0. \text{ So, } f(1) = f(3). \]
By Rolle’s theorem \(c\) such that \(f'(c) = 0\).
Now, \(f'(x) = 3x^2 - 12x + 11\)
\[ \Rightarrow f'(c) = 3c^2 - 12c + 11. \]
Now, \(f'(c) = 0 \Rightarrow 3c^2 - 12c + 11 = 0\)
\[ \Rightarrow c = \left(2 \pm \frac{1}{\sqrt{3}}\right) \]

2. (A) Since the sine function is continuous at each \(x \in \mathbb{R}\), so \(f(x) = \sin 2x\) is continuous in \([0, \frac{\pi}{2}]\).
Also, \(f'(x) = 2\cos 2x\), which clearly exists for all \(x \in ]0, \frac{\pi}{2}].\) So, \(f(x)\) is differentiable in \(x \in ]0, \frac{\pi}{2}].\)
Also, \(f(0) = f\left(\frac{\pi}{2}\right) = 0.\) By Rolle’s theorem, there exists \(c \in ]0, \frac{\pi}{2}[\) such that \(f'(c) = 0.\)
\[ 2\cos 2c = 0 \Rightarrow 2c = \frac{\pi}{2} \Rightarrow c = \frac{\pi}{4}. \]

3. (C) Since a polynomial function as well as an exponential function is continuous and the product of two continuous functions is continuous, so \(f(x)\) is continuous in \([-3, 0].\)
\[ f'(x) = (2x + 3) \cdot e^{\frac{x}{2}} - \frac{1}{2} e^{\frac{x}{2}} (x^2 + 3x) = e^{\frac{x}{2}} \left[ \frac{x + 6 - x^2}{2} \right] \]
which clearly exists for all \(x \in ]-3, 0[.\)
\(f(x)\) is differentiable in \([-3, 0].\)
Also, \(f(-3) = f(0) = 0.\)
By Rolle’s theorem \(c \in ]-3, 0[\) such that \(f'(c) = 0.\)
Now, \(f'(c) = 0 \Rightarrow e^{\frac{c}{2}} \left[ \frac{c + 6 - c^2}{2} \right] = 0\)
\[ c + 6 - c^2 = 0 \text{ i.e. } c^2 - c - 6 = 0 \]
\[ \Rightarrow (c + 2)(c - 3) = 0 \Rightarrow c = -2, c = 3. \]
Hence, \(c = -2 \leftrightarrow ]-3, 0[.\)

4. (D) \(f'(x) = 3c^2 - 12x + k\)
\[ f'(c) = 0 \Rightarrow 3c^2 - 12c + k = 0 \]
13. (B) Let \( f(x) = \tan x \) then,
\[
f'(\frac{\pi}{4} + x) = f'(\frac{\pi}{4}) + \frac{x^2}{2!} + f''(\frac{\pi}{4}) + \frac{x^3}{3!} + \text{higher terms}
\]
Thus
\[
\tan (\frac{\pi}{4} + x) = 1 + 2x + x^2 + \frac{x^3}{2} + \frac{x^4}{6} + \text{higher terms}
\]

14. (C) Here \( u = e^{x+y} \) \( \Rightarrow \frac{\partial u}{\partial x} = e^{x+y} \cdot yz \)
\[
\frac{\partial^2 u}{\partial x \partial y} = ze^{x+y} + yze^{x+y} \cdot xz = e^{x+y} (z + xyz^2)
\]
\[
\frac{\partial^2 u}{\partial x \partial y} = e^{x+y} (1 + 2xyz) + (z + xyz^2) e^{x+y} \cdot xy
\]
\[
= e^{x+y}(1 + 3xyz + x^2y^2z^2)
\]

15. (B) \( z = f(x + ay) + \phi(x - ay) \)
\[
\frac{\partial z}{\partial x} = f'(x + ay) + \phi'(x - ay)
\]
\[
\frac{\partial^2 z}{\partial x^2} = f''(x + ay) + \phi''(x - ay)
\]
\[
\frac{\partial z}{\partial y} = af'(x + ay) - a\phi'(x - ay)
\]
\[
\frac{\partial^2 z}{\partial y^2} = a^2 f''(x + ay) + a^2 \phi''(x - ay)
\]
Hence from (1) and (2), we get
\[
\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}
\]

16. (B) \( u = \tan\left(\frac{x + y}{\sqrt{x^2 + y^2}}\right) \)
\[
\Rightarrow \tan u = \frac{x + y}{\sqrt{x^2 + y^2}} = f \quad \text{(say)}
\]
Which is a homogeneous equation of degree 1/2
By Euler's theorem,
\[
x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} f
\]
\[
\Rightarrow x \frac{\partial (\tan u)}{\partial x} + y \frac{\partial (\tan u)}{\partial y} = \frac{1}{2} \tan u
\]

17. (A) Here \( u = \frac{x^3 + y^3 + x^2y - xy^2}{x^2 - xy + y^2} = f \) (say)
Which is homogeneous of degree 1
Thus \( x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = f \)

As above question number 16 \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin u \cos u = \frac{1}{4} \sin 2u \)

18. (A) Let \( v = \frac{y}{x} \) and \( w = x^3 \left(\frac{y}{x}\right) \)
Now \( v \) is homogeneous of degree zero and \( w \) is homogeneous of degree one
\[
\Rightarrow x^2 \frac{\partial^2 w}{\partial x^2} + 2xy \frac{\partial^2 w}{\partial x \partial y} + y^2 \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{(1)}
\]
and \( x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{(2)}
\]
Adding (1) and (2), we get
\[
x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0
\]
\[
\Rightarrow \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0
\]

19. (B) \( z = e^x \sin y \Rightarrow \frac{\partial z}{\partial x} = e^x \sin y \)
And \( \frac{\partial z}{\partial y} = e^x \cos y \), \( x = \log z, t = a \)
\[
\frac{dx}{dt} = \frac{1}{t}
\]
And \( y = t^2 \Rightarrow \frac{dy}{dt} = 2t \)
\[
\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}
\]
\[
= e^x \sin y \cdot \frac{1}{t} + e^x \cos y \cdot 2t = e^x (\sin y + 2t^2 \cos y)
\]

20. (C) Given that
\( z = z(u, v), u = x^2 - 2xy - y^2, v = a \)
\[
\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial x} \quad \text{...(i)}
\]
\[
\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \frac{\partial v}{\partial y} \quad \text{...(ii)}
\]
\[
\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} \quad \text{...(iii)}
\]
From (i),
\[
\frac{\partial u}{\partial x} = 2x - 2y, \quad \frac{\partial u}{\partial y} = -2x - 2y, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial v}{\partial y} = 0
\]
Substituting these values in (ii) and (iii)
\[ \frac{cz}{cx} = \frac{cz}{cu} \left(2x - 2y\right) + \frac{cz}{cv} \cdot 0 \ldots (iv) \]
and \[ \frac{cz}{cy} = \frac{cz}{cu} \cdot (-2x - 2y) + \frac{cz}{cv} \cdot 0 \ldots (v) \]

From (iv) and (v), we get
\[ (x + y) \frac{cz}{cx} = (y - x) \frac{cz}{cy} \]

21. (C) Given that \( f(x, y) = 0, \ \phi(y, z) = 0 \)
These are implicit functions
\[ \frac{dy}{dx} = \frac{\partial f}{\partial x}, \quad \frac{dz}{dy} = -\frac{\partial \phi}{\partial y} \]

\[ \frac{dy}{dx} \cdot \frac{dz}{dy} \left(\frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y} - \frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial x}\right) \]

or, \[ \frac{\partial f}{\partial y} \cdot \frac{d\phi}{dx} \cdot \frac{dz}{dx} = \frac{\partial f}{\partial x} \cdot \frac{d\phi}{dx} \]

22. (B) Given that \( z = \sqrt{x^2 + y^2} \)
and \( x^2 + y^2 + 3axy = 5a^2 \ldots (i) \)
\[ \frac{dz}{dx} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dx} \]

from (i), \[ \frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x, \quad \frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y \]
and \( 3x^2 + 3y^2 \frac{dy}{dx} + 3ax \frac{dy}{dx} + 3ay = 0 \)
\[ \Rightarrow \frac{dy}{dx} = \frac{-(x^2 + ay)}{y^2 + ax} \]

Substituting these value in (ii), we get
\[ \frac{dz}{dx} = \frac{x}{\sqrt{x^2 + y^2}} + \frac{y}{\sqrt{x^2 + y^2}} \left(\frac{x^2 + ay}{y^2 + ax}\right) \]
\[ \left(\frac{dz}{dx}\right)_{(a,a)} = \frac{a}{\sqrt{a^2 + a^2}} + \frac{a}{\sqrt{a^2 + a^2}} \left(\frac{-a^2 + aa}{a^2 + a.a}\right) = 0 \]

23. (B) Given that \( x = r \cos \theta, \ y = r \sin \theta \ldots (i) \)
\[ \frac{dx}{dt} = \frac{\partial x}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial x}{\partial \theta} \cdot \frac{d\theta}{dt} \ldots (ii) \]

From (i), \[ \frac{\partial x}{\partial r} \cos \theta, \quad \frac{\partial x}{\partial \theta} = -r \sin \theta \]

Substituting these values in (ii), we get
\[ \frac{dx}{dt} = \cos \theta \frac{dr}{dt} - r \sin \theta \cdot \frac{d\theta}{dt} \]

24. (C) \( r^2 = x^2 + y^2 \Rightarrow \frac{\partial r}{\partial x} = 2x \text{ and } \frac{\partial r}{\partial y} = 2y \)
and \( \frac{\partial^2 r}{\partial x^2} = 2 \text{ and } \frac{\partial^2 r}{\partial y^2} = 2 \Rightarrow \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = 2 + 2 + 4 \)
and \( \left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 4x^2 + 4y^2 = 4r^2 \)
\[ \Rightarrow \frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r^2} \left[ \left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 \right] \]

25. (A) \( x = r \cos \theta, \ y = r \sin \theta \)
\[ \Rightarrow \tan \theta = \frac{y}{x} \quad \Rightarrow \theta = \tan^{-1} \left(\frac{y}{x}\right) \]
\[ \Rightarrow \frac{\partial \theta}{\partial x} = \frac{1}{1 + (y/x)^2} \left(-\frac{y}{x^2}\right) = \frac{-y}{x^2 + y^2} \]
and \( \frac{\partial^2 \theta}{\partial x^2} = \frac{-2xy}{(x^2 + y^2)^2} \)
Similarly \( \frac{\partial^2 \theta}{\partial y^2} = \frac{2xy}{(x^2 + y^2)^2} \)
and \( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \)

26. (D) Given that \( u = x^m y^n \)
Taking logarithm of both sides, we get
\[ \log u = m \log x + n \log y \]
Differentiating with respect to \( x, \) we get
\[ \frac{1}{u} \frac{du}{dx} = \frac{1}{x} + n \frac{dy}{dx} \quad \text{or,} \quad \frac{du}{dx} = \frac{m}{x} + n \frac{dy}{dx} \]

27. (D) Given that \( f(x, y) = y^3 - 3ax^2 + x^3 = 0 \)
\( f_x = -6ax + 3x^2, \quad f_y = 3y^2, \quad f_{xx} = -6a + 6x, \)
\( f_{yy} = 6y, \quad f_{xy} = 0 \)
\[ \frac{d^2 y}{dx^2} = -\left[ f_{xx}(f_y)^2 - 2f_x f_{xy} f_y + f_y(f_x)^2 \right] \]
\[ \left( f_y \right)^2 \]
\[ = \left[ \frac{6y^3 - 6a(3y^2)^2}{(3y^2)^3} \right] \]
\[ = \frac{-2y^3}{y^6} \left( -a^3 - ay^3 + 4a^2 x^2 \right) \]
\[ = \frac{-2y^3}{y^6} \left( -a^3 + ay^3 + 4a^2 x^2 \right) \]
\[ = \frac{-2y^3}{y^6} \left( -a(3ax^2) + 4a^2 x^2 \right) \]
\[ = \frac{-2y^3}{y^6} \left( -a(3x^3) + 4a^2 x^2 \right) \]
\[ = \frac{-2a^2 x^2}{y^3} \]

28. (A) Given that \( z = \tan^{-1} \frac{y}{x} \ldots (i) \)
\[
\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \frac{dy}{dx} \quad \text{(ii)}
\]

From (i) \[ \frac{\partial z}{\partial x} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(-\frac{y}{x^2}\right) = \frac{-y}{x^2 + y^2} \]

\[
\frac{\partial z}{\partial y} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \left(x \cdot \frac{x}{y} \cdot \frac{dy}{dx}\right) = \frac{x}{x^2 + y^2}
\]

Substituting these in (ii), we get
\[
\frac{dz}{dx} = -\frac{y}{x^2 + y^2} + \frac{x}{x^2 + y^2} \frac{dy}{dx}, \quad dz = xdy - ydx
\]

29. (B) \[u = \log \frac{x^2 + y^2}{x + y}, \quad e^u = \frac{x^2 + y^2}{x + y} = f \quad \text{(say)}\]

\[f\text{ is a homogeneous function of degree one}\]
\[x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = f \quad \Rightarrow \quad x \frac{\partial e^u}{\partial x} + y \frac{\partial e^u}{\partial y} = e^u\]

or \[xe^u \frac{\partial u}{\partial x} + ye^u \frac{\partial u}{\partial y} = e^u\]

or \[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1\]

30. (C) Given that \[u = x^{-1}\log \left(\frac{y}{x}\right)\]

It is a homogeneous function of degree \(n\)

Euler’s theorem \[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu\]

Differentiating partially w.r.t. \(x\), we get
\[
x \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y \frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 u}{\partial y^2} = n \frac{\partial u}{\partial x}
\]

\[\Rightarrow \quad x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y \partial x} + (n - 1) \frac{\partial u}{\partial x} = 0\]

31. (B) In (a) \[u = \frac{x^2}{x + y}\] It is a homogeneous function of degree 2.

\[x \frac{\partial^2 u}{\partial x^2} + y \frac{\partial^2 u}{\partial y \partial x} = (n - 1) \frac{\partial u}{\partial x} = 0\] (as in question 30)

In (b) \[u = \frac{x^{1/2} - y^{1/2}}{x^{1/2} + y^{1/2}}\] It is a homogeneous function of degree \(1/2\).

\[x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n - 1)u\]

\[= \frac{1}{4} \left(\frac{1}{4} - 1\right)u = -\frac{3}{16} u\]

In (c) \[u = x^{1/2} + y^{1/2}\] It is a homogeneous function of degree \(1/2\).

\[x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n - 1)u\]

\[= \frac{1}{2} \left(\frac{1}{2} - 1\right)u = -\frac{1}{4} u\]

In (d) \[u = f \left(\frac{y}{x}\right)\] It is a homogeneous function of degree zero.

\[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0, \quad u = 0\]

Hence correct match is a \quad b \quad c \quad d

2 \quad 1 \quad 3 \quad 4

32. (B) Let 2\(a\) and 2\(b\) be the major and minor axes of the ellipse

Area \(A = \pi ab\)

\[\Rightarrow \quad \log A = \log \pi + \log a + \log b\]

\[= \partial \log A = \frac{\partial}{\partial A} (\log \pi) + \frac{\partial}{\partial a} (\log a) + \frac{\partial}{\partial b} (\log b)\]

\[\Rightarrow \quad \frac{\partial A}{\partial A} = 0 + \frac{\partial a}{a} + \frac{\partial b}{b}\]

\[\Rightarrow \quad 100 \frac{\partial A}{A} = 100 \frac{\partial a}{a} + \frac{100}{b} \cdot \frac{\partial b}{b}\]

But it is given that \[\frac{100}{a} \cdot \frac{\partial a}{a} = 1, \quad \text{and} \quad \frac{100}{b} \cdot \frac{\partial b}{b} = 1\]

\[\Rightarrow \quad 100 \frac{\partial A}{A} = 1 + 1 = 2\]

Thus percentage error in \(A = 2\%\)

33. (A) Given that \[u = x y f \left(\frac{y}{x}\right)\] Since it is a homogeneous function of degree 2.

By Euler’s theorem \[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu\] (where \(n = 2\))

Thus \[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u\]

34. (A) Given that \[u = x \log xy\] \(\text{(i)}\)

\[x^2 + y^3 + 3xy = 1\] \(\text{(ii)}\)

we know that \[\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \frac{dy}{dx} \quad \text{(ii)}\]

From (i) \[\frac{\partial u}{\partial x} = x \cdot \frac{1}{xy} \cdot y + \log xy = 1 + \log xy\]

and \[\frac{\partial u}{\partial y} = \frac{1}{xy} \cdot x = \frac{x}{y}\]
From (ii), we get 
\[ 3x^2 + 3y^2 \frac{dy}{dx} + 3\left( x \frac{dy}{dx} + y \cdot 1 \right) = 0 \Rightarrow \frac{dy}{dx} = -\left( \frac{x^2 + y}{y^2 + x} \right) \]

Substituting these in (A), we get 
\[ \frac{du}{dx} = (1 + \log xy) + \frac{x}{y} \left( \frac{x^2 + y}{y^2 + x} \right) \]

35. (B) The given function is homogeneous of degree 2.
Euler’s theorem 
\[ x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z \]

36. (C) \( f(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3) \)
Clearly, \( f(x) > 0 \) when \( x < 2 \) and also when \( x > 3 \).
\( f(x) \) is increasing in \( ] -\infty, 2 [ \cup ] 3, \infty [ \).

37. (B) \( f'(x) = \frac{(x^2 + 1) - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2} \)
Clearly, \( x^2 + 1 > 0 \) for all \( x \).
So, \( f'(x) > 0 \) \( \Rightarrow \) \( (1 - x^2) > 0 \)
\( \Rightarrow \) \( (1 - x)(1 + x) > 0 \)
This happens when \( -1 < x < 1 \).
So, \( f(x) \) is increasing in \( ] -1, 1 [ \).

38. (A) \( f'(x) = 4x^3 - 4x = 4(x-1)(x+1) \).
Clearly, \( f'(x) < 0 \) when \( x < -1 \) and also when \( x > 1 \).
Sol. \( f(x) \) is decreasing in \( ] -\infty, -1 [ \cup ] 1, \infty [ \).

39. (C) \( f'(x) = 9x^8 + 21x^5 > 0 \) for all non-zero real values of \( x \).

40. (C) \( f'(x) = 3kx^2 - 18x + 9 = 3[kx^2 - 6x + 3] \)
This is positive when \( k > 0 \) and \( 36 - 12k < 0 \) or \( k > 3 \).

41. (A) \( f'(x) = (e^{ax} + e^{-ax}) = 2 \cosh ax \).
\( f'(x) = 2a \sinh ax < 0 \) When \( x > 0 \) because \( a < 0 \)

42. (D) \( f'(x) = -x^2e^{-x} + 2xe^{-x} = e^{-x}(2-x) \).
Clearly, \( f'(x) > 0 \) when \( x > 0 \) and \( x < 2 \).

43. (B) \( f'(x) = (2x + a) \)
\( 1 < x < 2 \) \( \Rightarrow \) \( 2 < 2x < 4 \) \( \Rightarrow \) \( 2 + a < 2x + a < 4 + a \)
\( \Rightarrow \) \( (2 + a) < f'(x) < (4 + a) \).
For \( f(x) \) increasing, we have \( f'(x) > 0 \).
\( 2 + a > 0 \) or \( a > -2 \). So, least value of \( a \) is \( -2 \).

44. (B) Let the point closest to \( (4, 2) \) be \( (2t^2, 4) \).

Now, \( D = \sqrt{(2t^2 - 4)^2 + (4t - 2)^2} \) is minimum when \( E = (2t^2 - 4)^2 + (4t - 2)^2 \) is minimum.

Now, \( E = 4t^4 - 16t + 20 \)
\( \Rightarrow \) \( \frac{dE}{dt} = 16t^3 - 16 = 16(t - 1)(t^2 + t + 1) \)
\( \frac{dE}{dt} = 0 \) \( \Rightarrow \) \( t = 1 \)
\( \frac{d^2E}{dt^2} = 48t^2 \). So, \( \left[ \frac{d^2E}{dt^2} \right]_{t=1} = 48 > 0 \).
So, \( t = 1 \) is a point of minima.
Thus Minimum distance \( = \sqrt{(2 - 4)^2 + (4 - 2)^2} = 2\sqrt{2} \).
47. (B) Let \( y = \left( \frac{1}{x} \right)^x \) then, \( y = x^{-x} \)

\[
\frac{dy}{dx} = -x^{-x}(1 + \log x)
\]

\[
d^2y \over dx^2 = x^{-x} (1 + \log x)^2 - x^{-x} \frac{1}{x}
\]

\[
\frac{dy}{dx} = 0 \Rightarrow 1 + \log x = 0 \Rightarrow x = \frac{1}{e}
\]

\[
\left[ \frac{d^2y}{dx^2} \right]_{x = \frac{1}{e}} = -\left( \frac{1}{e} \right)^2 < 0.
\]

So, \( x = \frac{1}{e} \) is a point of maxima. Maximum value = \( e^{1/e} \).

48. (A) \( f'(x) = 2x - \frac{250}{x^2} \) and \( f''(x) = \left( 2 + \frac{500}{x^3} \right) \)

\( f'(x) = 0 \Rightarrow 2x - \frac{250}{x^2} = 0 \Rightarrow x = 5. \)

\( f''(5) = 6 > 0. \) So, \( x = 5 \) is a point of minima.

Thus minimum value = \( \left( 25 + \frac{250}{5} \right) = 75. \)

49. (D) \( f'(x) = 2(\cos x - 1)(\cos x + 1) \) and

\( f''(x) = -\sin x(1 + 4 \cos x). \)

\( f'(x) = 0 \Rightarrow \cos x = \frac{1}{2} \) or \( \cos x = -1 \Rightarrow x = \pi/3 \) or

\( x = \pi. \)

\( f'\left( \frac{\pi}{3} \right) = -\frac{3\sqrt{3}}{2} < 0. \) So, \( x = \pi/3 \) is a point of maxima.

Maximum value = \( \left( \sin \frac{\pi}{3} \right) \left( 1 + \cos \frac{\pi}{3} \right) = \frac{3\sqrt{3}}{4}. \)

50. (C) \( f(x) = \frac{2 \sin x \cos x}{\sin x + \cos x} \)

\[
= \frac{2\sqrt{2}}{\sec x + \csc x} = \frac{2\sqrt{2}}{x} \quad \text{(say),}
\]

where \( z = (\sec x + \csc x). \)

\[
\frac{dz}{dx} = \sec x \tan x - \csc x \cot x = \frac{\cos x}{\sin^2 x} (\tan^3 x - 1).
\]

\[
\frac{dz}{dx} = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} \text{ in } \left[ 0, \frac{\pi}{2} \right].
\]

Sign of \( \frac{dz}{dx} \) changes from \( -ve \) to \( +ve \) when \( x \) passes through the point \( \pi/4. \) So, \( z \) is minimum at \( x = \pi/4 \) and therefore, \( f(x) \) is maximum at \( x = \pi/4. \)

Maximum value = \( \frac{2\sqrt{2}}{[\sec(\pi/4) + \csc(\pi/4)]} = 1. \)

51. (C) \( \frac{dy}{dx} = \frac{a}{x} + 2bx + 1 \)

\[
\left[ \frac{dy}{dx} \right]_{x = 1} = 0 \Rightarrow -a - 2b + 1 = 0 \Rightarrow a + 2b = 1 \ldots (i)
\]

\[
\left[ \frac{dy}{dx} \right]_{x = 2} = 0 \Rightarrow \frac{a}{2} + 4b + 1 = 0 \Rightarrow a + 8b = -2 \ldots (ii)
\]

Solving (i) and (ii) we get \( b = -\frac{1}{2} \) and \( a = -2. \)

52. (C) The given curve is \( \frac{x^2}{5} + \frac{y^2}{4} = 1 \) which is an ellipse.

Let the required point be \((\sqrt{5} \cos \phi, 2 \sin \phi). \) Then,

\( D = (\sqrt{5} \cos \phi - 0)^2 + (2 \sin \phi + 2)^2 \) is maximum when \( z = D^2 \) is maximum

\( z = 5 \cos^2 \phi + 4(1 + \sin \phi)^2 \)

\( \Rightarrow \frac{dz}{d\phi} = -10 \cos \phi \sin \phi + 8(1 + \sin \phi) \cos \phi \)

\( \Rightarrow \cos \phi = 0 \Rightarrow \phi = \frac{\pi}{2} \)

\( \frac{dz}{d\phi} = -8 \cos 2\phi + 8 \sin \phi \Rightarrow \frac{d^2z}{d\phi^2} = -2 \cos 2\phi - 8 \sin \phi \)

when \( \phi = \frac{\pi}{2}, \frac{d^2z}{d\phi^2} < 0. \)

\( z \) is maximum when \( \phi = \frac{\pi}{2}. \) So, the required point is \( \left( \sqrt{5} \cos \frac{\pi}{2}, \sin \frac{\pi}{2} \right) \) i.e. \((0, 2). \)

53. (D) Let \( z = \frac{1 + \tan x}{x} = \frac{1 + \tan x}{x} \)

Then, \( \frac{dz}{dx} = -\frac{1}{x^2} + \sec^2 x \) and \( \frac{d^2z}{dx^2} = \frac{2}{x^3} + 2 \sec^2 x \tan x \)

\( \frac{dz}{dx} = 0 \Rightarrow -\frac{1}{x^2} + \sec^2 x = 0 \Rightarrow x = \cos x. \)

\( \left[ \frac{d^2z}{dx^2} \right]_{x = \cos x} = 2 \cos^3 x + 2 \sec^2 x \tan x > 0. \)

Thus \( z \) has a minima and therefore \( y \) has a maxima at \( x = \cos x. \)

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CHAPTER 9.3

INTEGRAL CALCULUS

1. \( \int \frac{x}{x^2 + 1} \, dx \) is equal to
   (A) \( \frac{1}{2} \log (x^2 + 1) \)  
   (B) \( \log (x^2 + 1) \)  
   (C) \( \tan^{-1} \frac{x}{2} \)  
   (D) \( 2 \tan^{-1} x \)

2. If \( F(a) = \frac{1}{\log a}, \ a > 1 \) and \( F(x) = \int a^2 \, dx + K \) is equal to
   (A) \( \frac{1}{\log a} (a^x - a^a + 1) \)  
   (B) \( \frac{1}{\log a} (a^x - a^a) \)  
   (C) \( \frac{1}{\log a} (a^a + a^x + 1) \)  
   (D) \( \frac{1}{\log a} (a^a + a^x - 1) \)

3. \( \int \frac{dx}{1 + \sin x} \) is equal to
   (A) \(-\cot x + \cosec x + c\)  
   (B) \(\cot x + \cosec x + c\)  
   (C) \(\tan x - \sec x + c\)  
   (D) \(\tan x + \sec x + c\)

4. \( \int \frac{3x + 1}{2x^2 - 2x + 3} \, dx \) is equal to
   (A) \( \frac{3}{4} \log (2x^2 - 2x + 3) + \frac{\sqrt{5}}{2} \tan^{-1} \left( \frac{2x - 1}{\sqrt{5}} \right) \)  
   (B) \( \frac{4}{3} \log (2x^2 - 2x + 3) + \sqrt{5} \tan^{-1} \left( \frac{2x - 1}{\sqrt{5}} \right) \)  
   (C) \( \frac{4}{3} \log (2x^2 - 2x + 3) + \frac{2}{\sqrt{5}} \tan^{-1} \left( \frac{2x - 1}{\sqrt{5}} \right) \)  
   (D) None of these

5. \( \int \frac{dx}{1 + 3\sin^2 x} \) is equal to
   (A) \( \frac{1}{2} \tan^{-1} (\tan x) \)  
   (B) \( 2 \tan^{-1} (\tan x) \)  
   (C) \( \frac{1}{2} \tan^{-1} (2 \tan x) \)  
   (D) \( 2 \tan^{-1} (\frac{1}{2} \tan x) \)

6. \( \int 2 \sin x + 3 \cos x \cdot 3 \sin x + 4 \cos x \, dx \) is equal to
   (A) \( \frac{9}{25} x + \frac{1}{25} \log (3 \sin x + 4 \cos x) \)  
   (B) \( \frac{18}{25} x + \frac{2}{25} \log (3 \sin x + 4 \cos x) \)  
   (C) \( \frac{18}{25} x + \frac{1}{25} \log (3 \sin x + 4 \cos x) \)  
   (D) None of these

7. \( \int \sqrt{3 + 8x - 3x^2} \, dx \) is equal to
   (A) \( \frac{3x - 4}{3\sqrt{3}} \sqrt{3 + 8x - 3x^2} - \frac{25}{18 \sqrt{3}} \sin^{-1} \left( \frac{3x - 4}{5} \right) \)  
   (B) \( \frac{3x - 4}{6} \sqrt{3 + 8x - 3x^2} + \frac{25 \sqrt{3}}{18} \sin^{-1} \left( \frac{3x - 4}{5} \right) \)  
   (C) \( \frac{3x - 4}{6 \sqrt{3}} \sqrt{3 + 8x - 3x^2} - \frac{25}{18 \sqrt{3}} \sin^{-1} \left( \frac{3x - 4}{5} \right) \)  
   (D) None of these

8. \( \int \frac{dx}{\sqrt{2x^2 + 3x + 4}} \) is equal to
   (A) \( \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{4x + 3}{\sqrt{23}} \right) \)  
   (B) \( \frac{1}{\sqrt{2}} \sinh^{-1} \left( \frac{4x + 3}{\sqrt{23}} \right) \)  
   (C) \( \frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{4x + 3}{\sqrt{23}} \right) \)  
   (D) None of these
9. \[ \int \frac{2x + 3}{\sqrt{x^2 + x + 1}} \, dx \] is equal to
(A) \( 2\sqrt{x^2 + x + 1} + 2 \sinh^{-1} \frac{2x + 1}{\sqrt{3}} \)
(B) \( \sqrt{x^2 + x + 1} + 2 \sinh^{-1} \frac{2x + 1}{\sqrt{3}} \)
(C) \( 2\sqrt{x^2 + x + 1} + \sinh^{-1} \frac{2x + 1}{\sqrt{3}} \)
(D) \( 2\sqrt{x^2 + x + 1} - \sinh^{-1} \frac{2x + 1}{\sqrt{3}} \)

10. \( \int \frac{dx}{\sqrt{x - x^2}} \) is equal to
(A) \( \sqrt{x - x^2} + c \)  
(B) \( \sin^{-1}(2x - 1) + c \)
(C) \( \log(2x - 1) + c \)  
(D) \( \tan^{-1}(2x - 1) + c \)

11. \( \int \frac{1}{(x + 1)\sqrt{1 - 2x - x^2}} \, dx \) is equal to
(A) \( \sqrt{2} \cosh^{-1} \left( \frac{\sqrt{2}}{1 + x} \right) \)  
(B) \( \frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\sqrt{2}}{1 + x} \right) \)
(C) \( -\sqrt{2} \cosh^{-1} \left( \frac{\sqrt{2}}{1 + x} \right) \)  
(D) \( -\frac{1}{\sqrt{2}} \cosh^{-1} \left( \frac{\sqrt{2}}{1 + x} \right) \)

12. \( \int \frac{dx}{\sin x + \cos x} \) is equal to
(A) \( \frac{1}{\sqrt{2}} \log \tan \left( x + \frac{\pi}{4} \right) \)  
(B) \( \frac{1}{\sqrt{2}} \log \tan \left( x + \frac{\pi}{6} \right) \)
(C) \( \frac{1}{\sqrt{2}} \log \tan \left( \frac{x}{2} + \frac{\pi}{8} \right) \)  
(D) \( \frac{1}{\sqrt{2}} \log \tan \left( \frac{x}{4} + \frac{\pi}{4} \right) \)

13. \( \int \frac{dx}{\sin(x - a)\sin(x - b)} \) is equal to
(A) \( \sin(x - a) \log \sin(x - b) \)  
(B) \( \log \sin \left( \frac{x - a}{x - b} \right) \)
(C) \( \sin(a - b) \log \left( \frac{\sin(x - a)}{\sin(x - b)} \right) \)  
(D) \( \frac{1}{\sin(a - b)} \log \left( \frac{\sin(x - a)}{\sin(x - b)} \right) \)

14. \( \int \frac{dx}{e^x - 1} \) is equal to
(A) \( \log(e^x - 1) \)  
(B) \( \log(1 - e^x) \)
(C) \( \log(e^x - 1) \)  
(D) \( \log(1 - e^x) \)

15. \( \int \frac{dx}{1 + x + x^2 + x^3} \) is equal to
(A) \( \frac{1}{2} \log \left( \frac{(x + 1)^2}{x^2 + 1} + \tan^{-1} x \right) \)  
(B) \( \frac{1}{4} \log \left( \frac{(x + 1)^2}{x^2 + 1} + 2 \tan^{-1} x \right) \)
(C) \( \frac{1}{2} \log \left( \frac{(x + 1)^2}{x^2 + 1} - 2 \tan^{-1} x \right) \)  
(D) None of these

16. \( \int \frac{\sin x \, dx}{1 - \sin x} \) is equal to
(A) \( -x + \sec x + \tan x + k \)  
(B) \( -x + \sec x + \tan x \)
(C) \( -x + \sec x - \tan x \)  
(D) \( -x - \sec x - \tan x \)

17. \( \int e^{f(x)} + f'(x) \, dx \) is equal to
(A) \( e^{f(x)} \)  
(B) \( e^{f(x)} \)
(C) \( e^{\tan x} \)  
(D) None of these

18. The value of \( \int e^{\left( \frac{1 + \sin x}{1 + \cos x} \right)} \, dx \) is equal to
(A) \( e^{\tan \left( \frac{x}{2} + c \right)} \)  
(B) \( e^{\cot \left( \frac{x}{2} + c \right)} \)
(C) \( e^{\tan x + c} \)  
(D) \( e^{\cot x + c} \)

19. \( \int \frac{x^3}{x^2 + 1} \, dx \) is equal to
(A) \( x^2 + \log(x^2 + 1) + c \)  
(B) \( \log(x^2 + 1) - x^2 + c \)
(C) \( \frac{1}{2} x^2 - \frac{1}{2} \log(x^2 + 1) + c \)  
(D) \( \frac{1}{2} x^2 + \frac{1}{2} \log(x^2 + 1) + c \)

20. \( \int \frac{\sin x \, dx}{x} \) is equal to
(A) \( x \sin^{-1} x + \sqrt{1 - x^2} + c \)  
(B) \( x \sin^{-1} x - \sqrt{1 - x^2} + c \)
(C) \( x \sin^{-1} x + \sqrt{1 + x^2} + c \)  
(D) \( x \sin^{-1} x - \sqrt{1 + x^2} + c \)

21. \( \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} \, dx \) is equal to
(A) \( \sin x \)  
(B) \( x \)
(C) \( \cos x \)  
(D) \( \tan x \)

22. The value of \( \int_0^1 x - 3 \, dx \) is
(A) \(-1/2\)  
(B) \(13/10\)
(C) \(1/2\)  
(D) \(23/10\)
39. \[ \int_0^1 \int_{\sqrt{x}}^1 (x^2 + y^2) \, dy \, dx \] is equal to
(A) \[ \frac{7}{60} \]  
(B) \[ \frac{3}{35} \]  
(C) \[ \frac{4}{49} \]  
(D) None of these

40. The value of \[ \int_0^1 \int_0^{\sqrt{x^2 + y^2}} \, dy \, dx \] is
(A) \[ \frac{\pi}{4} \log (\sqrt{2} + 1) \]  
(B) \[ \frac{\pi}{4} \log (\sqrt{2} - 1) \]  
(C) \[ \frac{\pi}{2} \log (\sqrt{2} + 1) \]  
(D) None of these

41. If A is the region bounded by the parabolas \[ y^2 = 4x \] and \[ x^2 = 4y \], then \[ \iint_A y \, dx \, dy \] is equal to
(A) \[ \frac{48}{5} \]  
(B) \[ \frac{36}{5} \]  
(C) \[ \frac{32}{5} \]  
(D) None of these

42. The area of the region bounded by the curves \[ x^2 + y^2 = a^2 \] and \[ x + y = a \] in the first quadrant is given by
(A) \[ \int_0^{\sqrt{a^2 - x^2}} x \, dx \]  
(B) \[ \int_0^{\sqrt{a^2 - x^2}} y \, dy \]  
(C) \[ \int_0^{\sqrt{a^2 - y^2}} x \, dx \]  
(D) None of these

43. The area bounded by the curves \[ y = 2\sqrt{x} \] and \[ x = 1 \] is given by
(A) \[ 25 \]  
(B) \[ \frac{33}{2} \]  
(C) \[ \frac{47}{4} \]  
(D) \[ \frac{101}{6} \]

44. The area bounded by the curves \[ y^2 = 9x \] is given by
(A) \[ 1 \]  
(B) \[ \frac{1}{2} \]  
(C) \[ \frac{3}{2} \]  
(D) \[ \frac{5}{4} \]

45. The area of the cardioid \[ r = a (1 + \cos \theta) \] is given by
(A) \[ 2\int_0^a \int_{\theta=0}^{\theta=\pi} r \, dr \, d\theta \]  
(B) \[ 2\int_0^a \int_{\theta=0}^{\theta=\pi} r \, dr \, d\theta \]  
(C) \[ 2\int_0^a \int_{\theta=0}^{\theta=\pi} r \, d\theta \, dr \]  
(D) \[ 2\int_0^a \int_{\theta=0}^{\theta=\pi} r \, dr \, d\theta \]

46. The area bounded by the curve \[ r = \theta \cos \theta \] and the lines \[ \theta = 0 \] and \[ \theta = \frac{\pi}{2} \] is given by
(A) \[ \frac{\pi}{4} \left( \frac{\pi^2}{16} - 1 \right) \]  
(B) \[ \frac{\pi}{16} \left( \frac{\pi^2}{6} - 1 \right) \]  
(C) \[ \frac{\pi}{6} \left( \frac{\pi^2}{16} - 1 \right) \]  
(D) None of these

47. The area of the region bounded by the curve \[ y = x^2 + 2 \] and \[ 4y = x^2 \] is given by
(A) \[ \int_0^1 x^2 \, dx \]  
(B) \[ \int_0^1 y^2 \, dy \]  
(C) \[ \int_0^1 y^2 \, dy \]  
(D) None of these

48. The area of the region bounded by the curve \[ x^2 + y^2 = 4 \] and \[ x + y = 1 \] is given by
(A) \[ \pi \]  
(B) \[ \pi a^2 \]  
(C) \[ \frac{1}{3} \pi a^3 \]  
(D) None of these

49. The volume of the cylinder \[ x^2 + y^2 = a^2 \] bounded below by \[ z = 0 \] and bounded above by \[ z = h \] is given by
(A) \[ \pi ah \]  
(B) \[ \pi a^2 h \]  
(C) \[ \frac{1}{3} \pi a^3 h \]  
(D) None of these

50. \[ \int_0^1 \int_0^1 e^{x+y+z} \, dx \, dy \] is equal to
(A) \[ (e - 1)^3 \]  
(B) \[ \frac{3}{2} (e - 1) \]  
(C) \[ (e - 1)^2 \]  
(D) None of these

51. \[ \int_0^1 \int_0^{x+y+z} (x+y+z) \, dy \, dx \] is equal to
(A) \[ 4 \]  
(B) \[ -4 \]  
(C) \[ 0 \]  
(D) None of these
SOLUTIONS

1. (A) \[ \int \frac{x}{x^2 + 1} \, dx \]
   
   Put \( x^2 + 1 = t \) \( \Rightarrow \) \( 2x \, dx = dt \)

   \[ \int \frac{x}{x^2 + 1} \, dx = \int \frac{1}{2} \, \frac{1}{t} \, dt \]

   \[ = \frac{1}{2} \log t = \frac{1}{2} \log (x^2 + 1) \]

2. (A) \[ F(x) = \int a^x \, dx + K = \frac{a^x}{\log a} + K \]

   \[ \Rightarrow F(a) = \frac{a^a}{\log a} + K \]

   \[ K = \frac{1}{\log a} - \frac{a^a}{\log a} = \frac{1-a^a}{\log a} \]

   \[ F(x) = \frac{a^x}{\log a} + \frac{1-a^a}{\log a} = \frac{1}{\log a} [a^x - a^a + 1] \]

3. (C) \[ \int \frac{dx}{1 + \sin x} \]

   \[ = \int \frac{dx}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} \]

   \[ = \int \frac{\sec^2 \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} \, dx \]

   Put \( 1 + \tan \frac{x}{2} = t \)

   \[ \Rightarrow \sec^2 \frac{x}{2} \, dx = 2 \, dt \]

   \[ = \int \frac{2 \, dt}{t^2} \, dt = -\frac{2}{t} + K \]

   \[ = -\frac{2}{1 + \tan \frac{x}{2}} + K \]

   \[ = \frac{-2 \cos \frac{x}{2} + K}{\cos \frac{x}{2} + \sin \frac{x}{2}} \]

4. (A) Let \( I = \int \frac{3x + 1}{2x^2 - 2x + 3} \, dx \)

   Let \( 3x + 1 = p(4x - 2) + q \) \( \Rightarrow \) \( p = \frac{3}{4}, \quad q = \frac{5}{2} \)

   \[ I = \frac{3}{4} \int \frac{4x - 2}{2x^2 - 2x + 3} \, dx + \frac{5}{2} \int \frac{dx}{2x^2 - 2x + 3} \]

   \[ = \frac{3}{4} \log(2x^2 - 2x + 3) + \frac{5}{4} \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2} \]

   \[ = \frac{3}{4} \log(2x^2 - 2x + 3) + \frac{5}{4} \left(\frac{1}{\sqrt{5}}\right) \tan^{-1} \left(x - \frac{1}{2}\right) \]

5. (C) Let \( I = \int \frac{dx}{1 + 3 \sin^2 x} \)

   \[ = \int \frac{\csc^2 x \, dx}{\csc^2 x + 3} = \int \frac{\csc^2 x \, dx}{(1 + \cot^2 x) + 3} \]

   Put \( \cot x = t \) \( \Rightarrow \) \(-\csc^2 x \, dx = dt \)

   \[ I = \int \frac{-dt}{4 + t^2} = \frac{1}{2} \cot^{-1} \left(\frac{t}{2}\right) \]

   \[ = \frac{1}{2} \tan^{-1}(2 \tan x) \]

6. (C) Let \( I = \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} \, dx \)

   Let \( (2 \sin x + 3 \cos x) = p(3 \sin x - 4 \cos x) + q(3 \sin x + 4 \cos x) \)

   \[ p = \frac{1}{25}, \quad q = \frac{18}{25} \]

   \[ I = \frac{1}{25} \int \frac{3 \cos x - 4 \sin x \, dx}{3 \sin x + 4 \cos x} + \frac{18}{25} \int \frac{3 \sin x + 4 \cos x \, dx}{3 \sin x + 4 \cos x} \]

   \[ = \frac{1}{25} \log(3 \sin x + 4 \cos x) + \frac{18}{25} x \]

7. (B) \[ \int \left(3 + 8x - 3x^2\right) \, dx = \sqrt{3} \int \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2 \, dx \]

   \[ = \sqrt{3} \left[ \left(\frac{5}{3}\right)^2 - \left(\frac{4}{3}\right)^2 \right] - \sin^{-1} \left(\frac{4}{3}\right) \]

   \[ = 3 \left(\frac{5}{3}\right)^2 + 18 \sin^{-1} \left(\frac{3}{4}\right) \]

8. (B) \[ \int \frac{dx}{\sqrt{2x^2 + 3x + 4}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2 + \frac{3}{4} + \frac{\sqrt{23}}{4}}} \]

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11. \( \int \frac{dx}{\sqrt{x^2 + x + 1}} = I \)

Put \( x = \sin^2 \theta \Rightarrow dx = 2 \sin \theta \cos \theta \, d\theta \\
I = \int \frac{2 \sin \theta \cos \theta}{\sqrt{1 - \sin^2 \theta}} \, d\theta = \int 2 \sin \theta \cos \theta \, d\theta \\
I = \int 2 \sin \theta \, d\theta = 2 \left( -\cos \theta + c \right) \\
I = \sin^{-1} (2x - 1) + c

12. \( \int \frac{dx}{\sin x + \cos x} \)

\[
\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \int \csc \left( x + \frac{\pi}{4} \right) \, dx
\]

\[
\int \frac{dx}{\cos x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\cos x} = \frac{1}{\sqrt{2}} \int \csc \left( x + \frac{\pi}{4} \right) \, dx
\]

13. \( \int \frac{dx}{\sin(x-a) \sin(x-b)} \)

\[
\int \frac{dx}{\sin(x-a) \sin(x-b)} = \frac{1}{\sin(a-b)} \int \frac{df}{\sin(x-a) \sin(x-b)}
\]

\[
\frac{1}{\sin(a-b)} \left( \int \frac{\cos(x-a) - \cos(x-b) \sin(x-a)}{\sin(x-a) \sin(x-b)} \, dx \right)
\]

\[
= \frac{1}{\sin(a-b)} \int \left( \cot(x-a) - \cot(x-b) \right) \, dx
\]

\[
= \frac{1}{\sin(a-b)} \left( \log \sin(x-a) - \log \sin(x-b) \right)
\]

14. \( \int \frac{dx}{e^x - 1} = \int \frac{e^x \, dx}{1 - e^{-x}} \)

Put \( 1 - e^{-x} = t \Rightarrow e^x \, dx = dt \\
I = \int \frac{dt}{t} = \log t = \log (1 - e^{-x})

15. \( \int \frac{dx}{1 + x + x^2 + x^3} \)

\[
\int \frac{dx}{(1 + x)(1 + x^2)}
\]

Let \( \frac{1}{1 + x(1 + x^2)} = A \frac{1}{1 + x} + Bx + C \frac{1}{1 + x^2} \)

\[
1 = A(1 + x^2) + (Bx + C)(1 + x)
\]

Comparing the coefficients of \( x^2 \), \( x \) and constant terms, \( A + B = 0 \), \( B + C = 0 \), \( A + C = 1 \)

Solving these equations, we get

\[
A = \frac{1}{2}, \ B = -\frac{1}{2}, \ C = \frac{1}{2}
\]

\[
I = \frac{1}{2} \int \frac{1}{1 + x} \, dx - \frac{1}{2} \int \frac{x - 1}{x^2 + 1} \, dx
\]

\[
= \frac{1}{2} \log (1 + x) - \frac{1}{2} \log (x^2 + 1) + \frac{1}{2} \tan^{-1} x
\]

\[
= \frac{1}{4} \left[ \log \left( \frac{(x + 1)^2}{x^2 + 1} + 2 \tan^{-1} x \right) \right]
\]
16. (B) Let \( I = \int \frac{\sin x}{1 - \sin x} \, dx \)

\[ = \int \frac{1 - (1 - \sin x)}{1 - \sin x} \, dx \]

\[ = \int \frac{1}{1 - \sin x} \, dx - \int \frac{1 + \sin x}{1 - \sin^2 x} \, dx \]

\[ = \int \frac{1 + \sin x}{\cos^2 x} \, dx - x = (\sec^2 x + \sec x \tan x)dx - x \]

\[ = \tan x + \sec x - x \]

17. (B) Let \( I = \int e^x(f(x) + f'(x)) \, dx \)

\[ = \int e^x f(x) \, dx + \int e^x f'(x) \, dx \]

\[ = (f(x)e^x - \int f'(x)e^x \, dx) + \int e^x f'(x) \, dx = f(x) \cdot e^x \]

18. (A) Let \( I = \int e^x \left( \frac{1 + \sin x}{1 + \cos x} \right) \, dx \)

\[ = \frac{1}{2} \int e^x \sec^2 \frac{x}{2} \, dx + \int e^x \tan \frac{x}{2} \, dx \]

\[ = \frac{1}{2} \left( e^x \cdot 2 \tan \frac{x}{2} - \int e^x \cdot 2 \tan \frac{x}{2} \, dx \right) + \int e^x \tan \frac{x}{2} \, dx \]

\[ = e^x \tan \frac{x}{2} + c \]

19. (C) \( I = \int \frac{x^3}{x^4 + 1} \, dx = \int \frac{x \cdot x^2}{x^4 + 1} \, dx \)

\[ = \int \frac{x(x^2 + 1) - x}{x^4 + 1} \, dx = \int x \, dx - \int \frac{x}{x^4 + 1} \, dx \]

\[ = \frac{1}{2} x^2 - \frac{1}{2} \log (x^2 + 1) + c \]

20. (A) Let \( I = \int \sin^{-1} x \, dx = \int \sin^{-1} x \cdot 1 \, dx \)

\[ = \sin^{-1} x \cdot x - \int \frac{1}{\sqrt{1 - x^2}} \cdot x \, dx \]

\[ = x \sin^{-1} x - \int \frac{x}{\sqrt{1 - x^2}} \, dx \]

In second part put \( 1 - x^2 = t^2 \)

\[ x \, dx = -tdt = x \sin^{-1} x + \int dt \]

\[ = x \sin^{-1} x + t = x \sin^{-1} x + \sqrt{1 - x^2} + c \]

21. \( I = \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} \, dx \)

\[ = \int \frac{\sin x + \cos x}{\sqrt{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x}} \, dx \]

\[ = \int \frac{\sin x + \cos x}{\sqrt{(\cos x + \sin x)^2}} \, dx \]

\[ = \int \frac{\sin x + \cos x}{\sin x + \cos x} \, dx = \int dx = x \]

22. (D) \( \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x - 3 \, dx = -\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x - 3 \, dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x - 3 \, dx \)

\[ = \left[ \frac{\sin^2 x}{2} + 3x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \]

\[ = \left[ \frac{9}{10} + \frac{9}{10} \right] = \frac{9}{10} \]

\[ = \frac{9}{10} + \left( \frac{1}{2} + \frac{9}{10} \right) = \frac{13}{10} \]

23. (B) \( \int_{0}^{1} \frac{1}{e^x + e^{-x}} \, dx = \int_{0}^{1} e^x \, dx \)

Put \( e^x = t \Rightarrow e^x \, dx = dt = \int_{1}^{t} \frac{dt}{t^2 + 1} = \tan^{-1} t \]

\[ = \tan^{-1} e - \tan^{-1} 1 = \tan^{-1} e - \frac{\pi}{4} \]

24. (D) \( \int_{0}^{1} x(1 - x) \, dx = \int_{0}^{1} (x - x^2) \, dx \)

\[ = \left[ \frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_{0}^{1} = \frac{1}{6} \]

\[ = \frac{1}{6} c^2(3 - 2c) = 0 \]

\[ \Rightarrow c = \frac{3}{2} \]

25. (D) Put \( x^2 + x = t \Rightarrow (2x + 1) \, dx = dt \)

\[ \int_{0}^{1} \frac{2x + 1}{\sqrt{x + x^2}} \, dx = \int_{0}^{1} \frac{dt}{\sqrt{t}} = 2(t^{1/2})_{0}^{1} = 2\sqrt{2} \]

26. (A) \( \int_{0}^{1} x^4 \sin^5 x \, dx \)

Since, \( f(-x) = (-x)^4 \sin^5 (-x) = -x^4 \sin^5 x \)

\( f(x) \) is odd function thus

\[ \int_{0}^{1} x^4 \sin^5 x \, dx = 0 \]

27. (A) \( \int_{0}^{1} \cos^2 x \, dx = \int_{0}^{1} \frac{1}{2} (\cos 2x + 1) \, dx \)
28. (B) Let \( I = \int_0^a \sqrt{a^2 - x^2} \, dx \)

Put \( x = a \sin \theta \Rightarrow dx = a \cos \theta \, d\theta \) when \( x = 0, \ \theta = 0, \)
when \( x = a, \ \theta = \frac{\pi}{2} \)

\[ I = \int_0^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 \theta} \, a \cos \theta \, d\theta \]

\[ = a^2 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta = a^2 \cdot \frac{1}{2} \cdot \frac{\pi}{2} \] (By Walli’s Formula)

\[ = \frac{\pi a^2}{4} \]

Aliter: \( \int_0^a \sqrt{a^2 - x^2} \, dx = \frac{\pi a^2}{4} \)

29. (D) Let \( I = \int_0^{\frac{\pi}{2}} \log (\tan x) \, dx \)

\[ I = \int_0^{\frac{\pi}{2}} \log \tan \left( \frac{\pi}{2} - x \right) \, dx \]

\[ I = \int_0^{\frac{\pi}{2}} \log (\cot x) \]...

Adding (1) and (2), we get

\[ 2I = \int_0^{\frac{\pi}{2}} \log (\tan x) + \log (\cot x) \, dx \]

\[ = \int_0^{\frac{\pi}{2}} \log (\tan x \cdot \cot x) \, dx \]

\[ = \int_0^{\frac{\pi}{2}} \log 1 \, dx = 0 \Rightarrow I = 0 \]

30. (D) Let \( I = \int_0^1 2 \sin \left( \frac{\pi t}{4} \right) \, dt \) ...

\[ = \int_0^1 2 \sin \left( \frac{\pi}{2} - \frac{\pi t}{4} \right) \, dt = \int_0^1 2 \sin \left( \frac{\pi}{4} - \frac{\pi t}{4} \right) \, dt \]

\[ = -\int_0^1 2 \sin \left( \frac{\pi}{4} - \frac{\pi t}{4} \right) \, dt = -1 \]

\[ 2I = 0 \Rightarrow I = 0 \]

31. (C) Let \( I = \int_0^2 f(x) + f(2a-x) \, dx \) ...

\[ I = \int_0^2 f(x) + f(2a-x) \, dx \]

Adding (1) and (2), we get

\[ 2I = \int_0^2 f(x) + f(2a-x) \, dx = \int_0^2 1 \cdot dx = [x]_0^a = 2a \]

\[ \Rightarrow I = a \]

32. (C) Let \( I = \int_0^\infty \frac{e^{-x^2}}{\sqrt{1-x^2}} \, dx \)

Put \( \sqrt{1-x^2} = t \)

\[ \Rightarrow \frac{1}{2\sqrt{1-x^2}} (-2x) \, dx = dt \]

when \( x = 0, \ t = 1, \) when \( x = 1, \ t = 0 \)

\[ I = \int_0^1 -e^t \, dt = -e^0 - e^1 = -1 \]

33. (B) Let \( I = \int_0^\infty \frac{dx}{x + x^2} \)

\[ = \int_0^\infty \frac{dx}{x + \frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \left[ \tan^{-1} \frac{x}{\sqrt{3}} - \frac{\pi}{6} \right]_0^\infty \]

\[ = \frac{2}{\sqrt{3}} \left[ \tan^{-1} \frac{1}{\sqrt{3}} - \frac{\pi}{3} \right] = \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{9} \]

34. (B) Let \( I = \int_{-1}^1 \frac{dx}{x} \)

\[ = \int_{-1}^0 \frac{dx}{x} + \int_0^1 \frac{dx}{x} \]

\[ = \ln(-1) - [x]_0^1 = -\ln(-1) + 1 \]

\[ = -[0 - (1)] + [1 - 0] = 0 \]

35. (C) \[ \int_0^{\pi} \sin x \, dx = 100 \int_0^{\pi} \sin x \, dx \]

[ ... sin x is periodic with period π]
\[ I = \int_0^1 \sin x \, dx = 100(-\cos x)^0_0 = 100(\cos \pi + \cos 0) = 100(1 + 1) = 200. \]

36. (C) Let \( I = \int_0^\pi \cos^m x \sin n x \, dx = \int_0^\pi f(x) \, dx \)

Where \( f(x) = \cos^m x \sin^n x \)
\[ f(\pi - x) = \cos^m(\pi - x) \sin^n(\pi - x) = (-\cos x)^m (\sin x)^n = \cos^m x \sin^n x, \text{ if } m \text{ is odd} \]

\[ I = \int_0^\pi \cos^m x \sin^n x \, dx = 0, \text{ if } m \text{ is odd} \]

37. (A) Let \( I = \int_0^\pi x F(\sin x) \, dx \) ...(1)

\[ = \left[ (x - \pi) F(\sin(\pi - x)) \right]_0^\pi \]

\[ I = \int_0^\pi (\pi - x) F(\sin x) \, dx \) ...(2)

Adding (1) and (2), we get

\[ 2I = \int_0^\pi \pi F(\sin x) \, dx \]

\[ \Rightarrow I = \frac{1}{2} \int_0^\pi \pi F(\sin x) \, dx \]

38. (B) Let \( I = \int_0^{\pi/2} e^x \left( \sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right) \, dx \)

\[ = \frac{1}{2} \int_0^{\pi/2} e^x \sec^2 \frac{x}{2} \, dx + \frac{1}{2} \int_0^{\pi/2} e^x \tan \frac{x}{2} \, dx = I_1 + I_2 \]

Here, \( I_1 = \int_0^{\pi/2} e^x \sec^2 \frac{x}{2} \, dx \)
\[ = \left[ \frac{1}{2} e^x \cdot 2 \tan \frac{x}{2} \right]_0^{\pi/2} - \frac{1}{2} \int_0^{\pi/2} e^x \cdot 2 \tan \frac{x}{2} \, dx \]

\[ = e^{\pi/2} \tan \frac{\pi}{4} - 0 - \frac{1}{2} \int_0^{\pi/2} e^x \cdot 2 \tan \frac{x}{2} \, dx \]

\[ = e^{\pi/2} - I_2, \quad I_1 + I_2 = e^{\pi/2} \]

\[ I = I_1 + I_2 = e^{\pi/2} \]

39. (B) \[ \int_0^1 (\sqrt{x^2 + y^2}) \, dy \, dx = \int_0^1 \left[ x^2 + \frac{y^3}{3} \right]^\pi_0 \, dx \]

\[ = \int_0^1 \left[ x^2 + \frac{y^3}{3} - x^3 - \frac{3}{3} \right] \, dx \]

\[ = \left[ \frac{2}{7} x^{4/3} + \frac{2}{15} x^{5/3} - \frac{1}{3} x^4 \right]_0^1 = \frac{3}{35} \]

40. (D) \[ \int_0^1 \int_0^{y^{3/2}} dy \, dx = \int_0^1 \left[ y \right]_0^{y^{3/2}} \, dx \]

\[ = \int_0^1 \left[ y^{3/2} - 0 \right] \, dx \]

\[ = \frac{1}{2} \left[ y^{1/2} + \log(y + \sqrt{1 + y^2}) \right]_0^{y^{3/2}} \]

\[ = \frac{1}{2} \left[ 2 + \log(1 + \sqrt{2}) \right] \]

41. (A) Let \( I = \int_A y \, dy \, dx \)

Solving the given equations \( y^2 = 4x \) and \( x^2 = 4y \), we get \( x = 0, x = 4 \). The region of integration A is given by

\[ A = \int_{x=0}^4 \int_{y=0}^{4x} \left[ y^{3/2} - x^{3/2} \right] \, dy \, dx \]

\[ = \frac{4}{5} \left[ 4x - \frac{x^4}{10} \right]_{x=0}^{4} \]

\[ = \frac{4}{5} \left[ 4 \times 4 - \frac{4^4}{10} \right] = \frac{48}{5} \]

42. (A) The curves are

\[ x^2 + y^2 = a^2 \]
\[ x + y = a \]

The curves (i) and (ii) intersect at A (a, 0) and B (0, a)

The required area \( A = \int_{x=0}^a \int_{y=-a-x}^{a-x} \, dy \, dx \)

43. (D) The given equations of the curves are

\[ y = 2\sqrt{x} \text{ i.e., } y^2 = 4x \text{....(i)} \quad \text{ and } \quad y = -x \text{....(ii)} \]

If a figure is drawn then from fig. the required area is

\[ A = \int_{x=-1}^1 \int_{y=-x}^{\sqrt{x+2}} \, dy \, dx \]

\[ = \left[ \frac{32}{3} + 8 \right] - \left( \frac{4}{3} + \frac{1}{3} \right) = \frac{101}{6} \]

44. (B) The equations of the given curves are

\[ y^2 = 9x \text{....(i)} \quad x - y + 2 = 0 \text{....(ii)} \]

The curves (i) and (ii) intersect at A(1, 3) and B(4, 6)

If a figure is drawn then from fig. the required area is

\[ A = \int_{x=1}^4 \int_{y=-x+2}^{x+2} \, dy \, dx \]

\[ = \int_{x=1}^4 \left[ 3\sqrt{x} - (x + 2) \right] \, dx \]

\[ = \left[ \frac{3}{2} x^{3/2} - \frac{1}{2} x^2 - 2x \right]_1^4 \]

\[ = (16 - 8 - 8) - \left( \frac{1}{2} - \frac{1}{2} - 2 \right) = \frac{1}{2} \]
45. (A) The equation of the cardioid is 
\[ r = a (1 + \cos \theta) \quad \text{.....(i)} \]
If a figure is drawn then from fig. the required area is 
Required area 
\[ A = 2 \int_0^{\pi} r^2 \cos \theta \, d\theta \]

46. (C) The equation of the given curve is 
\[ r = \theta \cos \theta \quad \text{...(i)} \]
The required area 
\[ A = \int_0^{\pi} \int_0^{\theta \cos \theta} r \, dr \, d\theta = \frac{\pi}{2} \int_0^{\pi} \frac{1}{2} r^2 \cos \theta \, d\theta \]
\[ = \frac{\pi}{2} \int_0^{\pi} \left[ 0 + \frac{\theta^2}{2} \sin 2\theta \right] \, d\theta \]
\[ = \frac{\pi^3}{96} \left( 1 - \frac{\pi}{4} \right) \]
\[ = \frac{\pi^3}{96} - \frac{\pi^3}{16} \]

47. (A) The curve is 
\[ r^2 = a^2 \cos \theta \]
If a figure is drawn then from fig. the required area is 
\[ A = 4 \int_0^{\pi/2} \pi r^2 \cos \theta \, d\theta = 4 \int_0^{\pi/2} \frac{1}{2} r^2 \cos \theta \, d\theta \]
\[ = 2 \int_0^{\pi/2} a^2 \cos \theta \cos \theta \, d\theta = 2a^2 \left[ \frac{\sin 2\theta}{2} \right]_0^{\pi/2} = a^2 \]

48. (C) The equations of given curves are 
\[ y(x^2 + 2) = 3x \quad \text{.....(i)} \quad \text{and} \quad 4y = x^2 \quad \text{...(ii)} \]
The curve (i) and (ii) intersect at A (2, 1). 
If a figure is drawn then from fig. the required area is 
The required area 
\[ A = \int_{x=0}^{\ln/(x^2 + 2)} \int_{y=x^{7/4}} dx \, dy \]

49. (B) The equation of the cylinder is 
\[ x^2 + y^2 = a^2 \]
The equation of surface CDE is 
\[ z = h. \]
If a figure is drawn then from fig. the required area is
10. The integration of \( f(z) = x^2 + ixy \) from A(1, 1) to B(2, 4) along the straight line AB joining the two points is
(A) \(-\frac{29}{3} + i11\)  
(B) \(\frac{29}{3} - i11\)  
(C) \(\frac{23}{5} + i6\)  
(D) \(\frac{23}{5} - i6\)

11. \( \int_{c} e^{2z} \frac{dz}{(z + 1)^2} \) where \( c \) is the circle of \( |z| = 3 \)
(A) \(\frac{4\pi i}{9} e^{-3}\)  
(B) \(\frac{4\pi i}{9} e^{3}\)  
(C) \(\frac{4\pi i}{3} e^{-3}\)  
(D) \(\frac{8\pi i}{3} e^{-3}\)

12. \( \int_{c} \frac{1 - 2z}{z(z - 1)(z - 2)} dz \) where \( c \) is the circle \( |z| = 15 \)
(A) \(2 + i6\pi\)  
(B) \(4 + i3\pi\)  
(C) \(1 + i\pi\)  
(D) \(i3\pi\)

13. \( \int_{c} (z - z^2) dz \) where \( c \) is the upper half of the circle \( z = 1 \)
(A) \(-\frac{2}{3}\)  
(B) \(\frac{2}{3}\)  
(C) \(\frac{3}{2}\)  
(D) \(-\frac{3}{2}\)

14. \( \int_{c} \frac{\cos \pi z}{z - 1} dz \) where \( c \) is the circle \( |z| = 3 \)
(A) \(i2\pi\)  
(B) \(-i2\pi\)  
(C) \(i6\pi^2\)  
(D) \(-i6\pi^2\)

15. \( \int_{c} \frac{\sin \pi z^2}{(z - 2)(z - 1)} dz \) where \( c \) is the circle \( |z| = 3 \)
(A) \(i6\pi\)  
(B) \(i2\pi\)  
(C) \(i4\pi\)  
(D) 0

16. The value of \( \frac{1}{2\pi i} \int_{c} \frac{\cos \pi z}{z - 1} dz \) around a rectangle with vertices at \( 2 \pm i, -2 \pm i \)
(A) 6  
(B) \(i2\pi\)  
(C) 8  
(D) 0

17. The value of \( f(3) \) is
(A) 6  
(B) \(4i\)  
(C) \(-4i\)  
(D) 0

18. The value of \( f'(1 - i) \) is
(A) \(7(\pi + i2)\)  
(B) \(6(2 + i\pi)\)  
(C) \(2\pi(5 + i13)\)  
(D) 0

Statement for Q. 19–21:
Expand the given function in Taylor’s series.

19. \( f(z) = \frac{z - 1}{z + 1} \) about the points \( z = 0 \)
(A) \(1 + 2(z + z^2 + z^3 \ldots \ldots \ldots)\)  
(B) \(-1 - 2(z - z^2 + z^3 \ldots \ldots \ldots)\)  
(C) \(-1 + 2(z - z^2 + z^3 \ldots \ldots \ldots)\)  
(D) None of the above

20. \( f(z) = \frac{1}{z + 1} \) about \( z = 1 \)
(A) \(\frac{1}{2} \left[ -1 \frac{1}{2}(z - 1) + \frac{1}{2^2}(z - 1)^2 \right] \ldots \ldots \ldots \)  
(B) \(\frac{1}{2} \left[ 1 - \frac{1}{2}(z - 1) + \frac{1}{2^2}(z - 1)^2 \right] \ldots \ldots \ldots \)  
(C) \(\frac{1}{2} \left[ 1 + \frac{1}{2}(z - 1) + \frac{1}{2^2}(z - 1)^2 \right] \ldots \ldots \ldots \)  
(D) None of the above

21. \( f(z) = \sin z \) about \( z = \frac{\pi}{4} \)
(A) \(\frac{1}{\sqrt{2}} \left[ 1 + \frac{1}{2\pi}(z - \frac{\pi}{4}) - \frac{1}{2}(z - \frac{\pi}{4})^2 \right] \ldots \ldots \ldots \)  
(B) \(\frac{1}{\sqrt{2}} \left[ 1 + \frac{1}{2\pi}(z - \frac{\pi}{4}) + \frac{1}{2}(z - \frac{\pi}{4})^2 \right] \ldots \ldots \ldots \)  
(C) \(\frac{1}{\sqrt{2}} \left[ 1 - \frac{1}{2\pi}(z - \frac{\pi}{4}) - \frac{1}{2}(z - \frac{\pi}{4})^2 \right] \ldots \ldots \ldots \)  
(D) None of the above

22. If \( |z + 1| < 1 \), then \( z^2 \) is equal to
(A) \(1 + \sum_{n=1}^{\infty} (n + 1)(z + 1)^{n-1}\)  
(B) \(1 + \sum_{n=1}^{\infty} (n + 1)(z + 1)^{n+1}\)  
(C) \(1 + \sum_{n=1}^{\infty} n(z + 1)^{n}\)  
(D) \(1 + \sum_{n=1}^{\infty} (n + 1)(z + 1)^{n}\)

Statement for Q. 17–18:
\( f(z_o) = \int_{c} \frac{3z^2 + 7z + 1}{z - z_o} dz \), where \( c \) is the circle \( x^2 + y^2 = 4 \).
Statement for Q. 23–25.

Expand the function \( \frac{1}{(z-1)(z-2)} \) in Laurent's series for the condition given in question.

23. \( 1 < |z| < 2 \)
   (A) \( \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} + ... \)
   (B) \( ... - z^3 - z^2 - z^1 - \frac{1}{2} - \frac{1}{4} z - \frac{1}{8} z^2 - \frac{1}{18} z^3 - ... \)
   (C) \( \frac{1}{z^2} + \frac{3}{z^3} + \frac{7}{z^4} + ... \)
   (D) None of the above

24. \(|z| > 2\)
   (A) \( \frac{6}{z^2} + \frac{13}{z^3} + \frac{20}{z^4} + ... \)
   (B) \( \frac{1}{z^2} + \frac{8}{z^3} + \frac{13}{z^4} + ... \)
   (C) \( \frac{1}{z^2} + \frac{3}{z^3} + \frac{7}{z^4} + ... \)
   (D) \( \frac{2}{z^2} + \frac{3}{z^3} + \frac{4}{z^4} + ... \)

25. \(|z| < 1\)
   (A) \( 1 + 3z + \frac{7}{2} z^2 + \frac{15}{4} z^3 + ... \)
   (B) \( \frac{1}{2} + \frac{3}{4} z + \frac{7}{8} z^2 + \frac{15}{16} z^3 + ... \)
   (C) \( \frac{1}{4} + \frac{3}{8} z + \frac{7}{16} z^2 + ... \)
   (D) None of the above

26. If \(|z-1| < 1\), the Laurent's series for \( \frac{1}{z(z-1)(z-2)} \) is
   (A) \( -(z-1) - \frac{(z-1)^3}{2!} - \frac{(z-1)^5}{5!} + ... \)
   (B) \( -(z-1)^{-1} - \frac{(z-1)^3}{2!} - \frac{(z-1)^5}{5!} + ... \)
   (C) \( -(z-1) - (z-1)^3 - (z-1)^5 - ... \)
   (D) \( -(z-1)^{-1} - (z-1) - (z-1)^3 - (z-1)^5 - ... \)

27. The Laurent's series of \( \frac{1}{z(e^z-1)} \) for \(|z| < 2\) is
   (A) \( \frac{1}{z^2} + \frac{1}{2z} + \frac{1}{12} + 6z + \frac{1}{720} z^2 + ... \)
   (B) \( \frac{1}{z^2} - \frac{1}{2z} + \frac{1}{12} - \frac{1}{720} z^2 + ... \)
   (C) \( \frac{1}{z} + \frac{1}{12} + \frac{1}{634} z^2 + \frac{1}{720} z^2 + ... \)
   (D) None of the above

28. The Laurent's series of \( f(z) = \frac{z}{(z^2 + 1)(z^2 + 4)} \) is,
where \(|z| < 1\)
   (A) \( \frac{1}{4} z - \frac{5}{16} z^3 + \frac{21}{64} z^5 + ... \)
   (B) \( \frac{1}{2} + \frac{1}{4} z^2 + \frac{5}{16} z^4 + \frac{21}{64} z^6 + ... \)
   (C) \( \frac{1}{2} - \frac{3}{4} z^3 + \frac{15}{8} z^5 + ... \)
   (D) \( \frac{1}{2} + \frac{1}{2} z^2 + \frac{3}{4} z^4 + \frac{15}{8} z^6 + ... \)

29. The residue of \( \frac{1 - e^{z}}{z^3} \) at its pole is
   (A) \( \frac{4}{3} \)
   (B) \( -\frac{4}{3} \)
   (C) \( \frac{2}{3} \)
   (D) None of the above

30. The residue of \( z \cos \frac{1}{z} \) at \( z = 0 \) is
   (A) \( \frac{1}{2} \)
   (B) \( -\frac{1}{2} \)
   (C) \( \frac{1}{3} \)
   (D) None of the above

31. \( \int_{c} \frac{1-2z}{z(1-z)(z-2)} \, dz = ? \) where \( c \) is \(|z| = 15\)
   (A) \( -i3\pi \)
   (B) \( i3\pi \)
   (C) 2
   (D) -2

32. \( \int_{c} \frac{z \cos \frac{z}{2}}{z^2 - \pi^2} \, dz = ? \) where \( c \) is \(|z-1| = 1\)
   (A) \( 6\pi \)
   (B) \( -6\pi \)
   (C) \( i2\pi \)
   (D) None of the above

33. \( \int_{c} \frac{1}{z^2 - e^{z}} \, dz = ? \) where \( c \) is \(|z| = 1\)
   (A) \( i3\pi \)
   (B) \(-i3\pi \)
   (C) \( \frac{i\pi}{3} \)
   (D) None of the above

34. \( \int_{c} \frac{2\theta}{2 + \cos \theta} \, d\theta = ? \)
   (A) \( \frac{-2\pi}{\sqrt{2}} \)
   (B) \( \frac{2\pi}{\sqrt{3}} \)
   (C) \( 2\pi\sqrt{2} \)
   (D) \(-2\pi\sqrt{3} \)
35. \[ \int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} \, dx = ? \]
(A) \( \frac{\pi ab}{a + b} \) \hspace{1cm} (B) \( \frac{\pi(a + b)}{ab} \)
(C) \( \frac{\pi}{a + b} \) \hspace{1cm} (D) \( \pi(a + b) \)

36. \[ \int_{0}^{\infty} \frac{dx}{1 + x^6} = ? \]
(A) \( \frac{\pi}{6} \) \hspace{1cm} (B) \( \frac{\pi}{2} \)
(C) \( \frac{2\pi}{3} \) \hspace{1cm} (D) \( \frac{\pi}{3} \)

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**SOLUTIONS**

1. (C) Since, \( f(z) = u + iv = \frac{x^2(1 + i) - y^2(1 - i)}{x^2 + y^2} \); \( z \neq 0 \)

\[ \Rightarrow u = \frac{x^2 - y^2}{x^2 + y^2} ; \quad v = \frac{x^2 + y^2}{x^2 + y^2} \]

Cauchy Riemann equations are

\[ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \]

By differentiation the value of \( \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \) at \((0,0)\)

we get \( \frac{0}{0} \), so we apply first principle method.

At the origin,

\[ \frac{\partial u}{\partial x} = \lim_{h \to 0} \frac{u(0 + h, 0) - u(0,0)}{h} = \lim_{h \to 0} \frac{k^3/h^3}{h} = 1 \]

\[ \frac{\partial u}{\partial v} = \lim_{k \to 0} \frac{u(0, 0 + k) - u(0,0)}{k} = \lim_{k \to 0} \frac{-k^3/k^3}{k} = -1 \]

\[ \frac{\partial v}{\partial x} = \lim_{h \to 0} \frac{v(0 + h, 0) - v(0,0)}{h} = \lim_{h \to 0} \frac{h^3/h^3}{h} = 1 \]

\[ \frac{\partial v}{\partial y} = \lim_{k \to 0} \frac{v(0, 0 + k) - v(0,0)}{k} = \lim_{k \to 0} \frac{k^3/k^3}{k} = 1 \]

Thus, we see that \( \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \) and \( \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \)

Hence, Cauchy-Riemann equations are satisfied at \( z = 0 \).

Again, \( f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z} \)

\[ = \lim_{z \to 0} \left[ \frac{(x^2 - y^2) + i(x^2 + y^2)}{(x^2 + y^2)} \frac{1}{(x + iy)} \right] \]

Now let \( z \to 0 \) along \( y = x \), then

\[ f'(0) = \lim_{x \to 0} \left[ \frac{(x^2 - x^2) + i(x^2 + x^2)}{(x^2 + x^2)} \frac{1}{(x + x)} \right] = \frac{2i}{2(1 + i)} = \frac{1 + i}{2} \]

Again let \( z \to 0 \) along \( y = 0 \), then

\[ f'(0) = \lim_{x \to 0} \left[ \frac{x^2 + i(x^2)}{(x^2)} \frac{1}{x} \right] = 1 + i \]

So we see that \( f'(0) \) is not unique. Hence \( f'(0) \) does not exist.

2. (A) Since, \( f'(z) = \frac{df}{dz} = \lim_{\Delta z \to 0} \frac{\Delta f}{\Delta z} \)

or \( f'(z) = \lim_{\Delta z \to 0} \frac{\Delta u + i\Delta v}{\Delta x + i\Delta y} \) \hspace{1cm} \ldots(1) \)

Now, the derivative \( f'(z) \) exits of the limit in equation (1) is unique i.e. it does not depends on the path along which \( \Delta z \to 0 \).
Let $\Delta z \to 0$ along a path parallel to real axis
\[ \Rightarrow \Delta y = 0 ; \Delta z \to 0 \Rightarrow \Delta x \to 0 \]

Now equation (1)
\[ f'(z) = \lim_{\Delta z \to 0} \frac{\Delta u + i\Delta v}{\Delta x} = \lim_{\Delta y \to 0} \frac{\Delta u + i\Delta v}{\Delta x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \]
\[ f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad \text{...(2)} \]

Again, let $\Delta z \to 0$ along a path parallel to imaginary axis, then $\Delta x \to 0$ and $\Delta z \to 0 \Rightarrow \Delta y \to 0$

Thus from equation (1)
\[ \varphi(z) = \lim_{\Delta y \to 0} \frac{\Delta u + i\Delta v}{\Delta y} = \lim_{\Delta z \to 0} \frac{\Delta u + i\Delta v}{i\Delta z} = \frac{\partial u}{i\partial y} + \frac{\partial v}{\partial y} \]
\[ f'(z) = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \quad \text{...(3)} \]

Now, for existence of $f'(z)$ R.H.S. of equation (2) and (3) must be same i.e.,
\[ \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \]

\[ f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad \text{...(4)} \]

3. (A) Given $f(z) = x^2 + iy^2$ since, $f(z) = u + iv$

Here $u = x^2$ and $v = y^2$

Now, $u = x^2 \Rightarrow \frac{\partial u}{\partial x} = 2x$ and $\frac{\partial u}{\partial y} = 0$

and $v = y^2 \Rightarrow \frac{\partial v}{\partial x} = 0$ and $\frac{\partial v}{\partial y} = 2y$

we know that $f'(z) = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} \quad \text{...(1)}$

and $f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial y} \quad \text{...(2)}$

Now, equation (1) gives $f'(z) = 2x \quad \text{...(3)}$

and equation (2) gives $f'(z) = 2y \quad \text{...(4)}$

Now, for existence of $f'(z)$ at any point is necessary that the value of $f'(z)$ must be unique at that point, whatever be the path of reaching at that point.

From equation (3) and (4) $2x = 2y$

Hence, $f'(z)$ exists for all points lie on the line $x = y$.

4. (B) $\frac{\partial u}{\partial x} = 2(1 - y) ; \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{...(1)}$

$\frac{\partial u}{\partial y} = -2x ; \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{...(2)}$

$\Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, Thus $u$ is harmonic.

Now let $v$ be the conjugate of $u$ then
\[ dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy \]

(by Cauchy-Riemann equation)
\[ \Rightarrow dv = 2x dx + 2(1 - y)dy \]

On integrating $v = x^2 - y^2 + 2y + C$

5. (C) Given $f(z) = u + iv \quad \text{...(1)}$

$\Rightarrow i f(z) = -v + iu \quad \text{...(2)}$

add equation (1) and (2)
\[ (1 + i)f(z) = (u - v) + i(u + v) \]

$\Rightarrow F(z) = U + iV$

where, $F(z) = (1 + i)f(z); U = (u - v); V = u + v$

Let $F(z)$ be an analytic function.

Now, $U = u - v = e^r \cos y - \sin y$

$\frac{\partial U}{\partial x} = e^r \cos y - \sin y \quad \text{and} \quad \frac{\partial U}{\partial y} = e^r (-\sin y - \cos y)$

Now, $dv = \frac{-\partial U}{\partial y} dx + \frac{\partial U}{\partial x} dy = e^r (\sin y + \cos y) dy$

$\Rightarrow d[e^r (\sin y + \cos y)]$

on integrating $V = e^r (\sin y + \cos y) + c_1$

$F(z) = U + iV = e^r (\cos y - \sin y) + ie^r (\sin y + \cos y) + ic_1$

$= e^r (\cos y + i \sin y) + ie^r (\cos y + \sin y + i \sin y + ic_1$

$F(z) = (1 + i)e^{r+i} + ic_1 = (1 + i)e^r + ic_1$

$\Rightarrow f(z) = e^r + \frac{i}{1 + i} c_1 = e^r + c_1 \frac{i(1 - i)}{(1 + i)(1 - i)} = e^r + \frac{i + 1}{2} c_1$

$\Rightarrow f(z) = e^r + (1 + i)c$

6. (C) $u = \sinh x \cos y$

$\frac{\partial u}{\partial x} = \cosh x \cos y = \Phi(x, y)$

and $\frac{\partial u}{\partial y} = -\sinh x \sin y = \Psi(x, y)$

by Milne's Method

$f'(z) = \Phi(z, 0) - i\Psi(z, 0) = \cosh z - i \cdot 0 = \cosh z$

On integrating $f(z) = \sinh z + \text{constant}$
\[ \Rightarrow f(z) = u = \sinh z + ic \]

(As $u$ does not contain any constant, the constant $c$ is in the function $x$ and hence i.e. in $w$).

7. (A) $\frac{\partial v}{\partial x} = 2y = h(x, y), \frac{\partial v}{\partial y} = 2x = g(x, y)$

by Milne's Method

$f'(z) = g(z, 0) + ih(z, 0) = 2z + i 0 = 2z$

On integrating $f(z) = z^2 + c$
8. (D) \[ \frac{\partial v}{\partial y} = -(x^2 + y^2) - (x - y)2y \]
\[ \frac{\partial v}{\partial x} = (x^2 + y^2) - (x - y)2x \]
\[ g(x, y) = \frac{y^2 - x^2 - 2xy}{(x^2 + y^2)^2} \]
\[ h(x, y) = \frac{y^2 - x^2 + 2xy}{(x^2 + y^2)^2} \]

By Milne's Method
\[ f'(z) = g(z, 0) + ih(z, 0) = -\frac{1}{z^2} + i\left(-\frac{1}{z^2}\right) = -(1 + i) \frac{1}{z^2} \]

On integrating
\[ f(z) = (1 + i) \int \frac{1}{z^2} dz + c = (1 + i) \frac{1}{z} + c \]

9. (A) \[ \frac{\partial u}{\partial x} = 2 \cos 2x (\cosh 2y - \cos 2x) - 2 \sin^2 2x \]
\[ \frac{\partial u}{\partial y} = 2 \sin 2x \sinh 2y \]
\[ \phi(x, y) = \frac{2 \cos 2x \cosh 2y - 2}{(\cosh 2y - \cos 2x)^2} \]
\[ \psi(x, y) = \frac{2 \sin 2x \sinh 2y}{(\cosh 2y - \cos 2x)^2} \]

By Milne's Method
\[ f'(z) = \phi(z, 0) - i\psi(z, 0) \]
\[ f'(0) = \frac{2 \cos 2z - 2}{(1 - \cos 2z)^2} - i(0) = -\frac{2}{1 - \cos 2z} = -\csc^2 z \]

On integrating
\[ f(z) = -\int \csc^2 z dz + ic = \cot z + ic \]

10. \( x = at + b, \ y = ct + d \)

On \( A, z = 1 + i \) and on \( B, z = 2 + 4i \)

Let \( z = 1 + i \) corresponds to \( t = 0 \)

and \( z = 2 + 4i \) corresponding to \( t = 1 \)

then, \( t = 0 \) \( \Rightarrow \) \( x = b, \ y = d \)

\( \Rightarrow b = 1, \ d = 1 \)

and \( t = 1 \) \( \Rightarrow \) \( x = a + b, \ y = c + d \)

\( \Rightarrow 2 = a + 1, 4 = c + 1 \) \( \Rightarrow a = 1, c = 3 \)

AB is \( , \ y = 3t + 1 \) \( \Rightarrow dx = dt, \ dy = 3dt \)

\[ \int f(z)dz = \int \left( (t^2 + 1) + m(t + 1)(3t + 1)\right)dt = \frac{29}{3} + 11i \]

11. (D) We know by the derivative of an analytic function that

\[ f^{(n)}(z) = \frac{n!}{2\pi i} \int_{c} \frac{f(z)}{(z - z_o)^{n+1}}dz \]

Taking \( n = 3, \int \frac{f(z)}{(z - z_o)^4}dz = \frac{2\pi i}{3} f'''(z_o) \) ....(1)

Given \( f, f(z) = e^{2z} \) and \( z_o = -1 \) in (1), we have

\[ \int \frac{e^{2z}}{(z + 1)^3}dz = \frac{2\pi i}{3} e^{(1)} \]

Now, \( f(z) = e^{2z} \Rightarrow f'''(z) = 8e^{2z} \)

\[ \int \frac{e^{2z}}{(z + 1)^3}dz = \frac{8\pi i}{3} e^{2z} \]

12. (D) Since, \( \frac{1 - 2z}{z(z - 1)(z - 2)} = \frac{1}{2x} + \frac{1}{z - 1} - \frac{3}{2(z - 2)} \)

\[ \int \frac{1 - 2z}{z(z - 1)(z - 2)}dz = \frac{1}{2} I_1 + I_2 - \frac{3}{2} I_3 \] ...(1)

Since, \( z = 0 \) is the only singularity for \( I_1 = \int \frac{1}{z} dz \) and it lies inside \( |z| = 15 \), therefore by Cauchy's integral formula

\[ I_1 = \int \frac{1}{z} dz = 2\pi i \] ....(2)

\[ f(z_o) = \int \frac{f(z)}{z - z_o} \] [Here \( f(z) = 1 \) and \( z_o = 0 \)]

Similarly, for \( I_2 = \int \frac{1}{z - 1} dz \), the singular point \( z = 1 \) lies inside \( |z| = 15 \) therefore \( I_2 = 2\pi i \)...

For \( I_3 = \int \frac{1}{z - 2} dz \), the singular point \( z = 2 \) lies outside the circle \( |z| = 15 \), so the function \( f(z) \) is analytic everywhere in \( c \) i.e. \( |z| = 15 \), hence by Cauchy's integral theorem

\[ I_3 = \int \frac{1}{z - 2} dz = 0 \] ....(4)

using equations (2), (3), (4) in (1), we get

\[ \int \frac{1 - 2z}{z(z - 1)(z - 2)}dz = \frac{1}{2} (2\pi i) + 2\pi i - \frac{3}{2} (0) = 3\pi i \]

13. (B) Given contour \( c \) is the circle \( |z| = 1 \)

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14. (B) Let \( f(z) = \cos \pi z \) then \( f(z) \) is analytic within and on \( |z| = 3 \), now by Cauchy's integral formula

\[
f(z_0) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z - z_0} dz = \frac{\pi i}{2} [f(z_0)]
\]

take \( f(z) = \cos \pi z \), \( z_0 = 1 \), we have

\[
\int_{|z| = 3} \frac{\cos \pi z}{z - 1} dz = 2\pi i f(1) = 2\pi i \cos \pi = -2\pi i
\]

15. (D) \[
\int_c \frac{\sin \pi z^2}{z^2 - 1} dz = \int_c \frac{\sin \pi z^2}{z - 1} dz - \int_c \sin \pi z^2 dz
\]

\[
= 2\pi i f(2) - 2\pi i f(1) \text{ since, } f(z) = \sin \pi z^2
\]

\[
\Rightarrow f(2) = \sin 4\pi = 0 \text{ and } f(1) = \sin \pi = 0
\]

16. (D) Let, \( I = \frac{1}{2\pi i} \int_c \frac{\cos \pi z}{z^2 - 1} dz \)

\[
= \frac{1}{2 \cdot 2\pi i} \int_c \left( \frac{1}{z - 1} - \frac{1}{z + 1} \right) \cos \pi z dz
\]

Or \( I = \frac{1}{4\pi i} \int_c \left( \frac{\cos n z}{z - 1} - \frac{\cos n z}{z + 1} \right) dz \)

17. (D) \( f(3) = \frac{3z^2 + 7z + 1}{z - 3} \), since \( z_0 = 3 \) is the only singular point of \( \frac{3z^2 + 7z + 1}{z - 3} \) and it lies outside the circle \( x^2 + y^2 = 4 \) i.e., \( |z| = 2 \), therefore \( \frac{3z^2 + 7z + 1}{z - 3} \) is analytic everywhere within \( c \).

Hence by Cauchy's theorem—\( f(3) = \frac{3z^2 + 7z + 1}{z - 3} dz = 0 \)

18. (C) The point \((1 - i)\) lies within circle \(|z| = 2\) (... the distance of \(1 - i\) i.e., \((1, 1)\) from the origin is \(\sqrt{2}\) which is less than 2, the radius of the circle).

Let \( f(z) = 3z^2 + 7z + 1 \) then by Cauchy's integral formula

\[
\int_c \frac{3z^2 + 7z + 1}{z - z_0} dz = 2\pi i f(z_0)
\]

\[
f(z_0) = 2\pi i f(z_0) \Rightarrow f(z_0) = 2\pi i f(z_0)
\]

and \( f''(z_0) = 2\pi i f''(z_0) \)

since, \( \phi(z) = 3z^2 + 7z + 1 \)

\[
\Rightarrow \psi(z) = 6z + 7 \text{ and } \psi'(z) = 6
\]

\[
f'(1 - i) = 2\pi i [6(1 - i) + 7] = 2\pi (5 + 13i)
\]

19. (C) \( f(z) = \frac{z - 1}{z + 1} = 1 - \frac{2}{z + 1} \)

\[
\Rightarrow f(0) = -1, \; f(1) = 0\]

\[
f'(z) = \frac{2}{(z + 1)^2} \Rightarrow f'(0) = 2;
\]

\[
f''(z) = \frac{-4}{(z - 1)^3} \Rightarrow f''(0) = -4;
\]

\[
f'''(z) = \frac{12}{(z + 1)^4} \Rightarrow f'''(0) = 12; \text{ and so on.}
\]

Now, Taylor series is given by

\[
f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \frac{(z - z_0)^3}{3!} f'''(z_0) + \ldots
\]

about \( z = 0 \)

\[
f(z) = -1 + z(2) + \frac{z^2}{2!} (-4) + \frac{z^3}{3!} (12) + \ldots
\]

\[
= -1 + 2z - 2z^2 + 2z^3 + \ldots
\]

\[
f(z) = -1 + 2(z - z^2 + z^3 + \ldots)
\]

20. (B) \( f'(z)^2 = \frac{1}{z + 1} \Rightarrow f'(1) = \frac{1}{2}
\]

\[
f'(z) = \frac{-1}{(z + 1)^2} \Rightarrow f''(1) = \frac{-1}{4}
\]

\[
f''(z) = \frac{2}{(z + 1)^3} \Rightarrow f'''(1) = \frac{1}{4}
\]

\[
f'''(z) = \frac{-6}{(z + 1)^4} \Rightarrow f'''(1) = -\frac{3}{8}
\]

and so on.

Taylor series is

\[
f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \frac{(z - z_0)^3}{3!} f'''(z_0) + \ldots
\]

about \( z = 1 \)

\[
f(z) = \frac{1}{2} + (z - 1) \left( \frac{-1}{4} + \frac{(z - 1)^2}{2!} \left( \frac{1}{4} + \frac{(z - 1)^3}{3!} \left( -\frac{3}{8} + \ldots\right) \right) \right)
\]

\[
= \frac{1}{2} - \frac{1}{2^2} (z - 1) + \frac{1}{2^3} (z - 1)^2 - \frac{1}{2^4} (z - 1)^3 + \ldots
\]

or \( f(z) = \frac{1}{2} \left[ 1 - \frac{1}{2} (z - 1) + \frac{1}{2^2} (z - 1)^2 - \frac{1}{2^3} (z - 1)^3 + \ldots \right] \)
21. (A) \( f(z) = \sin z \Rightarrow f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \)
\( f'(z) = \cos z \Rightarrow f'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \)
\( f''(z) = -\sin z \Rightarrow f''\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} \)
\( f'''(z) = -\cos z \Rightarrow f'''\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} \) and so on.

Taylor series is given by
\[
f(z) = f(z_0) + (z - z_0)f'(z_0) + \frac{(z - z_0)^2}{2!}f''(z_0) + \frac{(z - z_0)^3}{3!}f'''(z_0) + \ldots
\]
about \( z = \frac{\pi}{4} \)
\[
f(z) = \frac{1}{\sqrt{2}} + (z - \frac{\pi}{4})\frac{1}{\sqrt{2}} - \frac{1}{2!}(z - \frac{\pi}{4})^2\frac{1}{\sqrt{2}}
+ \frac{(z - \frac{\pi}{4})^3}{3!}\frac{1}{\sqrt{2}} + \ldots
\]

22. (D) Let \( f(z) = z^2 = \frac{1}{\sqrt{2} \left[ 1 + (1 - (1 + z)^2) \right]^2} \)

Since, \( 1 + z < 1 \), so by expanding R.H.S. by binomial theorem, we get
\[
f(z) = 1 + 2(1 + z) + 3(1 + z)^2 + 4(1 + z)^3 + \ldots + (n + 1)(1 + z)^n + \ldots
\]
or \( f(z) = z^2 = 1 + \sum_{n=1}^{\infty} (n + 1)(1 + z)^n \)

23. (B) Here \( f(z) = \frac{1}{(z-1)(z-2)} = \frac{1}{z-2} - \frac{1}{z-1} \ldots \ldots (1) \)

Since, \( |z| > 1 \) \( \Rightarrow \) \( \frac{|z|}{2} < 1 \) and \( |z| < 2 \) \( \Rightarrow \) \( \frac{|z|}{2} < 1 \)
\[
\frac{1}{z-1} = \frac{1}{z} \left( \frac{1}{1 - \frac{1}{z}} \right) = \frac{1}{z} \left( \frac{1}{1 - \frac{1}{z}} \right)^{-1}
\]
\[
= \frac{1}{z} \left( 1 + \frac{1}{z} \right) + \frac{1}{z} \left( \frac{1}{4} \right) + \frac{1}{z} \left( \frac{1}{z} \right) + \ldots
\]
and \( \frac{1}{z-2} = \frac{-1}{2} \left( 1 - \frac{1}{z} \right)^{-1} = \frac{-1}{2} \left[ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{9} + \ldots \right] \)
equation (1) gives—

\[
f(z) = -\frac{1}{2} \left[ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{9} + \ldots \right] - \frac{1}{z} \left[ 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \ldots \right]
\]
or \( f(z) = -z^4 - z^3 - z - 1 - \frac{1}{4} - \frac{1}{8} - \frac{1}{18} - z^3 - \ldots \)

24. (C) \( \frac{2}{|z|} < 1 \Rightarrow \frac{1}{|z|} < 1 \)
\[
\frac{1}{z - 1} = \frac{1}{z} \left( 1 - \frac{1}{z} \right)^{-1} = \frac{1}{2} \left[ 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \ldots \right]
\]
and \( \frac{1}{z - 2} = \frac{1}{z} \left( 1 - \frac{2}{z} \right)^{-1} = \frac{1}{z} \left[ 1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \ldots \right] \)

Laurent's series is given by
\[
f(z) = \frac{1}{z} \left( 1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \ldots \right) - \frac{1}{z} \left( 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \ldots \right)
= \frac{1}{z} \left[ \frac{1}{z} + \frac{3}{z^2} + \frac{7}{z^3} + \ldots \right]
\Rightarrow f(z) = \frac{1}{z} + \frac{3}{z^2} + \frac{7}{z^3} + \ldots
\]

25. (B) \( |z| < 1 \), \( \frac{1}{z^2} - \frac{1}{z - 1} = \frac{1}{z} \left( 1 - \frac{1}{z} \right)^{-1} + (1 - z)^{-1}
= \frac{1}{z} \left[ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \ldots \right] + (1 + z + z^2 + z^3 + \ldots)
\]
\[
f(z) = \frac{1}{2} + \frac{3}{4} z + \frac{7}{8} z^2 + \frac{15}{16} z^3 + \ldots
\]

26. (D) Since, \( \frac{1}{z(1-z)} = \frac{1}{2z} - \frac{1}{z-1} + \frac{1}{2(z-2)} \)

For \( |z| < 1 \) Let \( z - 1 = u \)
\[
\Rightarrow \quad z = u + 1 \quad \text{and} \quad |u| < 1
\]
\[
\frac{1}{z(1-z)} = \frac{1}{2u} - \frac{1}{u} + \frac{1}{2(u-1)} = \frac{1}{2} (1 + u^{-1}) - u^{-1} - \frac{1}{2} (1 - u^{-1})
\]
\[
= \frac{1}{2} \left[ 1 + u + u^2 + u^3 + \ldots \right] - u^{-1} - \frac{1}{2} \left[ 1 + u + u^2 + u^3 + \ldots \right]
\]
\[
= \frac{1}{2} \left[ -2u + 2u + u^2 - u^3 - \ldots \right] - u^{-1} = -u - u^3 - \ldots - u^{-1}
\]

Required Laurent's series is
\[
f(z) = -(z - 1)^{-1} - (z - 1)^{-3} - (z - 1)^{-5} - \ldots
\]

27. (B) Let \( f(z) = \frac{1}{z(e^z - 1)} \)
\[
= \frac{1}{z} \left[ 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \ldots - 1 \right]
\]

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34. (B) Let \( z = e^{i\theta} \Rightarrow d\theta = -\frac{idz}{z} \); \( z \leq 0 \leq 2\pi \)

and \( \cos \theta = \frac{1}{2} \left( z + \frac{1}{z} \right) \)

\[
\int_C f(z) \, dz = 2\pi i \left( \text{Residue at } z = -2 + \sqrt{3} \right)
\]

Now, residue at \( z = -2 + \sqrt{3} \)

\[
= \lim_{z \to -2 + \sqrt{3}} \frac{z^2}{(z^2 + a^2)(z^2 + b^2)} = \frac{\pi i}{\sqrt{3}}
\]

\[
\int_C f(z) \, dz = 2\pi i \times \frac{1}{2\sqrt{3}} = \frac{\pi}{\sqrt{3}}
\]

\[
\int_C \frac{d\theta}{2 + \cos \theta} = -2i \times \frac{\pi}{\sqrt{3}} = \frac{2\pi}{\sqrt{3}}
\]

35. (C) \( I = \int_C \frac{z^2}{(z^2 + a^2)(z^2 + b^2)} \, dz \)

where \( C \) is the boundary of a semi-circle centered at \( 0 \) with radius \( r \) and \( \text{Re } d \)

\[
\int_C f(z) \, dz = \int_{-r}^r f(z) \, dz + \int_R^C f(z) \, dz
\]

\[
= \frac{2\pi i}{2i} \left( 1 + \sqrt{3}i \right) = \frac{1 + \sqrt{3}i}{12i}
\]

Residue at \( z = i \) is \( \frac{1}{6i} \)

Residue at \( z = 1 + \sqrt{3}i \) is \( \frac{1}{3(1 - \sqrt{3}i)} = \frac{1 - \sqrt{3}i}{12i} \)

\[
\int_C f(z) \, dz = \int_C f(z) \, dz + \int_{-R}^R f(z) \, dz
\]

\[
= \frac{2\pi i}{12i} (1 - \sqrt{3}i + 1 + \sqrt{3}i + 2i) = \frac{2\pi}{3}
\]

or \( \int_C f(z) \, dz + \int_R^C f(z) \, dz = \frac{2\pi}{3} \)

(1) \( \int_0^\pi \frac{ax}{1 + x^2} = \frac{2\pi}{3} \)

Now \( \int_C f(z) \, dz = \int_0^\pi \frac{ie^{i\theta} \, d\theta}{R^2 e^{i\theta} + a^2} \frac{R^2 e^{i\theta} + b^2}{R^2}
\]

\[
= \int_0^\pi \frac{\cos \theta \, d\theta}{R^2 e^{i\theta} + a^2} \frac{R^2 e^{i\theta} + b^2}{R^2}
\]

where \( R \to \infty \), \( \int_C f(z) \, dz \to 0 \)

*********
1. In a frequency distribution, the mid value of a class is 15 and the class interval is 4. The lower limit of the class is
   (A) 14  (B) 13  (C) 12  (D) 10

2. The mid value of a class interval is 42. If the class size is 10, then the upper and lower limits of the class are
   (A) 47 and 37  (B) 37 and 47  (C) 37.5 and 47.5  (D) 47.5 and 37.5

3. The following marks were obtained by the students in a test: 81, 72, 90, 90, 86, 85, 92, 70, 71, 83, 89, 95, 85, 79, 62. The range of the marks is
   (A) 9  (B) 17  (C) 27  (D) 33

4. The width of each of nine classes in a frequency distribution is 2.5 and the lower class boundary of the lowest class is 10.6. The upper class boundary of the highest class is
   (A) 35.6  (B) 33.1  (C) 30.6  (D) 28.1

5. In a monthly test, the marks obtained in mathematics by 16 students of a class are as follows: 0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8
   The arithmetic mean of the marks obtained is
   (A) 3  (B) 4  (C) 5  (D) 6

6. A distribution consists of three components with frequencies 45, 40 and 15 having their means 2, 2.5 and 2 respectively. The mean of the combined distribution is
   (A) 2.1  (B) 2.2  (C) 2.3  (D) 2.4

7. Consider the table given below
<table>
<thead>
<tr>
<th>Marks</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 10</td>
<td>12</td>
</tr>
<tr>
<td>10 – 20</td>
<td>18</td>
</tr>
<tr>
<td>20 – 30</td>
<td>27</td>
</tr>
<tr>
<td>30 – 40</td>
<td>20</td>
</tr>
<tr>
<td>40 – 50</td>
<td>17</td>
</tr>
<tr>
<td>50 – 60</td>
<td>6</td>
</tr>
</tbody>
</table>

   The arithmetic mean of the marks given above, is
   (A) 18  (B) 28  (C) 27  (D) 6

8. The following is the data of wages per day: 5, 4, 7, 5, 8, 8, 5, 7, 9, 5, 7, 9, 10, 8 The mode of the data is
   (A) 5  (B) 7  (C) 8  (D) 10

9. The mode of the given distribution is
<table>
<thead>
<tr>
<th>Weight (in kg)</th>
<th>40</th>
<th>43</th>
<th>46</th>
<th>49</th>
<th>52</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Children</td>
<td>5</td>
<td>8</td>
<td>16</td>
<td>9</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>
   (A) 55  (B) 46  (C) 40  (D) None
10. If the geometric mean of \(x\), 16, 50, be 20, then the value of \(x\) is
(A) 4  
(B) 10  
(C) 20  
(D) 40

11. If the arithmetic mean of two numbers is 10 and their geometric mean is 8, the numbers are
(A) 12, 18  
(B) 16, 4  
(C) 15, 5  
(D) 20, 5

12. The median of
0, 2, 2, 2, -3, 5, -1, 5, 5, -3, 6, 6, 5, 6 is
(A) 0  
(B) -1.5  
(C) 2  
(D) 3.5

13. Consider the following table

<table>
<thead>
<tr>
<th>Diameter of heart (in mm)</th>
<th>Number of persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>5</td>
</tr>
<tr>
<td>121</td>
<td>9</td>
</tr>
<tr>
<td>122</td>
<td>14</td>
</tr>
<tr>
<td>123</td>
<td>8</td>
</tr>
<tr>
<td>124</td>
<td>5</td>
</tr>
<tr>
<td>125</td>
<td>9</td>
</tr>
</tbody>
</table>

The median of the above frequency distribution is
(A) 122 mm  
(B) 123 mm  
(C) 122.5 mm  
(D) 122.75 mm

14. The mode of the following frequency distribution, is

<table>
<thead>
<tr>
<th>Class interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-6</td>
<td>2</td>
</tr>
<tr>
<td>6-9</td>
<td>5</td>
</tr>
<tr>
<td>9-12</td>
<td>21</td>
</tr>
<tr>
<td>12-15</td>
<td>23</td>
</tr>
<tr>
<td>15-18</td>
<td>10</td>
</tr>
<tr>
<td>18-21</td>
<td>12</td>
</tr>
<tr>
<td>21-24</td>
<td>3</td>
</tr>
</tbody>
</table>

(A) 11.5  
(B) 11.8  
(C) 12  
(D) 12.4

15. The mean-deviation of the data 3, 5, 6, 7, 8, 10, 11, 14 is
(A) 4  
(B) 3.25  
(C) 2.75  
(D) 2.4

16. The mean deviation of the following distribution is

<table>
<thead>
<tr>
<th>(x)</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f)</td>
<td>3</td>
<td>12</td>
<td>18</td>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

(A) 12  
(B) 0.75  
(C) 1.25  
(D) 26

17. The standard deviation for the data 7, 9, 11, 13, 15 is
(A) 2.4  
(B) 2.5  
(C) 2.7  
(D) 2.8

18. The standard deviation of 6, 8, 10, 12, 14 is
(A) 1  
(B) 0  
(C) 2.83  
(D) 2.73

19. The probability that an event A occurs in one trial of an experiment is 0.4. Three independent trials of experiment are performed. The probability that A occurs at least once is
(A) 0.936  
(B) 0.784  
(C) 0.964  
(D) None

20. Eight coins are tossed simultaneously. The probability of getting at least 6 heads is
(A) \(\frac{7}{64}\)  
(B) \(\frac{37}{256}\)  
(C) \(\frac{57}{64}\)  
(D) \(\frac{249}{256}\)

21. A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?
(A) 0.16  
(B) 0.63  
(C) 0.97  
(D) 0.20

22. A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident?
(A) 5%  
(B) 45%  
(C) 35%  
(D) 15%

23. The odds against a husband who is 45 years old, living till he is 70 are 7:5 and the odds against his wife who is 36, living till she is 61 are 5:3. The probability that at least one of them will be alive 25 years hence, is
(A) \(\frac{61}{64}\)  
(B) \(\frac{5}{8}\)  
(C) \(\frac{13}{64}\)  
(D) None
24. The probability that a man who is \( x \) years old will die in a year is \( p \). Then amongst \( n \) persons \( A_1, A_2, \ldots, A_n \) each \( x \) years old now, the probability that \( A_1 \) will die in one year is
   (A) \( \frac{1}{n^2} \)  
   (B) \( 1 - (1 - p)^n \)  
   (C) \( \frac{1}{n^2} [1 - (1 - p)^n] \)  
   (D) \( \frac{1}{n} [1 - (1 - p)^n] \)

25. A bag contains 4 white and 2 black balls. Another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, the probability that both are white is
   (A) \( \frac{1}{24} \)  
   (B) \( \frac{1}{4} \)  
   (C) \( \frac{5}{24} \)  
   (D) None

26. A bag contains 5 white and 4 red balls. Another bag contains 4 white and 2 red balls. If one ball is drawn from each bag, the probability that one is white and one is red, is
   (A) \( \frac{13}{27} \)  
   (B) \( \frac{5}{27} \)  
   (C) \( \frac{8}{27} \)  
   (D) None

27. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is
   (A) 0.76  
   (B) 0.4096  
   (C) 0.6976  
   (D) None of these

28. If the probabilities that A and B will die within a year are \( p \) and \( q \) respectively, then the probability that only one of them will be alive at the end of the year is
   (A) \( pq \)  
   (B) \( p(1 - q) \)  
   (C) \( q(1 - p) \)  
   (D) \( p + 1 - 2pq \)

29. In a binomial distribution, the mean is 4 and variance is 3. Then, its mode is
   (A) 5  
   (B) 6  
   (C) 4  
   (D) None

30. If 3 is the mean and \( \frac{3}{2} \) is the standard deviation of a binomial distribution, then the distribution is
   (A) \( \left( \frac{3}{4} + \frac{1}{4} \right)^{12} \)  
   (B) \( \left( \frac{1}{2} + \frac{3}{2} \right)^{12} \)  
   (C) \( \left( \frac{4}{5} + \frac{1}{5} \right)^{10} \)  
   (D) \( \left( \frac{1}{5} + \frac{4}{5} \right)^5 \)

31. The sum and product of the mean and variance of a binomial distribution are 24 and 18 respectively. Then, the distribution is
   (A) \( \left( \frac{1}{7} + \frac{1}{8} \right)^{12} \)  
   (B) \( \left( \frac{1}{4} + \frac{3}{4} \right)^{16} \)  
   (C) \( \left( \frac{1}{6} + \frac{5}{6} \right)^{24} \)  
   (D) \( \left( \frac{1}{2} + \frac{1}{2} \right)^{32} \)

32. A die is thrown 100 times. Getting an even number is considered a success. The variance of the number of successes is
   (A) 50  
   (B) 25  
   (C) 10  
   (D) None

33. A die is thrown thrice. Getting 1 or 6 is taken as a success. The mean of the number of successes is
   (A) \( \frac{3}{2} \)  
   (B) \( \frac{2}{3} \)  
   (C) 1  
   (D) None

34. If the sum of mean and variance of a binomial distribution is 4.8 for five trials, the distribution is
   (A) \( \left( \frac{1}{5} + \frac{4}{5} \right)^5 \)  
   (B) \( \left( \frac{1}{3} + \frac{2}{3} \right)^5 \)  
   (C) \( \left( \frac{2}{5} + \frac{3}{5} \right)^5 \)  
   (D) None of these

35. A variable has Poisson distribution with mean \( m \). The probability that the variable takes any of the values 0 or 2 is
   (A) \( e^{-m} \left( 1 + m + \frac{m^2}{2!} \right) \)  
   (B) \( e^{-m} (1 + m)^{-3/2} \)  
   (C) \( e^{32} (1 + m^2)^{-1/2} \)  
   (D) \( e^{-m} \left( 1 + \frac{m^2}{2!} \right) \)

36. If \( X \) is a Poisson variate such that \( P(2) = 9P(4) + 90P(6) \), then the mean of \( X \) is
   (A) \( \pm 1 \)  
   (B) \( \pm 2 \)  
   (C) \( \pm 3 \)  
   (D) None
37. When the correlation coefficient \( r = \pm 1 \), then the two regression lines
(A) are perpendicular to each other
(B) coincide
(C) are parallel to each other
(D) do not exist

38. If \( r = 0 \), then
(A) there is a perfect correlation between \( x \) and \( y \)
(B) \( x \) and \( y \) are not correlated.
(C) there is a positive correlation between \( x \) and \( y \)
(D) there is a negative correlation between \( x \) and \( y \)

39. If \( \Sigma x_i = 15, \Sigma y_i = 36, \Sigma x_i y_i = 110 \) and \( n = 5 \), then \( \text{cov} (x, y) \) is equal to
(A) 0.6
(B) 0.5
(C) 0.4
(D) 0.225

40. If \( \text{cov} (x, y) = -16.5 \), \( \text{var} (x) = 2.89 \) and \( \text{var} (y) = 100 \), then the coefficient of correlation \( r \) is equal to
(A) 0.36
(B) -0.64
(C) 0.97
(D) -0.97

41. The ranks obtained by 10 students in Mathematics and Physics in a class test are as follows

<table>
<thead>
<tr>
<th>Rank in Maths</th>
<th>Rank in Chem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
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<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

The coefficient of correlation between their ranks is
(A) 0.15
(B) 0.224
(C) 0.625
(D) None

42. If \( \Sigma x_i = 24, \Sigma y_i = 44, \Sigma x_i y_i = 306, \Sigma x_i^2 = 164, \Sigma y_i^2 = 574 \) and \( n = 4 \), then the regression coefficient \( b_{xy} \) is equal to
(A) 2.1
(B) 1.6
(C) 1.225
(D) 1.75

43. If \( \Sigma x_i = 30, \Sigma y_i = 42, \Sigma x_i y_i = 199, \Sigma x_i^2 = 184, \Sigma y_i^2 = 318 \) and \( n = 6 \), then the regression coefficient \( b_{xy} \) is
(A) -0.36
(B) -0.46
(C) 0.26
(D) None

44. Let \( r \) be the correlation coefficient between \( x \) and \( y \) and \( b_{xy}, b_{yx} \) be the regression coefficients of \( y \) on \( x \) and \( x \) on \( y \) respectively then
(A) \( r = b_{xy} + b_{yx} \)
(B) \( r = b_{xy} \times b_{yx} \)
(C) \( r = \frac{b_{xy} \times b_{yx}}{\sqrt{b_{xy} \times b_{yx}}} \)
(D) \( r = \frac{1}{2} (b_{xy} + b_{yx}) \)

45. Which one of the following is a true statement.
(A) \( \frac{1}{2} (b_{xy} + b_{yx}) = r \)
(B) \( \frac{1}{2} (b_{xy} + b_{yx}) < r \)
(C) \( \frac{1}{2} (b_{xy} + b_{yx}) > r \)
(D) None of these

46. If \( b_{xy} = 1.6 \) and \( b_{yx} = 0.4 \) and \( \theta \) is the angle between two regression lines, then \( \tan \theta \) is equal to
(A) 0.18
(B) 0.24
(C) 0.16
(D) 0.3

47. The equations of the two lines of regression are:

\[ 43x + 7y = 0 \]
\[ 348 + 0 = 0 \]

The correlation coefficient between \( x \) and \( y \) is
(A) 1.25
(B) 0.25
(C) \( \frac{3}{4} \)
(D) 0.92

48. If \( \text{cov}(X, Y) = 10, \text{var}(X) = 625 \) and \( \text{var}(Y) = 31.36 \), then \( \rho(X, Y) \) is
(A) \( \frac{1}{4} \)
(B) \( \frac{1}{2} \)
(C) \( \frac{3}{4} \)
(D) 0.256

49. If \( \Sigma x = \Sigma y = 15, \Sigma x^2 = \Sigma y^2 = 49, \Sigma xy = 44 \) and \( n = 5 \), then \( b_{xy} \) is
(A) \( -\frac{1}{3} \)
(B) \( -\frac{2}{3} \)
(C) \( -\frac{1}{4} \)
(D) \( -\frac{1}{2} \)

50. If \( \Sigma x = 125, \Sigma y = 100, \Sigma x^2 = 1650, \Sigma y^2 = 1500, \Sigma xy = 50 \) and \( n = 25 \), then the line of regression of \( x \) on \( y \) is
\( 22x + 9y = 146 \)
\( 22x - 9y = 74 \)
\( 22x - 9y = 146 \)
\( 22x + 9y = 74 \)

*********
1. (B) Let the lower limit be \( x \). Then, upper limit is \( x + 4 \). 
\[
x + \frac{(x + 4)}{2} = 15 \implies x = 13.
\]

2. (A) Let the lower limit be \( x \). Then, upper limit \( x + 10 \). 
\[
x + \frac{(x + 10)}{2} = 42 \implies x = 37.
\]
Lower limit = 37 and upper limit = 47.

3. (D) Range = Difference between the largest value 
\[95 - 62 = 33\] .

4. (B) Upper class boundary 
\[10 + 6 = 25\] .

5. (B) 
\[
\begin{array}{|c|c|c|}
\hline
\text{Marks} & \text{Frequency } f & f \cdot m \\
\hline
0 & 2 & 0 \\
2 & 2 & 4 \\
3 & 3 & 9 \\
4 & 1 & 4 \\
5 & 4 & 20 \\
6 & 2 & 12 \\
7 & 1 & 7 \\
8 & 1 & 8 \\
\hline
\end{array}
\]
\[
\Sigma f = 16 \quad \text{and} \quad \Sigma (f \cdot m) = 64
\]
\[
\text{A.M. } = \frac{\Sigma (f \cdot m)}{\Sigma f} = \frac{64}{16} = 4.
\]

6. (B) Mean 
\[
\frac{45 \times 2 + 40 \times 25 + 15 \times 2}{100} = \frac{220}{100} = 2.2.
\]

7. (B) 
\[
\begin{array}{|c|c|c|c|}
\hline
\text{Class} & \text{Mid value } x & \text{Frequency } f & f \cdot d \\
\hline
0–10 & 5 & 12 & -20 \\
10–20 & 15 & 18 & -10 \\
20–30 & 25 & 27 & 0 \\
30–40 & 35 & 20 & 10 \\
40–50 & 45 & 17 & 20 \\
50–60 & 55 & 6 & 30 \\
\hline
\text{Total} & \Sigma f = 100 & \Sigma (f \cdot d) = 390
\end{array}
\]

8. (C) Since 8 occurs most often, mode = 8.

9. (B) Clearly, 46 occurs most often. So, mode = 46.

10. (B) 
\[
x \times 16 \times 50)^{1/3} = 20 \implies x \times 16 \times 50 = (20)^3
\]
\[
\implies x = \frac{20 \times 20 \times 20}{16 \times 50} = 10.
\]

11. (B) Let the numbers be \( a \) and \( b \) Then, 
\[
a + \frac{b}{2} = 10 \implies (a + b) = 20 \quad \text{and} \quad \sqrt{ab} = 8 \implies ab = 64
\]
\[
a - b = \sqrt{(a+b)^2 - 4ab} = \sqrt{44 - 256} = \sqrt{144} = 12.
\]
Solving \( a + b = 20 \) and \( a - b = 12 \) we get \( a = 16 \) and \( b = 4 \).

12. (D) Observations in ascending order are 
\[-3, -3, -1, 0, 2, 2, 2, 5, 5, 5, 5, 6, 6, 6\]
Number of observations is 14, which is even.
Median = \[\frac{1}{2} (7 \text{ term} + 8 \text{ term}) = \frac{1}{2} (2 + 5) = 3.5.\]

13. (A) The given Table may be presented as
\[
\begin{array}{|c|c|c|}
\hline
\text{Diameter of heart (in mm)} & \text{Number of persons} & \text{Cumulative frequency} \\
\hline
120 & 5 & 5 \\
121 & 9 & 14 \\
122 & 14 & 28 \\
123 & 8 & 36 \\
124 & 5 & 41 \\
125 & 9 & 50 \\
\hline
\end{array}
\]

14. (B) Maximum frequency is 23. So, modal class is 12–15.
\( L_a = 12, \quad L_2 = 15, \quad f_1 = 23, \quad f_1 = 21 \) and \( f_2 = 10 \).
Thus Mode = \[
L_a + \frac{f_1 - f_2}{2f_1 - f_1 - f_2} (L_2 - L_a)
\]

A.M. = \[
A + \frac{\Sigma (f \cdot d)}{\Sigma f} = \left(25 + \frac{300}{100}\right) = 28.
\]
15. (C) Mean $= \left( \frac{3 + 5 + 6 + 7 + 8 + 10 + 11 + 14}{8} \right) = 8$.

$\Sigma \delta = |8 - 8| + |5 - 8| + |8 - 8| + |10 - 8| + |11 - 8| + |14 - 8| = 22$

Thus Mean deviation $= \frac{\Sigma \delta}{n} = \frac{22}{8} = 2.75$.

16. (B)

| $x$  | $f$  | $f \times x$ | $\delta = |x - M|$ | $f \times \delta$ |
|------|------|--------------|------------------|------------------|
| 10   | 3    | 30           | 2                | 6                |
| 11   | 2    | 132          | 1                | 2                |
| 12   | 1    | 216          | 0                | 0                |
| 13   | 2    | 156          | 1                | 2                |
| 14   | 3    | 42           | 2                | 6                |

$\Sigma f = 48$  $\Sigma f x = 576$  $\Sigma \delta = 36$

Thus $M = \frac{576}{48} = 12$.

So, Mean deviation $= \frac{\Sigma \delta}{n} = \frac{36}{48} = 0.75$

17. (D) $m = \frac{7 + 9 + 11 + 13 + 15}{5} = \frac{55}{5} = 11$.

$\Sigma \delta^2 = 6 - 11|^2 + 9 - 11|^2 + 11 - 11|^2 + 13 - 11|^2 + 15 - 11|^2 = 40$

$\sigma = \sqrt{\frac{\Sigma \delta^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2.828427$.

18. (C) $M = \frac{6 + 8 + 10 + 12 + 14}{5} = \frac{50}{5} = 10$.

$\Sigma \delta^2 = |6 - 10|^2 + |8 - 10|^2 + |10 - 10|^2 + |12 - 10|^2 + |14 - 10|^2 = 40$

$6 = \frac{\Sigma \delta^2}{n} = \frac{40}{5} = 8$

$\sigma = \sqrt{\frac{\Sigma \delta^2}{n}} = \sqrt{\frac{40}{5}} = 2.828427$ (approx).

19. (B) Here $p = 0.4$, $q = 0.6$ and $n = 3$.

Required probability $= P(A$ occurring at least once $)$

$= ^3C_1 \cdot (0.4)^1 \cdot (0.6)^2 + ^3C_2 \cdot (0.4)^2 \cdot (0.6) + ^3C_3 \cdot (0.4)^3$

$= \left( 3 \times \frac{4}{10} \times \frac{36}{100} + 3 \times \frac{16}{100} \times \frac{6}{10} + \frac{64}{1000} \right) = \frac{784}{1000} = 0.784$.

20. (B) $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 8$. Required probability $= P(6$ heads or $7$ heads or $8$ heads)$

$= ^8C_6 \cdot \left( \frac{1}{2} \right)^6 \cdot \left( \frac{1}{2} \right)^2 + ^8C_7 \cdot \left( \frac{1}{2} \right)^7 \cdot \frac{1}{2} + ^8C_8 \cdot \left( \frac{1}{2} \right)^8$

$= \frac{28}{256} + \frac{8}{256} + \frac{1}{256} = \frac{37}{256}$

21. (C) Let $E$ = the event that $A$ solves the problem. and $F$ = the event that $B$ solves the problem.

Clearly $E$ and $F$ are independent events.

$P(E) = \frac{90}{100} = 0.9$,  $P(F) = \frac{70}{100} = 0.7$.

$P(E \cup F) = P(E) \cdot P(F) = 0.9 \times 0.7 = 0.63$

$P(E = F) = P(E) \cdot P(F) = 0.9 \times 0.7 - 0.63 = 0.97$. Required probability $= P(E \cup F)$

22. (C) Let $E$ = event that $A$ speaks the truth. $F$ = event that $B$ speaks the truth.

Then,  $P(E) = \frac{75}{100} = 0.75$,  $P(F) = \frac{80}{100} = 0.8$

$P(\overline{E}) = \left( 1 - \frac{3}{4} \right) = \frac{1}{4}$,  $P(\overline{F}) = \left( 1 - \frac{4}{5} \right) = \frac{1}{5}$

$P$ (A and B contradict each other).

$= P[(A$ speaks truth and $B$ tells a lie) or ($A$ tells a lie and $B$ speaks the truth)]

$= P(E)$ and $\overline{F} + P(\overline{E})$ and $F$

$= P(E) \cdot P(\overline{F}) + P(\overline{E}) \cdot P(F)$

$= \frac{3}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} = \frac{3}{20} + \frac{1}{5} \times \frac{7}{20} = \left( \frac{7}{20} \times 100 \right) \% = 35\%$.

23. (A) Let $E$ = event that the husband will be alive 25 years hence and $F$ = event that the wife will be alive 25 years hence.

Then,  $P(E) = \frac{5}{12}$ and $P(F) = \frac{3}{8}$

Thus $P(\overline{E}) = \left( 1 - \frac{5}{12} \right) = \frac{7}{12}$ and $P(\overline{F}) = \left( 1 - \frac{3}{8} \right) = \frac{5}{8}$.

Clearly, $E$ and $F$ are independent events.

So, $\overline{E}$ and $\overline{F}$ are independent events.

$P(\text{at least one of them will be alive 25 years hence})$

$= 1 - P(\text{none will be alive 24 years hence})$

$= 1 - P(\overline{E} = \overline{F}) = 1 - P(\overline{E}) \cdot P(\overline{F}) = \left( 1 - \frac{7}{12} \times \frac{5}{8} \right) = \frac{61}{96}$

24. (D) $P(\text{none dies})$

$= (1 - p) (1 - p) \ldots n \text{ times} = (1 - p)^n$

$P(\text{at least one dies}) = 1 - (1 - p)^n$

$P(A_i \text{ dies}) = \frac{1}{n} [1 - (1 - p)^n]$.  

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39. (C) \( \bar{x} = \frac{\Sigma x_i}{n} = \frac{15}{5} = 3 \), \( \bar{y} = \frac{\Sigma y_i}{n} = \frac{36}{5} = 7.2 \)
\( \text{cov}(x, y) = \left( \frac{\Sigma x_i y_i}{n} - \bar{x} \bar{y} \right) = \left( \frac{110}{5} - 3 \times 7.2 \right) = 0.4 \)

40. (D) \( r = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \cdot \text{var}(y)}} = \frac{-165}{\sqrt{2.89 \times 100}} = -0.97. \)

41. (B) \( D_1 = -2, -8, -2, 3, -3, 3, 0, 2, 4. \)
\( \Sigma D_1^2 = (4 + 64 + 4 + 9 + 9 + 9 + 0 + 4 + 16) = 128. \)
\( R = 1 - \frac{6(\Sigma D_1^2)}{n(n^2 - 1)} = 1 - \frac{6 \times 128}{10 \times 99} = \frac{37}{165} = 0.224. \)

42. (A) \( b_{xy} = \frac{\Sigma x_i y_i - \left( \frac{\Sigma x_i}{n} \right) \left( \frac{\Sigma y_i}{n} \right)}{\Sigma x_i^2 - \left( \frac{\Sigma x_i}{n} \right)^2} \)
\( = \frac{\left( 306 - 24 \times 44 \right)}{4} = \frac{\left( 306 - 264 \right)}{4} = \frac{20}{4} = 5 \)

43. (B) \( b_{xy} = \frac{\Sigma x_i y_i - \left( \frac{\Sigma x_i}{n} \right) \left( \frac{\Sigma y_i}{n} \right)}{\Sigma y_i^2 - \left( \frac{\Sigma y_i}{n} \right)^2} = \frac{\left( 199 - 30 \times 42 \right)}{6} = \frac{318 - 42 \times 42}{6} \)
\( = \frac{318 - 294}{6} = -0.46. \)

44. (C) \( b_{xy} = \frac{\text{cov}(x, y)}{\text{var}(x)} \) and \( b_{xy} = \frac{\text{cov}(x, y)}{\text{var}(y)} \)
\( r^2 = b_{xy} \times b_{xy} \quad \Rightarrow \quad r = \sqrt{b_{xy} \times b_{xy}}. \)

45. (C) \( \frac{1}{2}(b_{xy} + b_{yx}) > r \) is true if \( \frac{1}{2} \left[ r \cdot \frac{\text{cov}(x, y)}{\text{var}(x)} + r \cdot \frac{\text{cov}(x, y)}{\text{var}(y)} \right] > r \)
i.e. if \( \sigma_x^2 + \sigma_y^2 > 2 \sigma_x \sigma_y \)
i.e. if \( (\sigma_x - \sigma_y)^2 > 0 \), which is true.

46. (A) \( r = \sqrt{1.6 \times 0.4} = \sqrt{0.64} = 0.8 \)
\( b_{xy} = \frac{\text{cov}(x, y)}{\text{var}(x)} = \frac{\sigma_y}{\sigma_x} = \frac{0.8}{2} = 0.4 \)
\( m_1 = \frac{1}{r} \cdot \frac{\sigma_y}{\sigma_x} = \frac{1}{0.8} \times 2 = \frac{5}{2}, \quad m_2 = r \cdot \frac{\sigma_y}{\sigma_x} = 0.8 \times 2 = 1.6. \)
\( \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{2.5 - 1.6}{1 + 2.5 \times 1.6} = \frac{0.9}{5} = 0.18. \)
12. For \( \frac{dy}{dx} = xy \) given that \( y = 1 \) at \( x = 0 \). Using Euler method taking the step size 0.1, the \( y \) at \( x = 0.4 \) is
(A) 1.0611  
(B) 2.6480  
(C) 1.6321  
(D) 2.4189


For \( \frac{dy}{dx} = x^2 + y^2 \) given that \( y = 1 \) at \( x = 0 \). Determine the value of \( y \) at given \( x \) in question using modified method of Euler. Take the step size 0.02.

13. \( y \) at \( x = 0.02 \) is
(A) 1.0468  
(B) 1.0204  
(C) 1.0346  
(D) 1.0348

14. \( y \) at \( x = 0.04 \) is
(A) 1.0316  
(B) 1.0301  
(C) 1.403  
(D) 1.0416

15. \( y \) at \( x = 0.06 \) is
(A) 1.0348  
(B) 1.0539  
(C) 1.0638  
(D) 1.0796

16. For \( \frac{dy}{dx} = x + y \) given that \( y = 1 \) at \( x = 0 \). Using modified Euler's method taking step size 0.2, the value of \( y \) at \( x = 1 \) is
(A) 3.401638  
(B) 3.405417  
(C) 9.164396  
(D) 9.168238

17. For the differential equation \( \frac{dy}{dx} = x - y^2 \) given that

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>0.02</td>
<td>0.0795</td>
</tr>
</tbody>
</table>

Using Milne predictor–correction method, the \( y \) at next value of \( x \) is
(A) 0.2498  
(B) 0.3046  
(C) 0.4648  
(D) 0.5114

Statement for Q. 18–19:

For \( \frac{dy}{dx} = 1 + y^2 \) given that

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.2027</td>
<td>0.4228</td>
<td>0.6841</td>
</tr>
</tbody>
</table>

Using Milne's method determine the value of \( y \) for \( x \) given in question.

18. \( y(0.8) = ? \)
(A) 1.0293  
(B) 0.4228  
(C) 0.6065  
(D) 1.4396

19. \( y(1.0) = ? \)
(A) 1.9428  
(B) 1.3428  
(C) 1.5555  
(D) 2.168

Statement for Q. 20–22:

Apply Runge Kutta fourth order method to obtain \( y(0.2) \), \( y(0.4) \) and \( y(0.6) \) from \( \frac{dy}{dx} = 1 + y^2 \), with \( y = 0 \) at \( x = 0 \). Take step size \( h = 0.2 \).

20. \( y(0.2) = ? \)
(A) 0.2027  
(B) 0.4396  
(C) 0.3846  
(D) 0.9341

21. \( y(0.4) = ? \)
(A) 0.1649  
(B) 0.8397  
(C) 0.4227  
(D) 0.1934

22. \( y(0.6) = ? \)
(A) 0.9348  
(B) 0.2935  
(C) 0.6841  
(D) 0.563

23. For \( \frac{dy}{dx} = x + y^2 \), given that \( y = 1 \) at \( x = 0 \). Using Runge Kutta fourth order method the value of \( y \) at \( x = 0.2 \) is \( (h = 0.2) \)
(A) 1.2735  
(B) 2.1635  
(C) 1.9356  
(D) 2.9468

24. For \( \frac{dy}{dx} = x + y \) given that \( y = 1 \) at \( x = 0 \). Using Runge Kutta fourth order method the value of \( y \) at \( x = 0.2 \) is \( (h = 0.2) \)
(A) 1.1384  
(B) 1.9438  
(C) 1.2428  
(D) 1.6389

*********
SOLUTIONS

1. (B) Let \( f(x) = x^3 - 4x - 9 \)
Since \( f(2) \) is negative and \( f(3) \) is positive, a root lies between 2 and 3.
First approximation to the root is 
\[ x_1 = \frac{1}{2} (2 + 3) = 2.5. \]
Then \( f(x_1) = 2.5^3 - 4(2.5) - 9 = -3.375 \)
i.e. negative: The root lies between \( x_1 \) and 3. Thus the second approximation to the root is 
\[ x_2 = \frac{1}{2} (x_1 + 3) = 2.75. \]
Then \( f(x_2) = (2.75)^3 - 4(2.75) - 9 = 0.7969 \) i.e. positive.
The root lies between \( x_1 \) and \( x_2 \). Thus the third approximation to the root is 
\[ x_3 = \frac{1}{2} (x_1 + x_2) = 2.625. \]
Then \( f(x_3) = (2.625)^3 - 4(2.625) - 9 = -1.4121 \) i.e. negative.
The root lies between \( x_2 \) and \( x_3 \). Thus the fourth approximation to the root is 
\[ x_4 = \frac{1}{2} (x_2 + x_3) = 2.6875. \]
Hence the root is 2.6875 approximately.

2. (B) Let \( f(x) = x^3 - 2x - 5 \)
So that \( f(2) = -1 \) and \( f(3) = 16 \)
i.e. a root lies between 2 and 3.
Taking \( x_0 = 2, x_1 = 3, f(x_0) = -1, f(x_1) = 16 \), in the method of false position, we get
\[ x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = 2 + \frac{1}{17} = 2.0588 \]
Now, \( f(x_2) = (2.0588)^3 = -0.3908 \) i.e., that root lies between 2.0588 and 3.
Taking \( x_0 = 2.0588, x_1 = 3, f(x_0) = -0.3908 \), \( f(x_1) = 16 \) in (i), we get
\[ x_3 = 2.0588 - \frac{0.9412}{16.3908} (-0.3908) = 2.0813 \]
Repeating this process, the successive approximations are
\[ x_4 = 2.0862, x_5 = 2.0915, x_6 = 2.0934, x_7 = 2.0941, \]
\[ x_8 = 2.0943 \text{ etc.} \]
Hence the root is 2.094 correct to 3 decimal places.

3. (C) Let \( f(x) = 2x - \log_{10} x - 7 \)
Taking \( x_0 = 35, x_1 = 4 \), in the method of false position, we get
\[ x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) \]
\[ = 35 - \frac{0.5}{0.3979 + 0.5441} (-0.5441) = 37.888 \]
Since \( f(3.7888) = -0.0009 \) and \( f(4) = 0.3979 \), therefore the root lies between 3.7888 and 4.
Taking \( x_0 = 3.7888, x_1 = 4 \), we obtain
\[ x_2 = 3.7888 - \frac{0.2112}{0.3988} (-0.009) = 3.7893 \]
Hence the required root correct to three places of decimal is 3.789.

4. (D) Let \( f(x) = x^e - 2 \), Then \( f(0) = -2 \), and \( f(1) = e - 2 = 0.7183 \)
So a root of \( (i) \) lies between 0 and 1. It is nearer to 1. Let us take \( x_0 = 1 \).
Also \( f(x) = x^e + e^x \) and \( f(1) = e + e = 5.4366 \)
By Newton's rule, the first approximation \( x_1 \) is
\[ x_1 = x_0 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{0.7183}{5.4366} = 0.8679 \]
\[ f(x_1) = 0.0672, f'(x_1) = 4.4491. \]
Thus the second approximation \( x_2 \) is
\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8679 - \frac{0.0672}{4.4491} = 0.8528 \]
Hence the required root is 0.853 correct to 3 decimal places.

5. (B) Let \( y = x + \log_{10} x - 3.375 \)
To obtain a rough estimate of its root, we draw the graph of \( (i) \) with the help of the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y  )</td>
<td>-2.375</td>
<td>-1.074</td>
<td>0.102</td>
<td>1.227</td>
</tr>
</tbody>
</table>

Taking 1 unit along either axis = 0.1, The curve crosses the x-axis at \( x_0 = 2.9 \), which we take as the initial approximation to the root.
Now let us apply Newton–Raphson method to
\[ f(x) = x + \log_{10} x - 3.375 \]
\[ f'(x) = 1 + \frac{1}{x} \log_{10} e \]
\[ f(2.9) = 2.9 + \log_{10} 2.9 - 3.375 = -0.0126 \]
\[ f'(2.9) = 1 + \frac{1}{2.9} \log_{10} e = 1.1497 \]
The first approximation \( x \) to the root is given by
(B) Let \( f(x) = x^2 - 28 \), hence the desired root, correct to four significant figures, is 2.911

6. Let \( x = \sqrt{28} \), so that \( x^2 - 28 = 0 \).

Taking \( f(x) = x^2 - 28 \), Newton’s iterative method gives

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{2x_n} = \frac{1}{2} x_n + \frac{28}{x_n}
\]

Now since \( f(5) = -3, f(6) = 8 \), a root lies between 5 and 6.

Taking \( x_0 = 5.5 \),

\[
x_1 = \frac{1}{2} (x_0 + \frac{28}{x_0}) = \frac{1}{2} \left( 5.5 + \frac{28}{5.5} \right) = 5.29545
\]

\[
x_2 = \frac{1}{2} \left( x_1 + \frac{28}{x_1} \right) = \frac{1}{2} \left( 5.29545 + \frac{28}{5.29545} \right) = 5.2915
\]

\[
x_3 = \frac{1}{2} \left( x_2 + \frac{28}{x_2} \right) = \frac{1}{2} \left( 5.2915 + \frac{28}{5.2915} \right) = 5.2915
\]

Since \( x_3 = x_4 \), up to 4 decimal places, we take \( \sqrt{28} = 5.2915 \).

7. Let \( h = 0.1 \), given \( x_0 = 0 \), \( x_1 = x_0 + h = 0.1 \)

\[
dy = x \frac{dy}{dx} + y
\]

\[
d^2y = x \frac{d^2y}{dx^2} + \frac{dy}{dx} \frac{d^2y}{dx^2} + \frac{d^3y}{dx^3}
\]

given that \( x = 0 \), \( y = 1 \)

\[
\frac{dy}{dx} = 1, \quad \frac{d^2y}{dx^2} = 2, \quad \frac{d^3y}{dx^3} = 3 \quad \text{and so on.}
\]

The Taylor series expression gives:

\[
y(x + h) = y(x) + h \frac{dy}{dx} + \frac{h^2}{2!} \frac{d^2y}{dx^2} + \frac{h^3}{3!} \frac{d^3y}{dx^3} + \ldots
\]

\[
y(0.1) = 1 + 0.1 \times 1 + \frac{(0.1)^2}{2!} \times 1 + \frac{(0.1)^3}{3!} 2 + \ldots
\]

\[
y(0.1) = 1 + 0.1 + \frac{0.01}{2} + \frac{0.001}{3} + \ldots
\]

\[
= 1 + 0.1 + 0.005 + 0.000033 \ldots \ldots = 1.1053
\]

8. Let \( h = 0.1 \), given \( x_0 = 0 \), \( y_0 = 1 \)

\[
x_1 = x_0 + h = 0.1, \quad \frac{dy}{dx} = x - y^2
\]

at \( x = 0 \), \( y = 1 \), \( \frac{dy}{dx} = -1 \)

\[
\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}
\]

at \( x = 0 \), \( y = 1 \), \( \frac{d^2y}{dx^2} = 1 + 2 = 3 \)

The Taylor series expression gives

\[
y(x + h) = y(x) + h \frac{dy}{dx} + \frac{h^2}{2!} \frac{d^2y}{dx^2} + \frac{h^3}{3!} \frac{d^3y}{dx^3} + \frac{h^4}{4!} \frac{d^4y}{dx^4} + \ldots
\]

\[
y(0.1) = 1 + 0.1 -1 + \frac{(0.1)^2}{2!} 3 + \frac{(0.1)^3}{3!} (-8) + \frac{(0.1)^4}{4!} 34 + \ldots
\]

\[
= 1 - 0.1 + 0.015 - 0.00133 + 0.0001417 = 0.9138
\]

9. Here \( f(x, y) = x^2 + y^2 \), \( x_0 = 0 \), \( y_0 = 0 \)

We have, by Picard’s method

\[
y = y_0 + \int_{x_0}^{x} f(x, y_0) dx \quad \ldots (1)
\]

The first approximation to \( y \) is given by

\[
y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0) dx
\]

Where \( y_0 = 0 + \int_{0}^{x} f(x, 0) dx = \int_{0}^{x} x^2 dx \) \ldots (2)

The second approximation to \( y \) is given by

\[
y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)}) dx = 0 + \int_{0}^{x} f \left( x, \frac{x^3}{3} \right) dx
\]

\[
= 0 + \int_{0}^{x} x^2 + \frac{x^6}{9} dx = \frac{x^3}{3} + \frac{x^7}{63}
\]

Now, \( y(0.4) = \frac{(0.4)^3}{3} + \frac{(0.4)^7}{63} = 0.02135 \)

10. Here \( f(x, y) = y - x \); \( x_0 = 0 \), \( y_0 = 2 \)

We have by Picard’s method

\[
y = y_0 + \int_{x_0}^{x} f(x, y) dx
\]

The first approximation to \( y \) is given by

\[
y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0) dx = 2 + \int_{0}^{x} f(x, 2) dx
\]
The third approximation is given by
\[ y(3) = y_0 + \int_0^x f(x, y(x)) \, dx \]
\[ = 2 + \int_0^x \left( 2 + 2x + \frac{x^2}{2} - \frac{x^3}{2} \right) \, dx \]
\[ = 2 + 2x + \frac{x^2}{2} + \frac{x^3}{6} \quad \text{...(2)} \]

The second approximation to \( y \) is given by
\[ y(2) = y_0 + \int_0^x f(x, y(x)) \, dx \]
\[ = 2 + \int_0^x \left( 2 + 2x + \frac{x^2}{2} - \frac{x^3}{2} - x \right) \, dx \]
\[ = 2 + 2x + \frac{x^2}{2} - \frac{x^3}{6} \]

The third approximation to \( y \) is given by
\[ y(3) = y_0 + \int_0^x f(x, y(x)) \, dx \]
\[ = 2 + \int_0^x \left( 2 + 2x + \frac{x^2}{2} - \frac{x^3}{2} - x \right) \, dx \]
\[ = 2 + 2x + \frac{x^2}{2} - \frac{x^3}{6} \]

11. (B) Here \( f(x, y) = x + y^2 \), \( x_0 = 0 \), \( y_0 = 0 \)

We have, by Picard’s method
\[ y = y_0 + \int_0^x f(x, y(x)) \, dx \]

The first approximation to \( y \) is given by
\[ y(1) = y_0 + \int_0^x f(x, y(x)) \, dx = 0 + \int_0^x f(x, 0) \, dx \]
\[ = 0 + \int_0^x x \, dx = \frac{x^2}{2} \]

The second approximation to \( y \) is given by
\[ y(2) = y_0 + \int_0^x f(x, y(x)) \, dx = 0 + \int_0^x f(x, x^2) \, dx \]
\[ = \frac{x^3}{4} + \frac{x^5}{50} \]

The third approximation is given by
\[ y(3) = y_0 + \int_0^x f(x, y(x)) \, dx \]
\[ = 0 + \int_0^x f(x, x^2) \, dx \]
\[ = \frac{x^3}{4} + \frac{x^5}{20} + \frac{x^7}{400} \]

12. (A) \( x \): 0.1 0.2 0.3 0.4

Euler’s method gives
\[ y_{n+1} = y_n + h(f(x_n, y_n)) \]
\[ n = 0 \text{ in (1) gives} \]
\[ y_1 = y_0 + hf(x_0, y_0) \]

Here \( x_0 = 0, \ y_0 = 1, \ h = 0.1 \)
\[ y_1 = 1 + 0.1 f(0, 1) = 1 + 0 = 1 \]

Thus \( y_1 = y_{(0.1)} = 101 \)

13. (B) The Euler’s modified method gives
\[ y_1^* = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^*)] \]

Now, here \( h = 0.02, \ y_0 = 1, \ x_0 = 0 \)
\[ y_1^* = 1 + 0.02f(0, 1), \ y_1^* = 1 + 0.02 = 1.02 \]

Next \( y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x, y_1^*)] \)
\[ = 1 + \frac{0.02}{2} [f(0, 1) + f(0.02, 1.02)] \]
\[ = 1 + 0.01 [1 + 10204] = 10202 \]

So, \( y_1 = y(0.02) = 10202 \)

14. (D) \( y_2^* = y_1 + h f(x_1, y_1) \)
\[ = 10202 + 0.02f(0.02, 10202)] \]
\[ = 10202 + 0.02 \quad = 10406 \]

Next \( y_2 = y_1 + \frac{h}{2} [f(x, y) + f(x_2, y_2^*)] \)
\[ = 10202 + \frac{0.02}{2} [f(0.02, 10202) + f(0.04, 10406)] \]
\[ = 10202 + 0.01 [10206 + 10422] = 10408 \]

15. (C) \( y_3^* = y_2 + hf(x, y_2) \)
\[ = 10416 + 0.02f(0.04, 10416)] \]
\[ = 10416 + 0.0217 = 10633 \]

Next \( y_3 = y_2 + \frac{h}{2} [f(x_2, y_2) + f(x_1, y_3^*)] \)
\[ = \]
\[ k_2 = hf\left( x_0 + \frac{1}{2} h, y_0 + \frac{1}{2} k_1 \right) = (0.2)f(0.1, 0.1) = 0.202 \]
\[ k_3 = hf\left( x_0 + \frac{1}{2} h, y_0 + \frac{1}{2} k_2 \right) = (0.2)f(0.1, 0.101) = 0.2020 \]
\[ k_4 = hf(x_0 + h, y_0 + k_3) = 0.2f(0.2, 0.2020) = 0.20816 \]
\[ k = \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \]
\[ = \frac{1}{6} [0.2 + (2.202) + 2(2.20204) + 0.20816], \]
\[ k = 0.2027 \]
\[ \text{such that} \quad y_1 = y(0.2) = y_0 + k = 0 + 0.2027 = 0.2027 \]

21. (C) We now to find \( y_2 = y(0.4) \), \( k_1 = hf(x_1, y_1) = (0.2)f(0.2, 0.2027) = 0.2 (10410) = 2082 \)
\[ k_2 = hf\left( x_1 + \frac{1}{2} h, y_1 + \frac{1}{2} k_1 \right) = (0.2)f(0.3, 0.3068) = 0.2188 \]
\[ k_3 = hf\left( x_1 + \frac{1}{2} h, y_1 + \frac{1}{2} k_2 \right) = (0.2)f(0.4, 0.4221) = 0.2356 \]
\[ k = \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \]
\[ = \frac{1}{6} [0.2082 + 2(0.2188) + 2(0.2194) + 0.356] = 0.2200 \]
\[ y_2 = y(0.4) = y_1 + k = 0.2200 + 0.2027 = 0.4227 \]

22. (C) We now to find \( y_3 = y(0.6) \), \( k_1 = hf(x_2, y_2) = (0.2)f(0.4, 0.4228) = 0.2357 \)
\[ k_2 = hf\left( x_2 + \frac{1}{2} h, y_2 + \frac{1}{2} k_1 \right) = (0.2)f(0.5, 0.5406) = 0.2584 \]
\[ k_3 = hf\left( x_2 + \frac{1}{2} h, y_2 + \frac{1}{2} k_2 \right) = (0.2)f(0.5, 0.5520) = 0.2609 \]
\[ k = \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \]
\[ = \frac{1}{6} [0.2357 + 2(0.2584) + 2(0.2609) + 0.2935] \]
\[ = \frac{1}{6} [0.2357 + 0.5168 + 0.5218 + 0.2935] = 0.2613 \]
\[ y_3 = y(0.6) = y_2 + k = 0.4228 + 0.2613 = 0.6841 \]

23. (A) Here given \( x_0 = 0 \), \( y_0 = 1 \), \( h = 0.2 \)
\( f(x, y) = x + y^2 \)
To find \( y_1 = y(0.2) \)
\[ k_1 = hf(x_0, y_0) = (0.2)f(0, 1) = (0.2) \times 1 = 0.2 \]
1. If \( E \) Denotes expectation, the variance of a random variable \( X \) is given by
   (A) \( E[X^2] - E^2[X] \)  
   (B) \( E[X^2] + E^2[X] \)  
   (C) \( E[X^2] \)  
   (D) \( E^2[X] \)

2. The following plot shows a function \( y \) which varies linearly with \( X \). The value of the integral \( I = \int_{1}^{2} y \, dx \) is
   (A) 1.0  
   (B) 2.5  
   (C) 4.0  
   (D) 5.0

3. For \(|x| < 1\), \( \coth(x) \) can be approximated as
   (A) \( x \)  
   (B) \( x^2 \)  
   (C) \( \frac{1}{x} \)  
   (D) \( \frac{1}{x^2} \)

4. \( \lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta} \) is
   (A) 0.5  
   (B) 1  
   (C) 2  
   (D) not defined

5. Which of the following functions is strictly bounded ?
   (A) \( \frac{1}{x^2} \)  
   (B) \( e^x \)  
   (C) \( x^2 \)  
   (D) \( e^{-x^2} \)

6. For the function \( e^{-x} \), the linear approximation around \( x = 2 \) is
   (A) \( (3-x)e^{-2} \)  
   (B) \( 1-x \)  
   (C) \( [3 + 2\sqrt{2} - 1(1 + \sqrt{2})]e^{-2} \)  
   (D) \( e^{-2} \)

7. An independent voltage source in series with an impedance \( Z_s = R_s + jX_s \) delivers a maximum average power to a load impedance \( Z_L \) when
   (A) \( Z_L = R_s + jX_s \)  
   (B) \( Z_L = R_s \)  
   (C) \( Z_L = jX_s \)  
   (D) \( Z_L = R_s - jX_s \)

8. The RC circuit shown in the figure is
   (A) a low-pass filter  
   (B) a high-pass filter  
   (C) a band-pass filter  
   (D) a band-reject filter

9. The electron and hole concentrations in an intrinsic semiconductor are \( n_i \) per cm\(^3\) at 300 K. Now, if acceptor impurities are introduced with a concentration of \( N_A \) per cm\(^3\) (where \( N_A \gg n_i \)) the electron concentration per cm\(^3\) at 300 K will be
   (A) \( n_i \)  
   (B) \( n_i + N_A \)  
   (C) \( N_A - n_i \)  
   (D) \( \frac{n_i^2}{N_A} \)
10. In a \( p' n \) junction diode under reverse biased the magnitude of electric field is maximum at
(A) the edge of the depletion region on the \( p \) -side
(B) the edge of the depletion region on the \( n \) -side
(C) the \( p' n \) junction
(D) the center of the depletion region on the \( n \)-side

11. The correct full wave rectifier circuit is

![Rectifier Circuit Options]

(A) (B) (C) (D)

12. In a trans-conductance amplifier, it is desirable to have
(A) a large input resistance and a large output resistance
(B) a large input resistance and a small output resistance
(C) a small input resistance and a large output resistance
(D) a small input resistance and a small output resistance

13. \( X = 01110 \) and \( Y = 11001 \) are two 5-bit binary numbers represented in two’s complement format. The sum of \( X \) and \( Y \) represented in two’s complement format using 6 bits is
(A) 100111  (B) 0010000
(C) 000111  (D) 101001

14. The Boolean function \( Y = AB + CD \) is to be realized using only 2-input NAND gates. The minimum number of gates required is
(A) 2  (B) 3  (C) 4  (D) 5

15. If closed-loop transfer function of a control system is given as \( T(s) = \frac{s^3}{(s+2)(s+3)} \), then It is
(A) an unstable system
(B) an uncontrollable system
(C) a minimum phase system
(D) a non-minimum phase system

16. If the Laplace transform of a signal \( \gamma(t) \) is \( Y(s) = \frac{1}{s^2 - 1} \), then its final value is
(A) -1  (B) 0  (C) 1  (D) unbounded

17. If \( R(\tau) \) is the auto correlation function of a real, wide-sense stationary random process, then which of the following is NOT true
(A) \( R(\tau) = R(-\tau) \)
(B) \( |R(\tau)| \leq R(0) \)
(C) \( R(\tau) = -R(-\tau) \)
(D) The mean square value of the process is \( R(0) \)

18. If \( S(f) \) is the power spectral density of a real, wide-sense stationary random process, then which of the following is ALWAYS true?
(A) \( S(0) \leq S(f) \)
(B) \( S(f) \geq 0 \)
(C) \( S(-f) = -S(f) \)
(D) \( \int S(f) df = 0 \)

19. A plane wave of wavelength \( \lambda \) is traveling in a direction making an angle 30° with positive \( x \) -axis. The \( \vec{E} \) field of the plane wave can be represented as \( (E_0 \) is constant)
(A) \( \vec{E} = yE_0 e^{\left( \frac{-\pi y}{\lambda} \right)} \)
(B) \( \vec{E} = yE_0 e^{\left( \frac{-\pi x}{\lambda} \right)} \)
(C) \( \vec{E} = yE_0 e^{\left( \frac{-\pi x}{\lambda} \right)} \)
(D) \( \vec{E} = yE_0 e^{\left( \frac{-\pi y}{\lambda} \right)} \)

20. If \( C \) is close curve enclosing a surface \( S \), then the magnetic field intensity \( \vec{H} \), the current density \( \vec{j} \) and the electric flux density \( \vec{D} \) are related by
(A) \( \int \oint \vec{H}.d\vec{s} = \oint \left[ j + \frac{\partial \vec{D}}{\partial t} \right].d\vec{l} \)
(B) \( \int \oint \vec{H}.d\vec{l} = \oint \left[ j + \frac{\partial \vec{D}}{\partial t} \right].d\vec{s} \)
29. For the circuit shown in the figure, the Thevenin voltage and resistance looking into X - Y are

(A) \( \frac{4}{3} \, V, 2\Omega \)  
(B) \( 4 \, V, \frac{2}{3} \Omega \)  
(C) \( \frac{4}{3} \, V, \frac{2}{3} \Omega \)  
(D) \( 4 \, V, 2\Omega \)

30. In the circuit shown, \( V_c \) is 0 volts at \( t=0 \) sec. for \( t>0 \), the capacitor current \( i_c(t) \), where \( t \) is in seconds, is given by

(A) \( 0.50 \exp(-25t) \) mA  
(B) \( 0.25 \exp(-25t) \) mA  
(C) \( 0.50 \exp(-25t) \) mA  
(D) \( 0.25 \exp(-25t) \) mA

31. In the AC network shown in the figure, the phasor voltage \( V_{ab} \) (in volts) is

(A) 0  
(B) 5\( \angle 30^\circ \)  
(C) 125\( \angle 30^\circ \)  
(D) 17\( \angle 30^\circ \)

32. A p' n junction has a built-in potential of 0.8 V. The depletion layer width at reverse bias of 1.2V is 2 \( \mu m \). For a reverse bias of 7.2 V, the depletion layer width will be

(A) 4 \( \mu m \)  
(B) 4.9 \( \mu m \)  
(C) 8 \( \mu m \)  
(D) 12 \( \mu m \)

33. Group I lists four types of \( p - n \) junction diodes. Match each device in Group I with one of the option in Group II to indicate the bias condition of the device in its normal mode of operation.

Group-I  
(P) Zener Diode  
(Q) Solar cell  
(R) LASER diode  
(S) Avalanche Photodiode

Group-II  
(1) Forward bias  
(2) Reverse bias

(A) P - 1 Q - 2 R - 1 S - 2  
(B) P - 2 Q - 1 R - 1 S - 2  
(C) P - 2 Q - 2 R - 1 S - 2  
(D) P - 2 Q - 1 R - 2 S - 2

34. The DC current gain (\( \beta \)) of a BJT is 50. Assuming that the emitter injection efficiency is 0.995, the base transport factor is

(A) 0.980  
(B) 0.985  
(C) 0.990  
(D) 0.995

35. Group I lists four different semiconductor devices. Match each device in Group I with its characteristic property in Group II.

Group-I  
(P) BJT  
(Q) MOS capacitor  
(R) LASER diode  
(S) JFET

Group-II  
(1) Population inversion  
(2) Pinch-off voltage  
(3) Early effect  
(4) Fat-band voltage

(A) P - 3 Q - 1 R - 4 S - 2  
(B) P - 1 Q - 4 R - 3 S - 2  
(C) P - 3 Q - 4 R - 1 S - 2  
(D) P - 3 Q - 2 R - 1 S - 4

36. For the Op-Amp circuit shown in the figure, \( V_o \) is

(A) -2 V  
(B) -1 V  
(C) -0.5 V  
(D) 0.5 V

37. For the BJT circuit shown, assume that the \( \beta \) of the transistor is very large and \( V_{be} = 0.7 \) V. The mode of operation of the BJT is
38. In the Op-Amp circuit shown, assume that the diode current follows the equation \( I = I_r \exp(V/V_T) \). For \( V_i = 2V, V_o = V_{o1} \), and for \( V_i = 4V, V_o = V_{o2} \). The relationship between \( V_{o1} \) and \( V_{o2} \) is

(A) \( V_{o2} = \sqrt{2} V_{o1} \)  
(B) \( V_{o2} = e^2 V_{o1} \)  
(C) \( V_{o2} = V_{o1} \ln 2 \)  
(D) \( V_{o1} - V_{o2} = V_i \ln 2 \)

39. In the CMOS inverter circuit shown, if the transconductance parameters of the NMOS and PMOS transistors are \( k_n = k_p = \mu_n C_{ox} \frac{W}{L_n} = \mu_p C_{ox} \frac{W}{L_p} = 40 \mu A/V^2 \) and their threshold voltages are \( V_{THn} = V_{THp} = 1V \), the current \( I \) is

(A) 0 A  
(B) 25 \mu A  
(C) 45 \mu A  
(D) 90 \mu A

40. For the Zener diode shown in the figure, the Zener voltage at knee is 7V, the knee current is negligible and the Zener dynamic resistance is 10\( \Omega \). If the input voltage \( (V_i) \) range is from 10 to 16V, the output voltage \( (V_o) \) ranges from

(A) 7.00 to 7.29 V  
(B) 7.14 to 7.29 V  
(C) 7.14 to 7.43 V  
(D) 7.29 to 7.43 V

41. The Boolean expression \( Y = \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} B \overline{C} \overline{D} + A B \overline{C} \overline{D} + A B \overline{C} \overline{D} \) can be minimized to

(A) \( Y = \overline{A} \overline{B} \overline{C} + \overline{A} B \overline{C} + A \overline{B} \overline{C} \overline{D} \)  
(B) \( Y = \overline{A} B \overline{C} D + B \overline{C} D + A \overline{B} \overline{C} \overline{D} \)  
(C) \( Y = A B C D + B \overline{C} D + A \overline{B} \overline{C} D \)  
(D) \( Y = \overline{A} B \overline{C} D + B \overline{C} D + A B \overline{C} \overline{D} \)

42. The circuit diagram of a standard TTL NOT gate is shown in the figure. \( V_i = 2.5V \), the modes of operation of the transistors will be

(A) \( Q_1 \): reverse active; \( Q_2 \): normal active; \( Q_4 \): saturation; \( Q_3 \): cut-off  
(B) \( Q_1 \): reverse active; \( Q_2 \): saturation; \( Q_3 \): saturation; \( Q_4 \): cut-off  
(C) \( Q_1 \): normal active; \( Q_2 \): cut-off; \( Q_3 \): cut-off; \( Q_4 \): saturation  
(D) \( Q_1 \): saturation; \( Q_2 \): saturation; \( Q_3 \): saturation; \( Q_4 \): normal active

43. In the following circuit, \( X \) is given by

(A) \( X = A \overline{B} \overline{C} + A B \overline{C} + \overline{A} B C + A B C \)  
(B) \( X = \overline{ABC} + A \overline{B} C + A \overline{B} \overline{C} + \overline{A} B \overline{C} \)
(C) $X = AB + BC + AC$

(D) $X = \overline{A}B + \overline{B}C + \overline{AC}$

44. The following binary values were applied to the X and Y inputs of NAND latch shown in the figure in the sequence indicated below:

$X = 0, Y = 1; X = 0, Y = 0; X = 1, Y = 1.$

![NAND Latch Diagram]

The corresponding stable $P, Q$ outputs will be

(A) $P = 1, Q = 0; P = 1, Q = 0; P = 1, Q = 0$ or $P = 0, Q = 1$

(B) $P = 1, Q = 0; P = 0, Q = 1; P = 0, Q = 1$

(C) $P = 1, Q = 0; P = 1, Q = 1; P = 1, Q = 0$ or $P = 0, Q = 1$

(D) $P = 1, Q = 0; P = 1, Q = 1; P = 1, Q = 1$

45. For the circuit shown, the counter state $(Q_1, Q_0)$ follows the sequence

(A) 00, 01, 10, 11, 00

(B) 00, 01, 10, 00, 01

(C) 00, 01, 11, 00, 01

(D) 00, 10, 11, 00, 10

46. An 8255 chip is interfaced to an 8085 microprocessor system as an I/O mapped I/O as shown in the figure. The address lines $A_3$ and $A_4$ of the 8085 are used by the 8255 chip to decode internally its three ports and the Control register. The address lines $A_3$ to $A_5$ as well as the $IO/M$ signal are used for address decoding. The range of addresses for which the 8255 chip would get selected is

(A) F8H - FBH

(B) F8H - FCH

(C) F8H - FFH

(D) F0H - F7H

47. (A) The 3-dB bandwidth of the low-pass signal $e^{-t}u(t)$, where $u(t)$ is the unit step function, is given by

$$\frac{1}{2\pi} \text{ Hz}$$

(B) $\frac{1}{2\pi}\sqrt{2 - 1} \text{ Hz}$

(C) $\infty$

(D) 1 Hz

48. A Hilbert transformer is a

(A) non-linear system

(B) non-causal system

(C) time-varying system

(D) low-pass system

49. The frequency response of a linear, time-invariant system is given by

$$H(f) = \frac{5}{1 + j\omega}.$$ The step response of the system is

(A) $\frac{5(1 - e^{-t})}{t}u(t)$

(B) $\frac{5(1 - e^{-\frac{t}{5}})}{\frac{t}{5}}u(t)$

(C) $\frac{1}{2}(1 - e^{-t})u(t)$

(D) $\frac{1}{5}(1 - e^{-\frac{t}{5}})u(t)$

50. A 5-point sequence $x[n]$ is given as $x[-3] = 1, x[-2] = 1, x[-1] = 0, x[0] = 5, x[1] = 1$. Let $X(e^{j\omega})$ denote the discrete-time Fourier transform of $x[n]$. The value of $X(e^{j\omega})$ is

(A) 5

(B) $10\pi$

(C) $16\pi$

(D) $5 + j10\pi$

51. The z-transform $x[z]$ of a sequence $x[n]$ is given by

$$X(z) = \frac{0.5}{1 - 0.5z^{-1}}.$$ It is given that the region of convergence of $x[n]$ includes the unit circle. The value of $x[0]$ is

(A) -0.5

(B) 0

(C) 0.25

(D) 0.5

52. A Control system with PD controller is shown in the figure. If the velocity error constant $K_v = 1000$ and the damping ration $\zeta = 0.5$, then the value of $K_p$ and $K_d$ are

(A) $K_p = 100, K_d = 0.09$

(B) $K_p = 100, K_d = 0.9$

(C) $K_p = 10, K_d = 0.09$

(D) $K_p = 10, K_d = 0.9$

53. The transfer function of a plant is

$$T(s) = \frac{5}{(s + 5)(s^2 + s + 1)}.$$
The second-order approximation of $T(s)$ using dominant pole concept is

(A) $\frac{1}{(s+5)(s+1)}$  
(B) $\frac{5}{(s+5)(s+1)}$  
(C) $\frac{5}{s^2+s+1}$  
(D) $\frac{1}{s^2+s+1}$

54. The open-loop transfer function of a plant is given as $G(s) = \frac{10(s-1)}{s+2}$. If the plant is operated in a unity feedback configuration, then the lead compensator that can stabilize this control system is

(A) $\frac{10(s+4)}{s+2}$  
(B) $\frac{10(s+2)}{s+10}$  
(C) $\frac{2(s+2)}{s+10}$  
(D) $\frac{10(s-1)}{s+2}$

55. A unity feedback control system has an open-loop transfer function

$G(s) = \frac{K}{s(s^2 + 7s + 12)}$

The gain $K$ for which $s = 1 + j1$ will lie on the root locus of this system is

(A) 4  
(B) 5.5  
(C) 6.5  
(D) 10

56. The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function $G(s)$ corresponding to this Bode plot is

(A) $\frac{1}{(s+1)(s+20)}$  
(B) $\frac{1}{s(s+1)(s+20)}$  
(C) $\frac{100}{s(s+1)(s+20)}$  
(D) $\frac{100}{s(s+1)(1+0.05s)}$

57. The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix} \frac{ds}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -10 & -1 \end{bmatrix} \begin{bmatrix} s \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$$

(A) $\frac{10}{s^2+11s+11}$  
(B) $\frac{1}{s^2+11s+11}$  
(C) $\frac{10s+10}{s^2+11s+11}$  
(D) $\frac{1}{s^2+s+1}$

58. In delta modulation, the slope overload distortion can be reduced by

(A) decreasing the step size  
(B) decreasing the granular noise  
(C) decreasing the sampling rate  
(D) increasing the step size

59. The raised cosine pulse $p(t)$ is used for zero ISI in digital communications. The expression for $p(t)$ with unity roll-off factor is given by

$p(t) = \frac{\sin 4\pi Wt}{4\pi Wt(1-16W^2t^2)}$

The value of $p(t)$ at $t = \frac{1}{4W}$ is

(A) -0.5  
(B) 0  
(C) 0.5  
(D) $\infty$

60. In the following scheme, if the spectrum $M(f)$ of $m(t)$ is as shown, then the spectrum $Y(f)$ of $y(t)$ will be

(A)  
(B)  
(C)  
(D)

61. During transmission over a certain binary communication channel, bit errors occurs independently with probability $p$. The probability of AT MOST one bit in error in a block of $n$ bits is given by

(A) $p^n$  
(B) $1 - p^n$  
(C) $np(1-p)^{n-1} + (1-p)^n$  
(D) $1 - (1-p)^n$
62. In a GSM system, 8 channels can co-exist in 200 KHz bandwidth using TDMA. A GSM based cellular operator is allocated 5 MHz bandwidth. Assuming a frequency reuse factor of \( \frac{1}{5} \), i.e. a five-cell repeat pattern, the maximum number of simultaneous channels that can exist in one cell is 
(A) 200  
(B) 40  
(C) 25  
(D) 5

63. In a Direct Sequence CDMA system the chip rate is \( 1.2288 \times 10^6 \) chips per second. If the processing gain is desired to be at least 100, the data rate must be less than or equal to \( 122.88 \times 10^3 \) bits/sec 
(A) must be less than or equal to \( 122.88 \times 10^3 \) bits/sec 
(B) must be greater than \( 122.88 \times 10^3 \) bits/sec 
(C) must be exactly equal to \( 122.88 \times 10^3 \) bits/sec 
(D) can take any value less than \( 122.88 \times 10^3 \) bits/sec

64. An air-filled rectangular waveguide has inner dimensions of 3 cm \( \times \) 2 cm. The wave impedance of the \( TE_{20} \) mode of propagation in the waveguide at a frequency of 30 GHz is (free space impedance \( \eta_0 = 377 \Omega \))
(A) 308 \( \eta_0 \)  
(B) 355 \( \eta_0 \)  
(C) 400 \( \eta_0 \)  
(D) 461 \( \eta_0 \)

65. The \( \vec{H} \) field (in A/m) of a plane wave propagating in free space is given by
\[
\vec{H} = \frac{x}{\eta_0} \cos(\omega t - \beta z) + \frac{y}{\eta_0} \sin(\omega t - \beta z + \frac{\pi}{2})
\]
The time average power flow density in Watts is 
(A) \( \frac{\eta_0}{100} \)  
(B) \( \frac{100}{\eta_0} \)  
(C) \( 50\eta_0^2 \)  
(D) \( \frac{50}{\eta_0} \)

66. The \( \vec{E} \) field in a rectangular waveguide of inner dimensions \( a \times b \) is given by
\[
\vec{E} = \frac{\omega \mu}{2 \pi} H_0 \sin \left( \frac{2\pi x}{a} \right)^2 \sin(\omega t - \beta z) y
\]
Where \( H_0 \) is a constant, and \( a \) and \( b \) are the dimensions along the \( x \)-axis and the \( y \)-axis respectively. The mode of propagation in the waveguide is 
(A) \( TE_{20} \)  
(B) \( TM_{11} \)  
(C) \( TM_{20} \)  
(D) \( TE_{10} \)

67. A load of 50\( \Omega \) is connected in shunt in a 2-wire transmission line of \( Z_0 = 50 \Omega \) as shown in the figure. The 2-port scattering parameter matrix (s-matrix) of the shunt element is 
\[
(A) \begin{bmatrix} \frac{j}{\sqrt{2}} & \frac{-j}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} 
(B) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} 
(C) \begin{bmatrix} -\frac{j}{\sqrt{2}} & \frac{j}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} 
(D) \begin{bmatrix} \frac{j}{\sqrt{4}} & \frac{-j}{\sqrt{4}} \\ -\frac{1}{\sqrt{4}} & \frac{1}{\sqrt{4}} \end{bmatrix}
\]

68. The parallel branches of a 2-wire transmission line are terminated in 100 \( \Omega \) and 200 \( \Omega \) resistors as shown in the figure. The characteristic impedance of the line is \( Z_0 = 50 \Omega \) and each section has a length of \( \frac{\lambda}{4} \). The voltage reflection coefficient \( \Gamma \) at the input is 
\[
(A) -\frac{j7}{5} 
(B) \frac{-5}{7} 
(C) \frac{5}{7} 
(D) \frac{50}{7}
\]

69. A \( \frac{\lambda}{2} \) dipole is kept horizontally at a height of \( \frac{\lambda}{4} \) above a perfectly conducting infinite ground plane. The radiation pattern in the lane of the dipole (\( \vec{E} \) plane) looks approximately as 
\[
(A) 
(B) 
(C) 
(D)
\]
A right circularly polarized (RCP) plane wave is incident at an angle of 60° to the normal, on an air-dielectric interface. If the reflected wave is linearly polarized, the relative dielectric constant \( \varepsilon_{r_2} \) is

\[
(A) \sqrt{2} \quad (B) \sqrt{3} \quad (C) 2 \quad (D) 3
\]

**Common Data for Questions 71, 72, 73:**

The figure shows the high-frequency capacitance-voltage (C-V) characteristics of a Metal/SiO\(_2\)/silicon (MOS) capacitor having an area of \( 1 \times 10^{-4} \) cm\(^2\). Assume that the permittivities (\( \varepsilon_0 \varepsilon_r \)) of silicon and SiO\(_2\) are \( 1 \times 10^{-12} \) F/cm and \( 3.5 \times 10^{-10} \) F/cm respectively.

![C-V Characteristics](image)

**71.** The gate oxide thickness in the MOS capacitor is

(A) 50 nm \hspace{1cm} (B) 143 nm \hspace{1cm} (C) 350 nm \hspace{1cm} (D) 1 \mu m

**72.** The maximum depletion layer width in silicon is

(A) 0.143 \mu m \hspace{1cm} (B) 0.857 \mu m \hspace{1cm} (C) 1 \mu m \hspace{1cm} (D) 1.143 \mu m

**73.** Consider the following statements about the C-V characteristics plot:

**S1:** The MOS capacitor has an n-type substrate.

**S2:** If positive charges are introduced in the oxide, the C-V plot will shift to the left.

Then which of the following is true?

(A) Both S1 and S2 are true

(B) S1 is true and S2 is false

(C) S1 is false and S2 is true

(D) Both S1 and S2 are false

**Common Data for Questions 74, 75:**

Two 4-ray signal constellations are shown. It is given that \( \phi_1 \) and \( \phi_2 \) constitute an orthonormal basis for the two constellations. Assume that the four symbols in both the constellations are equiprobable. Let \( \mathcal{N}_0 / 2 \) denote the power spectral density of white Gaussian noise.

![Constellations](image)

**74.** The ratio of the average energy of constellation 1 to the average energy of constellation 2 is

(A) \( 4 \alpha^2 \) \hspace{1cm} (B) 4 \hspace{1cm} (C) 2 \hspace{1cm} (D) 8

**75.** If these constellations are used for digital communications over an AWGN channel, then which of the following statements is true?

(A) Probability of symbol error for Constellation 1 is lower

(B) Probability of symbol error for Constellation 1 is higher

(C) Probability of symbol error is equal for both the constellations

(D) The value of \( \mathcal{N}_0 \) will determine which of the two constellations has a lower probability of symbol error,

**Linked Answer Questions: Q. 76 to Q. 85 Carry Two marks Each.**

**Statement for Linked Answer Questions 76 & 77:**

Consider the Op-Amp circuit shown in the figure.
76. The transfer function \( V_o(s)/V_i(s) \) is
   \[
   (A) \frac{1 - sRC}{1 + sRC} \qquad (B) \frac{1 + sRC}{1 - sRC} \\
   (C) \frac{1}{1 - sRC} \qquad (D) \frac{1}{1 + sRC}
   \]

77. If \( V_i = V_1 \sin(\omega t) \) and \( V_o = V_2 \sin(\omega t - \phi) \), then the minimum and maximum values of \( \phi \) (in radians) are respectively
   \[
   (A) -\frac{\pi}{2} \text{ and } \frac{\pi}{2} \qquad (B) 0 \text{ and } -\frac{\pi}{2} \qquad (C) -\pi \text{ and } 0 \qquad (D) \frac{\pi}{2} \text{ and } 0
   \]

Statement for Linked Answer Questions 78 & 79.

An 8085 assembly language program is given below.

Line 1: MVI A, B5H
2: MVI B, OEH
3: XRI 69H
4: ADD B
5: ANI 9BH
6: CPI 9FH
7: STA 3010H
8: HLT

78. The contents of the accumulator just after execution of the ADD instruction in line 4 will be
   \[
   (A) \text{C3H} \qquad (B) \text{EAH} \qquad (C) \text{DCH} \qquad (D) \text{69H}
   \]

79. After execution of line 7 of the program, the status of the CY and Z flags will be
   \[
   (A) \text{CY = 0, Z = 0} \qquad (B) \text{CY = 0, Z = 1} \qquad (C) \text{CY = A, Z = 0} \qquad (D) \text{CY = 1, Z = 1}
   \]

Statement for Linked Answer Questions 80 & 81.

Consider a linear system whose state space representation is \( x(t) = Ax(t) \). If the initial state vector of the system is \( x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \), then the system response is
   \[
   x(t) = \begin{bmatrix} e^{2t} \\ -2e^{-2t} \end{bmatrix}.
   \]

If the initial state vector of the system changes to \( x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \), then the system response becomes
   \[
   x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}.
   \]

80. The eigenvalue and eigenvector pairs \((\lambda, V)\) for the system are
   \[
   (A) \left[-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right] \text{ and } \left[-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right] \\
   (B) \left[-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right] \text{ and } \left[-2, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right] \\
   (C) \left[-1, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right] \text{ and } \left[1, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right] \\
   (D) \left[-2, \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right] \text{ and } \left[1, \begin{bmatrix} 1 \\ -2 \end{bmatrix}\right]
   \]

Statement for Linked Answer Questions 82 & 83:

An input to a 6-level quantizer has the probability density function \( f(x) \) as shown in the figure. Decision boundaries of the quantizer are chosen so as to maximize the entropy of the quantizer output. It is given that 3 consecutive decision boundaries are '+'1', '0' and '1'.

82. The values of \( a \) and \( b \) are
   \[
   (A) a = \frac{1}{6} \text{ and } b = \frac{1}{12} \qquad (B) a = \frac{1}{5} \text{ and } b = \frac{3}{40} \\
   (C) a = \frac{1}{4} \text{ and } b = \frac{1}{16} \qquad (D) a = \frac{1}{3} \text{ and } b = \frac{1}{24}
   \]

83. Assuming that the reconstruction levels of the quantizer are the mid-points of the decision boundaries, the ratio of signal power to quantization noise power is
   \[
   (A) \frac{152}{9} \\
   (B) \frac{64}{3} \\
   (C) \frac{76}{3} \\
   (D) 28
   \]
Statement for Linked Answer Questions 84 & 85.

In the digital-to-Analog converter circuit shown in the figure below, $V_R = 10 \text{ V}$ and $R = 10 \text{ k}\Omega$.

84. The current is
(A) $31.25 \mu\text{A}$
(B) $62.5 \mu\text{A}$
(C) $125 \mu\text{A}$
(D) $250 \mu\text{A}$

85. The voltage $V_0$ is
(A) $-0.781 \text{ V}$
(B) $-1.562 \text{ V}$
(C) $-3.125 \text{ V}$
(D) $-6.250 \text{ V}$
Q.1—30 carry one mark each

1. The minimum number of equations required to analyze the circuit shown in Fig. Q. 1 is
   (A) 3  (B) 4  (C) 6  (D) 7

2. A source of angular frequency 1 rad/sec has a source impedance consisting of 1Ω resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is
   (A) 1 Ω resistance
   (B) 1 Ω resistance in parallel with 1 H inductance
   (C) 1 Ω resistance in series with 1 F capacitor
   (D) 1 Ω resistance in parallel with 1 F capacitor

3. A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of $R$, $L$ and $C$ is doubled from its original value, the new $Q$ of the circuit is
   (A) 25  (B) 50  (C) 100  (D) 200

4. The Laplace transform of $i(t)$ is given by
   $$I(s) = \frac{2}{s(1 + s)}$$
   As $t \to \infty$, The value of $i(t)$ tends to
   (A) 0  (B) 1  (C) 2  (D) $\infty$

5. The differential equation for the current $i(t)$ in the circuit of Fig. Q.5 is
   (A) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + i(t) = \sin t$
   (B) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \cos t$
   (C) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + i(t) = \cos t$
   (D) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \sin t$

6. n-type silicon is obtained by doping silicon with
   (A) Germanium  (B) Aluminium  (C) Boron  (D) Phosphorus

7. The bandgap of silicon at 300 K is
   (A) 1.36 eV  (B) 1.10 eV  (C) 0.80 eV  (D) 0.67 eV
20. A 0 to 6 counter consists of 3 flip flops and a combination circuit of 2 input gate(s). The combination circuit consists of
(A) one AND gate
(B) one OR gate
(C) one AND gate and one OR gate
(D) two AND gates

21. The Fourier series expansion of a real periodic signal with fundamental frequency \( f_0 \) is given by
\[
g(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nf_0 t} \]
It is given that \( c_{-3} = 3 + j5 \). Then \( c_{-3} \) is
(A) 5 + j3
(B) -3 - j5
(C) -5 + j3
(D) 3 - j5

22. Let \( x(t) \) be the input to a linear, time-invariant system. The required output is \( 4x(t - 2) \). The transfer function of the system should be
(A) \( 4e^{j4\pi f} \)
(B) \( 2e^{-j8\pi f} \)
(C) \( 4e^{-j4\pi f} \)
(D) \( 2e^{j8\pi f} \)

23. A sequence \( x(n) \) with the \( z \)-transform
\[
X(z) = z^4 + z^2 - 2z + 2 - 3z^{-1}
\]
is applied as an input to a linear, time-invariant system with the impulse response \( h(n) = 2\delta(n - 3) \) where
\[
\delta(n) = \begin{cases} 
1, & n = 0 \\
0, & \text{otherwise}
\end{cases}
\]
The output at \( n = 4 \) is
(A) -6
(B) zero
(C) 2
(D) -4

24. Fig. Q.24 shows the Nyquist plot of the open-loop transfer function \( G(s)H(s) \) of a system. If \( G(s)H(s) \) has one right-hand pole, the closed-loop system is
(\( s \) is positive)

25. A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has
(A) a higher type number
(B) reduced damping
(C) higher noise amplification
(D) larger transient overshoot

26. The input to a coherent detector is DSB-SC signal plus noise. The noise at the detector output is
(A) the in-phase component
(B) the quadrature component
(C) zero
(D) the envelope

27. The noise at the input to an ideal frequency detector is white. The detector is operating above threshold. The power spectral density of the noise at the output is
(A) raised-cosine
(B) flat
(C) parabolic
(D) Gaussian

28. At a given probability of error, binary coherent FSK is inferior to binary coherent PSK by
(A) 6 dB
(B) 3 dB
(C) 2 dB
(D) 0 dB

29. The unit of \( \text{V} \times \text{H} \) is
(A) Ampere
(B) Ampere/meter
(C) Ampere/meter^2
(D) Ampere-meter

30. The depth of penetration of electromagnetic wave in a medium having conductivity \( \sigma \) at a frequency of 1 MHz is 25 cm. The depth of penetration at a frequency of 4 MHz will be
(A) 6.25 cm
(B) 12.50 cm
(C) 50.00 cm
(D) 100.00 cm

Q.31—90 carry two marks each.

31. Twelve 1 \( \Omega \) resistance are used as edges to form a cube. The resistance between two diagonally opposite corners of the cube is
(A) \( \frac{5}{6} \) \( \Omega \)
(B) 1 \( \Omega \)
(C) \( \frac{6}{5} \) \( \Omega \)
(D) \( \frac{3}{2} \) \( \Omega \)
32. The current flowing through the resistance $R$ in the circuit in Fig. Q.32 has the form $P \cos 4t$, where $P$ is

![Fig. Q32](image)

(A) $(0.18 + j 0.72)$  
(B) $(0.46 + j 190)$  
(C) $-(0.18 + j 190)$  
(D) $-(0.192 + j 0.144)$

The circuit for Q.33–34 are given in Fig. Q.33–34. For both the questions, assume that the switch $S$ is in position 1 for a long time and thrown to position 2 at $t = 0$.

![Fig. Q33-34](image)

33. At $t = 0^+$, the current $i_1$ is

(A) $-\frac{V}{2R}$  
(B) $-\frac{V}{R}$  
(C) $-\frac{V}{4R}$  
(D) zero

34. $I_1(s)$ and $I_2(s)$ are the Laplace transforms of $i_1(t)$ and $i_2(t)$ respectively. The equations for the loop currents $I_1(s)$ and $I_2(s)$ for the circuit shown in Fig. Q.33–34, after the switch is brought from position 1 to position 2 at $t = 0$, are

(A) \[
\begin{bmatrix}
R + Ls + \frac{1}{Cs} & -Ls \\
-Ls & R + \frac{1}{Cs}
\end{bmatrix}
\begin{bmatrix}
I_1(s) \\
I_2(s)
\end{bmatrix} = \begin{bmatrix}
\frac{V}{s} \\
0
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
R + Ls + \frac{1}{Cs} & -Ls \\
-Ls & R + \frac{1}{Cs}
\end{bmatrix}
\begin{bmatrix}
I_1(s) \\
I_2(s)
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{V}{s}
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
R + Ls + \frac{1}{Cs} & -Ls \\
-Ls & R + Ls + \frac{1}{Cs}
\end{bmatrix}
\begin{bmatrix}
I_1(s) \\
I_2(s)
\end{bmatrix} = \begin{bmatrix}
\frac{V}{s} \\
0
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
R + Ls + \frac{1}{Cs} & -Ls \\
-Ls & R + Ls + \frac{1}{Cs}
\end{bmatrix}
\begin{bmatrix}
I_1(s) \\
I_2(s)
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{V}{s}
\end{bmatrix}
\]

35. An input voltage

\[v(t) = 10 \sqrt{2} \cos (t + 10) + 10 \sqrt{3} \cos (2 + 10^\circ)\ V\]

is applied to a series combination of resistance $R = 1 \Omega$ and an inductance $L = 1 \ H$. The resulting steady state current $i(t)$ in ampere is

(A) $10 \cos (t + 55^\circ) + 10 \cos (2t + 10^\circ + \tan^{-1} 2)$  
(B) $1 - \cos (t + 55^\circ) + 10 \frac{3}{\sqrt{2}} \cos (2t + 55^\circ)$  
(C) $10 \cos (t - 55^\circ) + 10 \cos (2t + 10^\circ - \tan^{-1} 2)$  
(D) $1 - \cos (t - 55^\circ) + 10 \frac{3}{\sqrt{2}} \cos (2t - 35^\circ)$

36. The driving-point impedance $Z(s)$ of a network has the pole-zero locations as shown in Fig. Q.36. If $Z(0) = 3$, then $Z(s)$ is

![Fig. Q36](image)

(A) $\frac{3(s + 3)}{s^2 + 2s + 3}$  
(B) $\frac{2(s + 3)}{s^2 + 2s + 2}$  
(C) $\frac{3s - 3}{s^2 - 2s - 2}$  
(D) $\frac{2s - 3}{s^2 - 2s - 3}$

37. The impedance parameters $Z_{11}$ and $Z_{12}$ of the two-port network in Fig. Q.37 are

![Fig. Q37](image)

(A) $Z_{11} = 2.75 \Omega$ and $Z_{12} = 0.25 \Omega$  
(B) $Z_{11} = 3 \Omega$ and $Z_{12} = 0.5 \Omega$  
(C) $Z_{11} = 3 \Omega$ and $Z_{12} = 0.25 \Omega$  
(D) $Z_{11} = 2.25 \Omega$ and $Z_{12} = 0.5 \Omega$
38. An n-type silicon bar 0.1 cm long and 100 \( \mu m^2 \) in cross-sectional area has a majority carrier concentration of \( 5 \times 10^{20} / m^3 \) and the carrier mobility is 0.13 m\(^2\)/V-s at 300 K. If the charge of an electron is \( 1.6 \times 10^{-19} \) coulomb, then the resistance of the bar is
(A) \( 10^6 \) Ohm  
(B) \( 10^4 \) Ohm  
(C) \( 10 \) Ohm  
(D) \( 10^{-4} \) Ohm

39. The electron concentration in a sample of uniformly doped n-type silicon at 300 K varies linearly from \( 10^{17} \) cm\(^3\) at \( x = 0 \) to \( 6 \times 10^{16} \) cm\(^3\) at \( x = 2 \mu m \). Assume a situation that electrons are supplied to keep this concentration gradient constant with time. If electronic charge is \( 1.6 \times 10^{-19} \) coulomb and the diffusion constant \( D_n = 35 \) cm\(^2\)/s, the current density in the silicon, if no electric field is present, is
(A) zero  
(B) \( -112 \) A/cm\(^2\)  
(C) \( +1120 \) A/cm\(^2\)  
(D) \( -1120 \) A/cm\(^2\)

40. Match items in Group 1 with items in Group 2, most suitably.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. LED</td>
<td>1. Heavy doping</td>
</tr>
<tr>
<td>Q. Avalanche photo diode</td>
<td>2. Coherent radiation</td>
</tr>
<tr>
<td>R. Tunnel diode</td>
<td>3. Spontaneous emission</td>
</tr>
<tr>
<td>S. LASER</td>
<td>4. Current gain</td>
</tr>
</tbody>
</table>

(A) P  
(B) Q  
(C) R  
(D) S

(A) 1  
(B) 2  
(C) 3  
(D) 4

41. At 300 K, for a diode current of 1 mA, a certain germanium diode requires a forward bias of 0.1435 V, whereas a certain silicon diode requires a forward bias of 0.718 V. Under the conditions stated above, the closest approximation of the ratio of reverse saturation current in germanium diode to that in silicon diode is
(A) 1  
(B) 5  
(C) \( 4 \times 10^3 \)  
(D) \( 8 \times 10^3 \)

42. A particular green LED emits light of wavelength 5490 Å. The energy bandgap of the semiconductor material used there is (Plank’s constant = 6.626 \( \times 10^{-34} \) J – s)
(A) 2.26 eV  
(B) 1.98 eV  
(C) 1.17 eV  
(D) 0.74 eV

43. When the gate-to-source voltage \( V_{gs} \) of a MOSFET with threshold voltage of 400 mV, working in saturation is 900 mV, the drain current is observed to be 1 mA. Neglecting the channel width modulation effect and assuming that the MOSFET is operating at saturation, the drain current for an applied \( V_{gs} \) of 1400 mV is
(A) 0.5 mA  
(B) 2.0 mA  
(C) 3.5 mA  
(D) 4.0 mA

44. If P is Passivation, Q is n-well implant, R is metallization and S is source/drain diffusion, then the order in which they are carried out in a standard n-well CMOS fabrication process, is
(A) P–Q–R–S  
(B) Q–S–R–P  
(C) R–P–S–Q  
(D) S–R–Q–P

45. An amplifier without feedback has a voltage gain of 50, input resistance of 1 k\( \Omega \) and output resistance of 2.5 k\( \Omega \). The input resistance of the current-shunt negative feedback amplifier using the above amplifier with a feedback factor of 0.2, is
(A) \( 1/11 \) k\( \Omega \)  
(B) \( 1/5 \) k\( \Omega \)  
(C) 5 k\( \Omega \)  
(D) 11 k\( \Omega \)

46. In the amplifier circuit shown in Fig. Q.46, the values of \( R_1 \) and \( R_2 \) are such that the transistor is operating at \( V_{ce} = 3 \) V and \( I_c = 15 \) mA when its \( \beta \) is 150. For a transistor with \( \beta \) of 200, the operating point \( (V_{ce}, I_c) \) is

(A) (2 V, 2 mA)  
(B) (3 V, 2 mA)  
(C) (4 V, 2 mA)  
(D) (4 V, 1 mA)
47. The oscillator circuit shown in Fig. Q.47 has an ideal inverting amplifier. Its frequency of oscillation (in Hz) is

\[ \frac{1}{(2\pi \sqrt{RC})} \]  
\[ \frac{1}{(2\pi RC)} \]  
\[ \frac{1}{\sqrt{6RC}} \]  
\[ \frac{1}{(2\pi RC)} \]

![Fig. Q47](image)

48. The output voltage of the regulated power supply shown in Fig. Q.48 is

(A) 3 V  
(B) 6 V  
(C) 9 V  
(D) 12 V

![Fig. Q48](image)

49. The action of a JFET in its equivalent circuit can best be represented as a

(A) Current Controlled Current Source  
(B) Current Controlled Voltage Source  
(C) Voltage Controlled Voltage Source  
(D) Voltage Controlled Current Source

50. If the op-amp in Fig. Q.50 is ideal, the output voltage \( V_{out} \) will be equal to

(A) 1 V  
(B) 6 V  
(C) 14 V  
(D) 17 V

![Fig. Q50](image)

51. Three identical amplifiers with each one having a voltage gain of 50, input resistance of 1 k\( \Omega \) and output resistance of 250 \( \Omega \), are cascaded. The open circuit voltage gain of the combined amplifier is

(A) 49 dB  
(B) 51 dB  
(C) 98 dB  
(D) 102 dB

52. An ideal sawtooth voltage waveform of frequency 500 Hz and amplitude 3 V is generated by charging a capacitor of 2 \( \mu \)F in every cycle. The charging requires

(A) constant voltage source of 3 V for 1 ms  
(B) constant voltage source of 3 V for 2 ms  
(C) constant current source of 3 mA for 1 ms  
(D) constant current source of 3 mA for 2 ms

53. The circuit shown in Fig. Q.53 has 4 boxes each described by inputs, \( P, Q, R \) and outputs \( Y, Z \) with

\[ Y = P \oplus Q \oplus R, \quad Z = RQ + PR + QP \]

The circuit acts as a

(A) 4 bit adder giving \( P + Q \)  
(B) 4 bit subtracter giving \( P - Q \)  
(C) 4 bit subtracter giving \( Q - R \)  
(D) 4 bit adder giving \( P + Q + R \)

![Fig. Q53](image)

54. If the functions \( W, X, Y \) and \( Z \) are as follows

\[ W = R + \overline{PQ} + \overline{RS} \]  
\[ X = PQR + \overline{PQ} \overline{R} \overline{S} + PQ \overline{R} \overline{S} \]  
\[ Y = RS + PR + PQ = \overline{PQ} \]

\[ Z = R + S + PQ + R \cdot Q \cdot \overline{P} + PQ \cdot S \]

Then

(A) \( W = Z, \quad X = \overline{Z} \)  
(B) \( W = Z, \quad X = Y \)  
(C) \( W = Y \)  
(D) \( W = Y = \overline{Z} \)

55. A 4 bit ripple counter and a 4 bit synchronous counter are made using flip flops having a propagation delay of 10 ns each. If the worst case delay in the ripple
counter and the synchronous counter be R and S respectively, then
(A) R = 10 ns, S = 40 ns  (B) R = 40 ns, S = 10 ns
(C) R = 10 ns, S = 30 ns  (D) R = 30 ns, S = 10 ns

56. The DTL, TTL, ECL and CMOS families of digital ICs are compared in the following 4 columns

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fanout is minimum</td>
<td>DTL</td>
<td>DTL</td>
<td>TTL</td>
</tr>
<tr>
<td>Power consumption is minimum</td>
<td>TTL</td>
<td>CMOS</td>
<td>ECL</td>
</tr>
<tr>
<td>Propagation delay is minimum</td>
<td>CMOS</td>
<td>ECL</td>
<td>TTL</td>
</tr>
</tbody>
</table>

The correct column is
(A) P  (B) Q  (C) R  (D) S

57. The circuit shown in Fig. Q.57 is a 4 bit DAC. The input bits 0 and 1 are represented by 0 and 5 V respectively. The OP AMP is ideal, but all the resistance and the 5 V inputs have a tolerance of ±10%. The specification (rounded to the nearest multiple of 5%) for the tolerance of the DAC is
(A) ±35%  (B) ±20%
(C) ±10%  (D) ±5%

58. The circuit shown in Fig. Q.58 converts
(A) BCD to binary code
(B) Binary to excess -3 code
(C) Excess -3 to Gray code
(D) Gray to Binary code

59. In the circuit shown in Fig. Q.59, A is a parallel-in, parallel-out 4 bit register, which loads at the rising edge of the clock C. The input lines are connected to a 4 bit bus, W. Its output acts as the input to a 16 × 4 ROM whose output is floating when the enable input E is 0. A partial table of the contents of the ROM is as follows

<table>
<thead>
<tr>
<th>Address</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0011</td>
</tr>
<tr>
<td>2</td>
<td>1111</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
</tr>
<tr>
<td>6</td>
<td>1010</td>
</tr>
<tr>
<td>8</td>
<td>1011</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>12</td>
<td>0010</td>
</tr>
<tr>
<td>14</td>
<td>1000</td>
</tr>
</tbody>
</table>

The clock to the register is shown, and the data on the W bus at time t₁ is 0110. The data on the bus at time t₂ is
Data for Q.65–66 are given below. Solve the problems and choose the correct answers.

**Q.65**

Let \( H(f) \) denote the frequency response of the RC-LPF. Let \( f_1 \) be the highest frequency such that 
\[
0 < |f| < f_1 \left| \frac{H(f)}{H(0)} \right| 0.95. \text{ Then } f_1 \text{ (in Hz) is}
\]
(A) 327.8 (B) 163.9 (C) 52.2 (D) 104.4

**Q.66**

Let \( t_g(f) \) be the group delay function of the given RC-LPF and \( f_2 = 100 \text{ Hz} \). Then \( t_g(f_2) \) in ms, is
(A) 0.717 (B) 7.17 (C) 71.7 (D) 4.505
69. The root locus of the system
\[ G(s)H(s) = \frac{K}{\pi(s + 2)(s + 3)} \]
has the break-away point located at
(A) (-0.5, 0) (B) (-2.548, 0)
(C) (-4, 0) (D) (-0.784, 0)

70. The approximate Bode magnitude plot of a minimum phase system is shown in Fig. Q. 70. The transfer function of the system is
\[ (A) \frac{10^8}{(s + 0.1)^3(s + 100)} \]
\[ (B) \frac{10^7}{(s + 0.1)^3(s + 10)(s + 100)} \]
\[ (C) \frac{10^8}{(s + 0.1)^3(s + 10)^2(s + 100)} \]
\[ (D) \frac{10^9}{(s + 0.1)^3(s + 10)(s + 100)^2} \]

71. A second-order system has the transfer function
\[ \frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4} \]

With \( r(t) \) as the unit-step function, the response \( c(t) \) of the system is represented by
(A) \( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \)
(B) \( \begin{bmatrix} te^t \\ t \end{bmatrix} \)
(C) \( \begin{bmatrix} e^t \\ t e^t \end{bmatrix} \)
(D) \( \begin{bmatrix} t \\ te^t \end{bmatrix} \)

72. The gain margin and the phase margin of a feedback system with
\[ G(s)H(s) = \frac{s}{(s + 100)^3} \] are
(A) - dB, 0° (B) \( \infty, \infty \)
(C) \( \infty, 0° \) (D) 88.5 dB, \( \infty \)

73. The zero-input response of a system given by the state-space equation
\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]
\[ x_1(0) = 1, x_2(0) = 0 \]
is
(A) \( te^t \) (B) \( e^t \)
(C) \( e^t \) (D) \( t e^t \)

74. A DSB-SC signal is to be generated with a carrier frequency \( f_c = 1 \) MHz using a nonlinear device with the input-output characteristic \( v_i = a_0 v_i + a_1 v_i^3 \) where \( a_0 \) and \( a_1 \) are constants. The output of the nonlinear device can be filtered by an appropriate band-pass filter. Let \( v_i = A_1 \cos(2\pi f_c t + m(t)) \) where \( m(t) \) is the message signal. Then the value of \( f_c \) (in MHz) is
(A) 1.0 (B) 0.333
(C) 0.5 (D) 3.0

The data for Q. 75-76 are given below. Solve the problems and choose the correct answers.

Let \( m(t) = \cos((4 \times 10^3)t) \) be the message signal and \( c(t) = 5 \cos((2 \times 10^6)t) \) be the carrier.

75. \( c(t) \) and \( m(t) \) are used to generate an AM signal. The modulation index of the generated AM signal is 0.5. Then the quantity \( \frac{\text{Total side band power}}{\text{Carrier power}} \) is
(A) \( \frac{1}{7} \) (B) \( \frac{1}{8} \)
(C) \( \frac{1}{8} \) (D) \( \frac{1}{7} \)

76. \( c(t) \) and \( m(t) \) are used to generate an FM signal. If the peak frequency deviation of the generated FM is three times the transmission bandwidth of the AM signal, then the coefficient of the term \( \cos(2\pi 1008 \times 10^3 t) \) in the FM signal (in terms of the Bessel coefficients) is
(A) \( 5J_4(3) \) (B) \( \frac{5}{2}J_4(3) \)

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86. A uniform plane wave traveling in air is incident on the plane boundary between air and another dielectric medium with \( \varepsilon_r = 4 \). The reflection coefficient for the normal incidence, is

(A) zero  
(B) 0.5 \( \angle 180^\circ \)  
(C) 0.333 \( \angle 0^\circ \)  
(D) 0.333 \( \angle 180^\circ \)

87. If the electric field intensity associated with a uniform plane electromagnetic wave traveling in a perfect dielectric medium is given by \( E(z, t) = 10 \cos (2\pi \times 10^7 t - 0.1\pi z) \) volt/m, then the velocity of the traveling wave is

(A) 3.00 \( \times 10^8 \) m/sec  
(B) 2.00 \( \times 10^8 \) m/sec  
(C) 6.28 \( \times 10^7 \) m/sec  
(D) 2.00 \( \times 10^7 \) m/sec

88. A short-circuited stub is shunt connected to a transmission line as shown in Fig. Q.88. If \( Z_0 = 50 \) ohm, the admittance \( Y \) seen at the junction of the stub and the transmission line is

(A) \( (0.01 - j0.02) \) mho  
(B) \( (0.02 - j0.01) \) mho  
(C) \( (0.04 - j0.02) \) mho  
(D) \( (0.02 + j0) \) mho

89. A rectangular metal wave guide filled with a dielectric material of relative permittivity \( \varepsilon_r = 4 \) has the inside dimensions 3.0 cm \( \times \) 1.2 cm. The cut-off frequency for the dominant mode is

(A) 2.5 GHz  
(B) 5.0 GHz  
(C) 10.0 GHz  
(D) 12.5 GHz

90. Two identical antennas are placed in the \( \theta = \pi/2 \) plane as shown in Fig. Q.90. The elements have equal amplitude excitation with 180º polarity difference, operating at wavelength \( \lambda \). The correct value of the magnitude of the far-zone resultant electric field strength normalized with that of a single element, both computed for \( \phi = 0 \), is

(A) \( 2 \cos \left( \frac{2\pi s}{\lambda} \right) \)  
(B) \( 2 \sin \left( \frac{2\pi s}{\lambda} \right) \)  
(C) \( 2 \cos \left( \frac{\pi s}{\lambda} \right) \)  
(D) \( 2 \sin \left( \frac{\pi s}{\lambda} \right) \)
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(B)</td>
</tr>
<tr>
<td>2</td>
<td>(C)</td>
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<td>(A)</td>
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<td>(B)</td>
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<td>(D)</td>
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<td>(B)</td>
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<td>14</td>
<td>(C)</td>
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</table>
5. For the R-L circuit shown in Fig. Q.5, the input voltage \( v_i(t) = u(t) \). The current \( i(t) \) is

\[
\begin{align*}
\text{Fig Q.5} \\
(A) & \quad (B) & \quad (C) & \quad (D)
\end{align*}
\]

\[
\begin{align*}
\text{Fig Q.5} \\
0.5 & \quad 0.31 \\
2 & \quad 2
\end{align*}
\]

6. The impurity commonly used for realizing the base region of a silicon n-p-n transistor is

(A) Gallium  
(B) Indium  
(C) Boron  
(D) Phosphorus

7. If for a silicon n-p-n transistor, the base-to-emitter voltage \( V_{BE} \) is 0.7 V and the collector-to-base voltage \( V_{CB} \) is 0.2 V, then the transistor is operating in the

(A) normal active mode  
(B) saturation mode  
(C) inverse active mode  
(D) cutoff mode

8. Consider the following statements S1 and S2.

S1 : The \( \beta \) of a bipolar transistor reduces if the base width is increased.

S2 : The \( \beta \) of a bipolar transistor increases if the doping concentration in the base is increased.

Which one of the following is correct?

(A) S1 is FALSE and S2 is TRUE  
(B) Both S1 and S2 are TRUE  
(C) Both S1 and S2 are FALSE  
(D) S1 is TRUE and S2 is FALSE

9. An ideal op-amp is an ideal

(A) voltage controlled current source  
(B) voltage controlled voltage source  
(C) current controlled current source  
(D) current controlled voltage source

10. Voltage series feedback (also called series-shunt feedback) results in

(A) increase in both input and output impedances  
(B) decrease in both input and output impedances  
(C) increase in input impedance and decrease in output impedance  
(D) decrease in input impedance and increase in output impedance

11. The circuit in Fig. Q.11 is a

\[
\begin{align*}
\text{Fig Q.11} \\
(A) & \quad (B) & \quad (C) & \quad (D)
\end{align*}
\]

12. Assuming \( V_{CE\text{sat}} = 0.2 \text{ V} \) and \( \beta = 50 \), the minimum base current \( I_B \) required to drive the transistor in Fig. Q.12 to saturation is

(A) 56 \( \mu \text{A} \)  
(B) 140 mA  
(C) 60 \( \mu \text{A} \)  
(D) 3 mA

13. A master-slave flip-flop has the characteristic that

(A) change in the input is immediately reflected in the output  
(B) change in the output occurs when the state of the master is affected  
(C) change in the output occurs when the state of the slave is affected  
(D) both the master and the slave states are affected at the same time
14. The range of signed decimal numbers that can be represented by 6-bit 1’s complement numbers is
(A) –31 to +31  (B) –63 to +64  
(C) –64 to +63  (D) –32 to +31

15. A digital system is required to amplify a binary-encoded audio signal. The user should be able to control the gain of the amplifier from a minimum to a maximum in 100 increments. The minimum number of bits required to encode, in straight binary, is
(A) 8  (B) 6  
(C) 5  (D) 7

16. Choose the correct one from among the alternatives A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
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<tbody>
<tr>
<td>P: Shift register</td>
<td>1: Frequency division</td>
</tr>
<tr>
<td>Q: Counter</td>
<td>2: Addressing in memory chips</td>
</tr>
<tr>
<td>R: Decoder</td>
<td>3: Serial to parallel data conversion</td>
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</tbody>
</table>

(A) P–3  (B) P–3  (C) P–2  (D) P–1

17. Fig. Q.17 shows the internal schematic of a TTL AND-OR-Invert (AOI) gate. For the inputs shown in Fig. Q.17, the output \( Y \) is
\[ y = (A + B) \overline{A} \overline{B} \]

(A) 0  (B) 1  (C) AB  (D) \( AB \)

18. Fig. Q.18 is the voltage transfer characteristic of
(A) an NMOS inverter with enhancement mode transistor as load  
(B) an NMOS inverter with depletion mode transistor as load  
(C) a CMOS inverter  
(D) a BJT inverter

19. The impulse response \( h[n] \) of a linear time-invariant system is given by
where \( u[n] \) is the unit step sequence. The above system is

(A) stable but not causal  (B) stable and causal  
(C) causal but unstable  (D) unstable and not causal

20. The distribution function \( F_X(x) \) of a random variable \( X \) is shown in Fig. Q.20. The probability that \( X = 1 \) is

(A) zero  (B) 0.25  
(C) 0.55  (D) 0.30

21. The z-transform of a system is
\[ H(z) = \frac{z}{z - 0.2} \]
If the ROC is \( |z| < 0.2 \), then the impulse response of the system is

(A) \((0.2)^n u[n]\)  (B) \((0.2)^n u[-n - 1]\)  
(C) \(-(0.2)^n u[n]\)  (D) \(-(0.2)^n u[-n - 1]\)

22. The Fourier transform of a conjugate symmetric function is always
(A) imaginary  (B) conjugate anti-symmetric  
(C) real  (D) conjugate symmetric
33. Consider the Bode magnitude plot shown in Fig. Q.33. The transfer function \( H(s) \) is

\[
\begin{align*}
&\text{(A) } \frac{(s + 10)}{(s + 1)(s + 100)} \\
&\text{(B) } \frac{10(s + 1)}{(s + 10)(s + 100)} \\
&\text{(C) } \frac{10^2(s + 1)}{(s + 10)(s + 100)} \\
&\text{(D) } \frac{10^3(s + 100)}{(s + 1)(s + 10)}
\end{align*}
\]

34. The transfer function \( H(s) = \frac{V_c(s)}{V_i(s)} \) of an R–L–C circuit is given by

\[ H(s) = \frac{10^6}{s^2 + 20s + 10^6} \]

The Quality factor (Q-factor) of this circuit is

(A) 25  \hspace{1cm} (B) 50  \hspace{1cm} (C) 100  \hspace{1cm} (D) 5000

35. For the circuit shown in Fig. Q.35, the initial conditions are zero. Its transfer function \( H(s) = \frac{V_c(s)}{V_i(s)} \) is

\[
\begin{align*}
&\text{(A) } \frac{1}{s^2 + 10^3s + 10^6} \\
&\text{(B) } \frac{10^6}{s^2 + 10^3s + 10^6} \\
&\text{(C) } \frac{10^3}{s^2 + 10^3s + 10^6} \\
&\text{(D) } \frac{10^6}{s^2 + 10^6s + 10^6}
\end{align*}
\]

36. A system described by the following differential equation

\[
\frac{d^2y}{dt^2} + 3 \frac{dy}{dt} + 2y = x(t)
\]

is initially at rest. For input \( x(t) = 2u(t) \), the output \( y(t) \) is

\[
\begin{align*}
&\text{(A) } (1 - 2e^{-t} + e^{-2t})u(t) \\
&\text{(B) } (1 + 2e^{-t} - 2e^{-2t})u(t) \\
&\text{(C) } (0.5 + e^{-t} + 1.5e^{-2t})u(t) \\
&\text{(D) } (0.5 + 2e^{-t} + 2e^{-2t})u(t)
\end{align*}
\]

37. Consider the following statements S1 and S2.

S1 : At the resonant frequency the impedance of a series R–L–C circuit is zero.

S2 : In a parallel G–L–C circuit, increasing the conductance \( G \) results in increase in its Q factor.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE  \hspace{1cm} (B) Both S1 and S2 are TRUE  \hspace{1cm} (C) S1 is TRUE and S2 is FALSE  \hspace{1cm} (D) Both S1 and S2 are FALSE

38. In an abrupt p–n junction, the doping concentrations on the p-side and n-side are \( N_A = 9 \times 10^{16} \text{ cm}^{-3} \) respectively. The p–n junction is reverse biased and the total depletion width is 3 \( \mu \text{m} \). The depletion width on the p-side is

(A) 2.7 \( \mu \text{m} \)  \hspace{1cm} (B) 0.3 \( \mu \text{m} \)  \hspace{1cm} (C) 2.25 \( \mu \text{m} \)  \hspace{1cm} (D) 0.75 \( \mu \text{m} \)

39. The resistivity of a uniformly doped n-type silicon sample is 0.5 \( \Omega \text{ cm} \). If the electron mobility \( \mu_e \) is 1250 \( \text{cm}^2/\text{V s} \) and the charge of an electron is \( 1.6 \times 10^{-19} \text{ Coulomb} \), the donor impurity concentration \( N_D \) in the sample is

(A) 2.1 \( \times 10^{16} \text{ cm}^{-3} \)  \hspace{1cm} (B) 1 \( \times 10^{16} \text{ cm}^{-3} \)  \hspace{1cm} (C) 2.5 \( \times 10^{15} \text{ cm}^{-3} \)  \hspace{1cm} (D) 5 \( \times 10^{15} \text{ cm}^{-3} \)

40. Consider an abrupt p–n junction. Let \( V_{bi} \) be the built-in potential of this junction and \( V_R \) be the applied reverse bias. If the junction capacitance \( C_j \) is 1 pF for \( V_R = 1 \text{ V} \), then for \( V_R = 4 \text{ V} \), \( C_j \) will be

(A) 4 pF  \hspace{1cm} (B) 2 pF  \hspace{1cm} (C) 0.25 pF  \hspace{1cm} (D) 0.5 pF

41. Consider the following statements S1 and S2.

S1 : The threshold voltage \( V_T \) of a MOS capacitor decreases with increase in gate oxide thickness.

S2 : The threshold voltage \( V_T \) of a MOS capacitor decreases with increase in substrate doping concentration.

Which one of the following is correct ?

(A) S1 is FALSE and S2 is TRUE  \hspace{1cm} (B) Both S1 and S2 are TRUE  \hspace{1cm} (C) S1 is TRUE and S2 is FALSE  \hspace{1cm} (D) Both S1 and S2 are FALSE
42. The drain of an n-channel MOSFET is shorted to the gate so that \( V_{GS} = V_{DS} \). The threshold voltage \( (V_T) \) of the MOSFET is 1 V. If the drain current \( (I_D) \) is 1 mA for \( V_{GS} = 2 \text{ V} \), then for \( V_{GS} = 3 \text{ V} \), \( I_D \) is
   (A) 2 mA  
   (B) 3 mA  
   (C) 9 mA  
   (D) 4 mA

43. The longest wavelength that can be absorbed by silicon, which has the bandgap of 1.12 eV, is 1.1 \( \mu \text{m} \). If the longest wavelength that can be absorbed by another material is 0.87 \( \mu \text{m} \), then the bandgap of this material is
   (A) 1.416 eV  
   (B) 0.886 eV  
   (C) 0.854 eV  
   (D) 0.706 eV

44. The neutral base width of a bipolar transistor, biased in the active region, is 0.5 \( \mu \text{m} \). The maximum electron concentration and the diffusion constant in the base are \( 10^{14} \text{ cm}^{-3} \) and \( D_n = 25 \text{ cm}^2/\text{sec} \) respectively. Assuming negligible recombination in the base, the collector current density is \( (1.6 \times 10^{-19} \text{ Coulomb}) \)
   (A) 800 A/cm\(^2\)  
   (B) 9 A/cm\(^2\)  
   (C) 200 A/cm\(^2\)  
   (D) 2 A/cm\(^2\)

45. Assume that the \( \beta \) of the transistor is extremely large and \( V_{BE} = 0.7 \text{ V} \), \( I_c \) and \( V_{CE} \) in the circuit shown in Fig. Q.45 are
   \[ \begin{align*}
   I_c &= 1 \text{ mA}, \\
   V_{CE} &= 4.7 \text{ V} \\
   I_c &= 0.5 \text{ mA}, \\
   V_{CE} &= 3.75 \text{ V} \\
   I_c &= 1 \text{ mA}, \\
   V_{CE} &= 2.5 \text{ V} \\
   I_c &= 0.5 \text{ mA}, \\
   V_{CE} &= 3.9 \text{ V}
   \end{align*} \]

46. A bipolar transistor is operating in the active region with a collector current of 1 mA. Assuming that the \( \beta \) of the transistor is 100 and the thermal voltage \( (V_T) \) is 25 mV, the transconductance \( (g_m) \) and the input resistance \( (r_i) \) of the transistor in the common emitter configuration, are
   (A) \( g_m = 25 \text{ mA/V} \) and \( r_i = 15.625 \text{ k}\Omega \)  
   (B) \( g_m = 40 \text{ mA/V} \) and \( r_i = 4.0 \text{ k}\Omega \)  
   (C) \( g_m = 25 \text{ mA/V} \) and \( r_i = 2.5 \text{ k}\Omega \)  
   (D) \( g_m = 40 \text{ mA/V} \) and \( r_i = 2.5 \text{ k}\Omega \)

47. The value of \( C \) required for sinusoidal oscillations of frequency 1 kHz in the circuit of Fig. Q.47 is
   \[ (A) \frac{1}{2\pi} \mu\text{F} \quad (B) 2\pi \mu\text{F} \quad (C) \frac{1}{2\pi\sqrt{6}} \mu\text{F} \quad (D) 2\pi\sqrt{6} \mu\text{F} \]

48. In the op-amp circuit given in Fig. Q.48, the load current \( i_L \) is
   \[ \begin{align*}
   (A) &-\frac{v_o}{R_2} \\
   (B) &\frac{v_o}{R_2} \\
   (C) &-\frac{v_o}{R_L} \\
   (D) &\frac{v_o}{R_1}
   \end{align*} \]
57. Consider the sequence of 8085 instructions given below:
LXI H, 9258
MOV A, M
CMA
MOV M, A

Which one of the following is performed by this sequence?
(A) Contents of location 9258 are moved to the accumulator
(B) Contents of location 9258 are compared with the contents of the accumulator
(C) Contents of location 8529 are complemented and stored in location 8529
(D) Contents of location 5892 are complemented and stored in location 5892

58. A Boolean function \( f \) of two variables \( x \) and \( y \) is defined as follows:
\[
f(0,0) = f(0,1) = f(1,1) = 1; \quad f(1,0) = 0
\]
Assuming complements of \( x \) and \( y \) are not available, a minimum cost solution for realizing \( f \) using only 2-input NOR gates and 2-input OR gates (each having unit cost) would have a total cost of
(A) 1 unit
(B) 4 units
(C) 3 units
(D) 2 units

59. It is desired to multiply the numbers 0AH by 0BH and store the result in the accumulator. The numbers are available in registers B and C respectively. A part of the 8085 program for this purpose is given below:

```
MVI A, 00H
LOOP: ~~~~~~~~~~~~~~~~~
       ~~~~~~~~~~~~~~~
       ~~~~~~~~~~~~~~~
       ~~~~~~~~~~~~~~~
       ~~~~~~~~~~~~~~~
      HLT
      END
```

The sequence of instructions to complete the program would be
(A) JNZ LOOP, ADD B, DCR C
(B) ADD B, JNZ LOOP, DCR C
(C) DCR C, JNZ LOOP, ADD B
(D) ADD B, DCR C, JNZ LOOP

60. A 1 kHz sinusoidal signal is ideally sampled at 1500 samples/sec and the sampled signal is passed through an ideal low-pass filter with cut-off frequency 800 Hz. The output signal has the frequency
(A) zero Hz
(B) 0.75 kHz
(C) 0.5 kHz
(D) 0.25 kHz

61. A rectangular pulse train \( s(t) \) as shown in Fig. Q.61 is convolved with the signal \( \cos^2(4\pi \times 10^4 t) \). The convolved signal will be a
\[
\text{Fig Q.61}
\]

(A) DC
(B) 12 kHz sinusoid
(C) 8 kHz sinusoid
(D) 14 kHz sinusoid

62. Consider the sequence
\[
x_n[j] = [\text{4, 1, 2, 5}]
\]
The conjugate anti-symmetric part of the sequence is
(A) \([4, 1, 2, 5]^\star\]
(B) \([1, 2, 5, 4]^\star\]
(C) \([2, 5, 1, 4]^\star\]
(D) \([5, 2, 4, 1]^\star\]

63. A causal LTI system is described by the difference equation
\[
2y[n] = \alpha y[n-2] - 2x[n] + \beta x[n-1]
\]
The system is stable only if
(A) \(|\alpha| < 2, \quad |\beta| < 2\]
(B) \(|\alpha| > 2, \quad |\beta| > 2\]
(C) \(|\alpha| < 2, \quad \text{any value of } \beta\]
(D) \(|\beta| < 2, \quad \text{any value of } \alpha\]

64. A causal system having the transfer function
\[
H(s) = \frac{1}{s + 2}
\]
is excited with \(10u(t)\). The time at which the output reaches 99% of its steady state value is
(A) 2.7 sec
(B) 2.5 sec
(C) 2.3 sec
(D) 2.1 sec
The impulse response \( h[n] \) of a linear time invariant system is given as

\[
\begin{cases}
-2\sqrt{2} & n = 1, -1 \\
4\sqrt{2} & n = 2, -2 \\
0 & \text{otherwise}
\end{cases}
\]

If the input to the above system is the sequence \( e^{jn/4} \), then the output is

(A) \( 4\sqrt{2}e^{jn/4} \)  
(B) \( 4\sqrt{2}e^{-jn/4} \)  
(C) \( 4e^{jn/4} \)  
(D) \( -4e^{jn/4} \)

Let \( x(t) \) and \( y(t) \) with Fourier transforms \( F(f) \) and \( Y(f) \) respectively be related as shown in Fig. Q.66. Then \( Y(f) \) is

(A) \( -\frac{1}{2}X(f/2)e^{-jt/2} \)  
(B) \( -\frac{1}{2}X(f/2)e^{jt/2} \)  
(C) \( -X(f/2)e^{jt/2} \)  
(D) \( -X(f/2)e^{-jt/2} \)

A system has poles at 0.01 Hz, 1 Hz and 80 Hz; zeros at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

(A) -90°  
(B) 0°  
(C) 90°  
(D) -180°

Consider the signal flow graph shown in Fig. Q.68. The gain \( x_4 \) is

(A) \( \frac{1-(be+cf+dg)}{abcd} \)  
(B) \( \frac{bedg}{1-(be+cf+dg)} \)  
(C) \( \frac{abcd}{1-(be+cf+dg)+bedg} \)  
(D) \( \frac{1-(be+cf+dg)+bedg}{abcd} \)

If \( A = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix} \) then \( \sin At \) is

(A) \( \begin{bmatrix} \sin(-4t) + 2\sin(-t) & -\sin(-4t) + 2\sin(-t) \\ -\sin(-4t) + \sin(-t) & 2\sin(-4t) + \sin(-t) \end{bmatrix} \)  
(B) \( \begin{bmatrix} \sin(-2t) & \sin(2t) \\ \sin(t) & \sin(-3t) \end{bmatrix} \)  
(C) \( \begin{bmatrix} \sin(4t) + 2\sin(t) & 2\sin(-4t) - 2\sin(-t) \\ -\sin(-4t) + \sin(t) & 2\sin(4t) + \sin(t) \end{bmatrix} \)  
(D) \( \begin{bmatrix} \cos(-t) + 2\cos(t) & 2\cos(-4t) - 2\sin(-t) \\ -\cos(-4t) + \sin(t) & -2\cos(4t) + \cos(t) \end{bmatrix} \)

The open-loop transfer function of a unity feedback system is

\[
G(s) = \frac{K}{s(s^2 + s + 2)(s + 3)}
\]

The range of \( K \) for which the system is stable is

(A) \( \frac{21}{4} \)  
(B) 13 > \( K \) > 0  
(C) \( \frac{21}{4} \) < \( K \) < \( \infty \)  
(D) -6 < \( K \) < \( \infty \)

For the polynomial

\[
P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15
\]

the number of roots which lie in the right half of the \( s \)-plane is

(A) 4  
(B) 2  
(C) 3  
(D) 1

The state variable equations of a system are:

\[
\dot{x}_1 = -3x_1 - x_2 + u, \quad \dot{x}_2 = 2x_1, \quad y = x_1 + u
\]

The system is

(A) controllable but not observable  
(B) observable but not controllable  
(C) neither controllable nor observable  
(D) controllable and observable

Given \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), the state transition matrix \( e^{At} \) is given by

(A) \( \begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 0 & e^t \\ e^t & 0 \end{bmatrix} \)  
(C) \( \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix} \)  
(D) \( \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix} \)

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74. Consider the signal $x(t)$ shown in Fig. Q.74. Let $h(t)$ denote the impulse response of the filter matched to $x(t)$, with $h(t)$ being non-zero only in the interval 0 to 4 sec. The slope of $h(t)$ in the interval $3 < t < 4$ sec is

(A) $\frac{1}{2}$ sec\(^{-1}\) 
(B) $-1$ sec\(^{-1}\) 
(C) $-\frac{1}{2}$ sec\(^{-1}\) 
(D) 1 sec\(^{-1}\)

75. A 1 mW video signal having a bandwidth of 100 MHz is transmitted to a receiver through a cable that has 40 dB loss. If the effective one-sided noise spectral density at the receiver is $10^{-20}$ Watt/Hz, then the signal-to-noise ratio at the receiver is

(A) 50 dB 
(B) 30 dB 
(C) 40 dB 
(D) 60 dB

76. A 100 MHz carrier of 1V amplitude and a 1 MHz modulating signal of 1V amplitude are fed to a balanced modulator. The output of the modulator is passed through an ideal high-pass filter with cut-off frequency of 100 MHz. The output of the filter is added with 100 MHz signal of 1V amplitude and 90° phase shift as shown in Fig. Q.76. The envelope of the resultant signal is

(A) constant 
(B) $\sqrt{1 + \sin(2\pi \times 10^6 t)}$ 
(C) $\sqrt{5/4 - \sin(2\pi \times 10^6 t)}$ 
(D) $\sqrt{5/4 + \cos(2\pi \times 10^6 t)}$

77. Two sinusoidal signals of same amplitude and frequencies 10 kHz and 10.1 kHz are added together. The combined signal is given to an ideal frequency detector. The output of the detector is

(A) 0.1 kHz sinusoid 
(B) 20.1 kHz sinusoid 
(C) a linear function of time 
(D) a constant

78. Consider a binary digital communication system with equally likely 0's and 1's. When binary 0 is transmitted the voltage at the detector input can lie between the levels -0.25 V and +0.25 V with equal probability; when binary 1 is transmitted, the voltage at the detector can have any value between 0 and 1 V with equal probability. If the detector has a threshold of 0.2 V (i.e. if the received signal is greater than 0.2 V, the bit is taken as 1), the average bit error probability is

(A) 0.15 
(B) 0.2 
(C) 0.05 
(D) 0.5

79. A random variable $X$ with uniform density in the interval 0 to 1 is quantized as follows:

if $0 \leq X \leq 0.3$, $x_q = 0$
if $0.3 \leq X \leq 1$, $x_q = 0.7$

where $x_q$ is the quantized value of $X$. The root-mean square value of the quantization noise is

(A) 0.573 
(B) 0.198 
(C) 2.205 
(D) 0.266

80. Choose the correct one from among the alternatives A, B, C, D after matching an item from Group 1 with the most appropriate item in Group 2.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 : FM</td>
<td>P : Slope overload</td>
</tr>
<tr>
<td>2 : DM</td>
<td>Q : $\mu$-law</td>
</tr>
<tr>
<td>3 : PSK</td>
<td>R : Envelope detector</td>
</tr>
<tr>
<td>4 : PCM</td>
<td>S : Capture effect</td>
</tr>
<tr>
<td></td>
<td>T : Hilbert transfer</td>
</tr>
<tr>
<td></td>
<td>U : Matched filter</td>
</tr>
</tbody>
</table>

(A) 1–T 1–S 1–S 1–U 
(B) 2–P 2–U 2–P 2–R 
(C) 3–U 3–P 3–U 3–S 
(D) 4–S 4–T 4–Q 4–Q

81. Three analog signals, having bandwidth 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. The bit rate for the multiplexed signal is

(A) 1, 15.2 kbps 
(B) 28.8 kbps 
(C) 27.6 kbps 
(D) 38.4 kbps
Consider a system shown in Fig. Q.82. Let $X(f)$ and $Y(f)$ denote the Fourier transforms of $x(t)$ and $y(t)$ respectively. The ideal HPF has the cutoff frequency 10 kHz.

The positive frequencies where $Y(f)$ has spectral peaks are

(A) 1 kHz and 24 kHz  
(B) 2 kHz and 24 kHz  
(C) 1 kHz and 14 kHz  
(D) 2 kHz and 14 kHz

A parallel plate air-filled capacitor has plate area of $10^{-4} \text{m}^2$ and plate separation of $10^{-3} \text{m}$. It is connected to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is $(\varepsilon_r = 1/36 \pi \times 10^{-9} \text{F/m})$

(A) 10 mA  
(B) 100 mA  
(C) 10 A  
(D) 1.59 mA

A source produces binary data at the rate of 10 kbps. The binary symbols are represented as shown in Fig.Q.84

The source output is transmitted using two modulation schemes, namely Binary PSK (BPSK) and Quadrature PSK (QPSK). Let $B_1$ and $B_2$ be the bandwidth requirements of BPSK respectively. Assuming that the bandwidth of the above rectangular pulses is 10 kHz, $B_1$ and $B_2$ are

(A) $B_1 = 20$ kHz, $B_2 = k$ kHz  
(B) $B_1 = 10$ kHz, $B_2 = 10$ kHz

Consider a 300 $\Omega$, quarter-wave long (at 1 GHz) transmission line as shown in Fig. Q.85. It is connected to a 10 V, 5$\Omega$ source at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is

(A) 10 V  
(B) 5 V  
(C) 60 V  
(D) 60/7 V

In a microwave test bench, why is the microwave signal amplitude modulated at 1 kHz ?

(A) To increase the sensitivity of measurement  
(B) To transmit the signal to a far-off place  
(C) To study amplitude modulation  
(D) Because crystal detector fails at microwave frequencies

If $\tilde{E} = (\hat{a}_x + j\hat{a}_y) e^{jkx - j\omega t}$ and $\tilde{H} = (k/\omega \mu)(\hat{a}_y + j\hat{a}_x) e^{jkx - j\omega t}$, the time-averaged Poynting vector is

(A) null vector  
(B) $(k/\omega \mu)\hat{a}_x$  
(C) $(2k/\omega \mu)\hat{a}_x$  
(D) $(k/2\omega \mu)\hat{a}_x$

Consider an impedance $Z = R + jX$ marked with point P in an impedance Smith chart as shown in Fig. Q.88. The movement from point P along a constant resistance circle in the clockwise direction by an angle $45^\circ$ is equivalent to

\[ Z = R + jX \]
Question 1- 30 Carry one Mark each.

1. The following differential equation has
\[ 3 \frac{d^2y}{dt^2} + 4 \left( \frac{dy}{dt} \right)^3 + y^2 + 2 = x \]
(A) degree = 2, order = 1
(B) degree = 3, order = 2
(C) degree = 4, order = 3
(D) degree = 2, order = 3

2. Choose the function \( f(t) \); \(-\infty < t < \infty \) for which a Fourier series cannot be defined.
(A) \( 3\sin(25t) \)
(B) \( 4\cos(20t + 3) + 2\sin(710t) \)
(C) \( e^{-|t|}\sin(25t) \)
(D) \( 1 \)

3. A fair dice is rolled twice. The probability that an odd number will follow on even number is
(A) \( \frac{1}{2} \)
(B) \( \frac{1}{6} \)
(C) \( \frac{1}{3} \)
(D) \( \frac{1}{4} \)

4. A solution of the following differential equation is given by
\[ \frac{d^2y}{dt^2} - 5 \frac{dy}{dt} + 6y = 0 \]
(A) \( y = e^{2x} + e^{-3x} \)
(B) \( y = e^{2x} + e^{3x} \)
(C) \( y = e^{-2x} + e^{-3x} \)
(D) \( y = e^{-2x} + e^{3x} \)

5. The function \( x(t) \) is shown in the figure. Even and odd parts of a unit step function \( u(t) \) are respectively,

(A) \( \frac{1}{2} \), \( \frac{1}{2} x(t) \)
(B) \( -\frac{1}{2} \), \( -\frac{1}{2} x(t) \)
(C) \( \frac{1}{2} \), \( -\frac{1}{2} x(t) \)
(D) \( -\frac{1}{2} \), \( -\frac{1}{2} x(t) \)

6. The region of convergence of \( z \) - transform of the sequence \( \left\{ \frac{5}{6} \right\} u(n) - \left\{ \frac{6}{5} \right\} u(-n - 1) \) must be
(A) \( |z| < \frac{5}{6} \)
(B) \( |z| > \frac{5}{6} \)
(C) \( \frac{5}{6} < |z| < \frac{6}{5} \)
(D) \( \frac{6}{5} < |z| < \infty \)

7. The condition on \( R, L \) and \( C \) such that the step response \( y(t) \) in the figure has no oscillations, is

(A) \( R \geq 2 \sqrt{LC} \)
(B) \( R > \frac{L}{\sqrt{C}} \)
(C) \( R \geq \frac{L}{\sqrt{C}} \)
(D) \( R = \frac{1}{\sqrt{LC}} \)
3. The $ABCD$ parameters of an ideal $n:1$ transformer shown in the figure are $\begin{bmatrix} n & 0 \\ 0 & X \end{bmatrix}$. The value of $x$ will be

(A) $n$  
(B) $\frac{1}{n}$  
(C) $n^2$  
(D) $\frac{1}{n^2}$

4. In a series $RLC$ circuit, $R = 2 \, k\Omega$, $L = 1 \, H$ and $C = \frac{1}{400} \, \mu$. The resonant frequency is

(A) $2 \times 10^4 \, Hz$  
(B) $\frac{1}{\pi} \times 10^4 \, Hz$  
(C) $10^4 \, Hz$  
(D) $2\pi \times 10^4 \, Hz$

10. The maximum power that can be transferred to the load resistor $R_L$ from the voltage source in the figure is

(A) 1 W  
(B) 10 W  
(C) 0.25 W  
(D) 0.5 W

11. The bandgap of Silicon at room temperature is

(A) 1.3 eV  
(B) 0.7 eV  
(C) 1.1 eV  
(D) 1.4 eV

12. A Silicon PN junction at a temperature of $20^\circ C$ has a reverse saturation current of 10 pico - Amperes (pA). The reserve saturation current at $40^\circ C$ for the same bias is approximately

(A) 30 pA  
(B) 40 pA  
(C) 50 pA  
(D) 60 pA

13. The primary reason for the widespread use of Silicon in semiconductor device technology is

(A) abundance of Silicon on the surface of the Earth.  
(B) larger bandgap of Silicon in comparison to Germanium.  
(C) favorable properties of Silicon - dioxide (SiO2)  
(D) lower melting point.

14. The effect of current shunt feedback in an amplifier is to

(A) increase the input resistance and decrease the output resistance.  
(B) increase both input and output resistance  
(C) decrease both input and output resistance.  
(D) decrease the input resistance and increase the output resistance.

15. The input resistance of the amplifier shown in the figure is

(A) $\frac{30}{4} \, k\Omega$  
(B) 10 k$\Omega$  
(C) 40 k$\Omega$  
(D) infinite

16. The first and the last critical frequency of an $RC$ - driving point impedance function must respectively be

(A) a zero and a pole  
(B) a zero and a zero  
(C) a pole and a pole  
(D) a pole and a zero

17. The cascode amplifier is a multistage configuration of

(A) CC - CB  
(B) CE - CB  
(C) CB - CC  
(D) CE - CC

18. Decimal 43 in Hexadecimal and BCD number system is respectively

(A) B2, 0100 011  
(B) 2B, 0100 0011  
(C) 2B, 0011 0100  
(D) B2, 0100 0100
19. The Boolean function \( f \) implemented in the figure using two input multiplexes is

\[ f = \overline{A} \overline{B} \overline{C} + A \overline{B} \overline{C} \]

(A) \( A \overline{B} \overline{C} + A \overline{B} \overline{C} \)  
(B) \( A \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \)  
(C) \( \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \)  
(D) \( \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} \)

20. Which of the following can be impulse response of a causal system?

(A) \( h(t) \)  
(B) \( h(t) \)  
(C) \( h(t) \)  
(D) \( h(t) \)

21. Let \( x(n) = \left( \frac{1}{2} \right)^n u(n) \), \( y(n) = x^2(n) \) and \( Y(e^{i\theta}) \) be the Fourier transform of \( y(n) \) then \( Y(e^{i\theta}) \) is

(A) \( \frac{1}{4} \)  
(B) \( 2 \)  
(C) \( 4 \)  
(D) \( \frac{4}{3} \)

22. Find the correct match between group 1 and group 2

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. ( 1 + km(t) ) ( A \sin(\omega t) )</td>
<td>W. Phase Modulation</td>
</tr>
<tr>
<td>Q. ( km(t) A \sin(\omega t) )</td>
<td>X. Frequency Modulation</td>
</tr>
<tr>
<td>R. ( A \sin(\omega t + km(t)) )</td>
<td>Y. Amplitude Modulation</td>
</tr>
<tr>
<td>(A) P-Z, Q-Y, R-X, S-W</td>
<td>(B) P-W, Q-X, R-Y, S-Z</td>
</tr>
<tr>
<td>(C) P-X, Q-W, R-Z, S-Y</td>
<td>(D) P-Y, Q-Z, R-W, S-X</td>
</tr>
</tbody>
</table>

23. The power in the signal \( s(t) = 8 \cos \left( 20 \pi \frac{t}{2} \right) + 4 \sin(15\pi t) \) is

(A) 40  
(B) 41  
(C) 42  
(D) 82

24. Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel bandwidth?

(A) VSB  
(B) DSB - SC  
(C) SSB  
(D) AM

25. A linear system is equivalently represented by two sets of state equations:

\[ \dot{X} = AX + BU \quad \dot{W} = CW + DU \]

The eigenvalues of the representations are also computed as \([\lambda] \) and \([\mu] \). Which one of the following statements is true?

(A) \([\lambda] = [\mu] \) and \( X \neq W \)  
(B) \([\lambda] = [\mu] \) and \( X \neq W \)  
(C) \([\lambda] \neq [\mu] \) and \( X = W \)  
(D) \([\lambda] \neq [\mu] \) and \( X \neq W \)

26. Which one of the following polar diagrams corresponds to a lag network?

(A)  
(B)  
(C)  
(D)  

27. Despite the presence of negative feedback, control systems still have problems of instability because the

(A) Components used have non-linearities  
(B) Dynamic equations of the subsystem are not known exactly.  
(C) Mathematical analysis involves approximations.  
(D) System has large negative phase angle at high frequencies.
37. Given an orthogonal matrix

\[
A = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 \\
\end{bmatrix}
\]

\[\text{[AA}^T\text{]}^{-1} \text{ is}
\]

(A) \[
\begin{bmatrix}
\frac{1}{4} & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{1}{4} \\
\end{bmatrix}
\]

(B) \[
\begin{bmatrix}
\frac{1}{4} & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{1}{4} \\
\end{bmatrix}
\]

(C) \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(D) \[
\begin{bmatrix}
\frac{1}{4} & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 0 \\
0 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{1}{4} \\
\end{bmatrix}
\]

38. For the circuit shown in the figure, the instantaneous current \(i(t)\) is

\[
\begin{align*}
5 \text{ 0}^{\circ} \text{A} & \quad \text{Fig. Q38} \\
\text{(A)} & \quad \frac{10\sqrt{3}}{2} \angle 90^\circ \text{ Amps.} \\
\text{(B)} & \quad \frac{10\sqrt{3}}{2} \angle -90^\circ \text{ Amps.} \\
\text{(C)} & \quad 5 \angle 60^\circ \text{ Amps} \\
\text{(D)} & \quad 5 \angle -60^\circ \text{ Amps}
\end{align*}
\]

39. Impedance \(Z\) as shown in the given figure is

\[
\text{(A)} \quad j29 \Omega \\
\text{(B)} \quad j9 \Omega \\
\text{(C)} \quad j19 \Omega \\
\text{(D)} \quad j39 \Omega
\]

40. For the circuit shown in figure, Thevenin’s voltage and Thevenin’s equivalent resistance at terminals a - b is

\[
\text{(A)} \quad 5 \text{ V and 2 } \Omega \\
\text{(B)} \quad 7.5 \text{ V and 2.5 } \Omega \\
\text{(C)} \quad 4 \text{ V and 2 } \Omega \\
\text{(D)} \quad 3 \text{ V and 2.5 } \Omega
\]

41. If \(R_1 = R_2 = R_3 = R\) and \(R_4 = 1.1R\) in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is

\[
\text{(A)} \quad 0.238 \text{ V} \\
\text{(B)} \quad 0.138 \text{ V} \\
\text{(C)} \quad -0.238 \text{ V} \\
\text{(D)} \quad 1 \text{ V.}
\]

42. The \(h\) parameters of the circuit shown in the figure are

\[
\text{(A)} \quad \begin{bmatrix}
0.1 & 1 \text{.} \\
-0.1 & 0.3 \text{.}
\end{bmatrix} \\
\text{(B)} \quad \begin{bmatrix}
10 & -1 \text{.} \\
1 & 0.05 \text{.}
\end{bmatrix} \\
\text{(C)} \quad \begin{bmatrix}
30 & 20 \text{.} \\
20 & 20 \text{.}
\end{bmatrix} \\
\text{(D)} \quad \begin{bmatrix}
10 & 1 \text{.} \\
-1 & 0.05 \text{.}
\end{bmatrix}
\]

43. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage \(V_o\) at time \(t = 2\) sec is

\[
\text{(A)} \quad 3 \text{ V} \\
\text{(B)} \quad -3 \text{ V} \\
\text{(C)} \quad 4 \text{ V} \\
\text{(D)} \quad -4 \text{ V}
\]

44. A Silicon sample A is doped with \(10^{18}\) atoms/cm\(^3\) of boron. Another sample b of identical dimension is doped with \(10^{18}\) atoms/cm\(^3\) phosphorus. The ratio of electron to...
hole mobility is 3. The ratio of conductivity of the sample A to B is

(A) 3  
(B) \( \frac{1}{3} \)  
(C) \( \frac{2}{3} \)  
(D) \( \frac{3}{2} \)

45. A Silicon PN junction diode under reverse bias has depletion region of width 10 \( \mu \)m. The relative permittivity of Silicon, \( \varepsilon_r = 11.7 \) and the preemptively of free space \( \varepsilon_0 = 8.85 \times 10^{-12} \) F/m. The depletion capacitance of the diode per square meter is

(A) 100 \( \mu \)F  
(B) 10 \( \mu \)F  
(C) 1 \( \mu \)F  
(D) 20 \( \mu \)F

46. For an npn transistor connected as shown in figure \( V_{BE} = 0.7 \) volts. Given that reverse saturation current of the junction at room temperature 300 K is \( 10^{-12} \) A, the emitter current is

(A) 30 mA  
(B) 39 mA  
(C) 49 mA  
(D) 20 mA

47. The voltage \( e_o \) is indicated in the figure has been measured by an ideal voltmeter. Which of the following can be calculated ?

(A) Bias current of the inverting input only  
(B) Bias current of the inverting and non-inverting inputs only  
(C) Input offset current only  
(D) Both the bias currents and the input offset current.

48. The OP-amp circuit shown in the figure is filter. The type of filter and its cut off frequency are respectively.

(A) high pass, 1000 rad/sec.  
(B) Low pass, 1000 rad/sec.  
(C) high pass, 1000 rad/sec.  
(D) low pass, 10000 rad/sec.

49. In an ideal differential amplifier shown in the figure, a large value of \( R_E \)

(A) increase both the differential and common - mode gains  
(B) increases the common mode gain only  
(C) decreases the differential mode gain only  
(D) decreases the common mode gain only.

50. For an n-channel MOSFET and its transfer curve shown in the figure, the threshold voltage is

(A) 1 V and the device is in active region  
(B) -1 V and the device is in saturation region  
(C) 1 V and the device is in saturation region  
(D) -1 V and the device is in active region.
57. The given figure shows a ripple counter using positive edge triggered flip-flops. If the present state of the counter is $Q_2Q_1Q_0 = 001$ then its next state $Q_2Q_1Q_0$ will be

(A) 010  (D) 100
(B) 111  (D) 101

58. What memory address range is NOT represents by chip # 1 and chip # 2 in the figure $A_0$ to $A_{16}$ in this figure are the address lines and CS means chip select.

(A) 0100 - 02FF.  (B) 1500 - 16FF
(C) F900-FAFF   (D) F800 - F9FF

59. The output $y(t)$ of a linear time invariant system is related to its input $x(t)$ by the following equation

$$y(t) = 0.5x(t - t_d + T) + x(t - t_0) + 0.5x(t - t_d + T)$$

The filter transfer function $H(\omega)$ of such a system is given by

(A) $(1 + \cos \omega T)e^{-j\omega t_d}$  (B) $(1 + 0.5 \cos \omega T)e^{-j\omega t_d}$
(C) $(1 - \cos \omega T)e^{-j\omega t_d}$  (D) $(1 - 0.5 \cos \omega T)e^{-j\omega t_d}$

60. Match the following and choose the correct combination.

**Group 1**
E. Continuous and periodic signal
F. Continuous and periodic signal
G. Discrete and aperiodic signal
H. Discrete and periodic signal

**Group 2**
1. Fourier representation is continuous and aperiodic
2. Fourier representation is discrete and aperiodic
3. Fourier representation is continuous
4. Fourier representation is discrete and periodic

(A) E-3, F-2, G-4, H-1  (A) E-1, F-3, G-2, H-4
(C) E-1, F-2, G-3, H-4  (D) E-2, F-1, G-4, H-3

61. A signal $x(n) = \sin(\omega_0 n + \phi)$ is the input to a linear time-invariant system having a frequency response $H(e^{j\omega})$. If the output of the system $Ax(n - n_0)$ then the most general form of will be

(A) $-n_0 w_0 + \beta$ for any arbitrary real
(B) $-n_0 w_0 + 2\pi k$ for any arbitrary integer $k$
(C) $n_0 w_0 + 2\pi k$ for any arbitrary integer $k$
(D) $-n_0 w_0 \phi$

62. For a signal the Fourier transform is $X(f)$. Then the inverse Fourier transform of $X(3f + 2)$ is given by

(A) $\frac{1}{2} e^{j3\pi d}
(B) \frac{1}{3} x \left( t \frac{3}{2} \right) e^{j4\pi t}$
(C) $3x(3t)e^{-j4\pi t}$  (D) $x(3t + 2)$

63. The polar diagram of a conditionally stable system for open loop gain $K = 1$ is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for
66. A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively
(A) 1 and 20  
(B) 0 and 20  
(C) 0 and $\frac{1}{20}$  
(D) 1 and $\frac{1}{20}$

67. A MOS capacitor made using P type substrate is in the accumulation mode. The dominant charge in the channel is due to the presence of
(A) holes  
(B) electrons  
(C) positively charged ions  
(D) negatively charged ions

68. A device with input $x(t)$ and output $y(t)$ is characterized $y(t) = x^2(t)$. An FM signal with frequency deviation of 90 kHz and modulating signal bandwidth of 5 kHz is applied to this device. The bandwidth of the output signal is
(A) 370 kHz  
(B) 190 kHz  
(C) 380 kHz  
(D) 95 kHz
COMMON DATA QUESTION 78, 79, 80:
Given, \( r_d = 20 \, \text{k}\Omega \), \( I_{\text{DS}} = 10 \, \text{mA} \), \( V_p = -8 \, \text{V} \)

\[ 20 \, \text{V} \]
\[ 2 \, \text{k}\Omega \]
\[ v_i \]
\[ v_o \]
\[ 2 \, \text{M}\Omega \]
\[ 2 \, \text{V} \]
\[ Z_i \]
\[ Z_o \]

**Fig. Q78**

**78.** \( Z_i \) and \( Z_o \) of the circuit are respectively
(A) 2 M\( \Omega \) and 2 k\( \Omega \) (B) 2 M\( \Omega \) and \( \frac{20}{11} \) k\( \Omega \)
(C) \( \infty \) and 2 k\( \Omega \) (D) \( \infty \) and \( \frac{20}{11} \) k\( \Omega \)

**79.** \( I_D \) and \( V_{DS} \) under DC conditions are respectively
(A) 5.625 mA and 8.75 V (B) 7.500 mA and 5.00 V
(C) 4.500 mA and 11.00 V (D) 6.250 mA and 7.50 V

**80.** Transconductance in milli-Siemens (mS) and voltage gain of the amplifier are respectively
(A) 1.875 mS and 3.41 (B) 1.875 mS and -3.41
(C) 3.3 mS and -6 (D) 3.3 mS and 6

Linked Answer Questions : Q.81a to 85b Carry Two Marks Each

**Statement For Linked Answer Questions 81a and 81b:**

Consider an 8085 microprocessor system.

**81a.** The following program starts at location 0100H.
LXI SP, OOFF
LXI H, 0701
MVI A, 20H
SUB M

The content of accumulator when the program counter reaches 0109 H is
(A) 20 H (B) 02 H
(C) 00 H (D) FF H

**81b.** If in addition following code exists from 019H onwards,
ORI 40 H
ADD M

What will be the result in the accumulator after the last instruction is executed?
(A) 40 H (B) 20 H
(C) 60 H (D) 42 H

**Statement for Linked Answer Question 82a and 82b:**

The dopen loop transfer function of a unity feedback system is given by

\[ G(s) = \frac{20}{s^2 + 2s + 1} \]

**82a.** The gain and phase crossover frequencies in rad/sec are, respectively
(A) 0.632 and 1.26 (B) 0.632 and 0.485
(C) 0.485 and 0.632 (D) 1.26 and 0.632

**82b.** Based on the above results, the gain and phase margins of the system will be
(A) −7.09 dB and 87.5° (B) 7.09 dB and 87.5°
(C) 7.09 dB and −87.5° (D) −7.09 and −87.5°

**Statement for linked answer question 83a and 83b:**

Asymmetric three-level midtread quantizer is to be designed assuming equiprobable occurrence of all quantization levels.

**83a.** If the probability density function is divided into three regions as shown in the figure, the value of \( a \) in the figure is
(A) \( \frac{1}{3} \) (B) \( \frac{2}{3} \)
(C) \( \frac{1}{2} \) (D) \( \frac{1}{4} \)

**83b.** The quantization noise power for the quantization region between \(-a\) and \(+a\) in the figure is
(A) \( \frac{1}{8} \) (B) \( \frac{1}{6} \)
(C) \( \frac{1}{8} \) (D) \( \frac{1}{16} \)
Statement of Linked Answer Questions 84a and 84b

Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 and a resistive load is shown in the figure.

84a. The value of the load resistance is
(A) 50 Ω  (B) 200 Ω  (C) 125 Ω  (D) 0

84b. The reflection coefficient is given by
(A) −0.6  (B) −1  (C) 0.6  (D) 0

Statement of Linked Answer Question 85a and 85b:

A sequence x(n) has non-zero values as shown in the figure.

85a. The sequence

\[ y[n] = \begin{cases} x \left( \frac{n}{2} - 1 \right) & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases} \]

will be

(A)  

(B) 

85b. The Fourier transform of y[2n] will be

(A) \( e^{-j\frac{\pi}{2}} [\cos 4\omega + 2\cos 2\omega + 2] \)
(B) \( [\cos 2\omega + 2\cos \omega + 2] \)
(C) \( e^{j\pi} [\cos 2\omega + 2\cos \omega + 2] \)
(D) \( e^{-j\pi} [\cos 2\omega + 2\cos \omega + 2] \)
CHAPTER

10.4

EC-06

Duration: Three Hours  Maximum Marks: 150

Q.1 to carry Q.20 one marks each and Q.21 to Q.85 carry two marks each.

1. The rank of the matrix
\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 0 \\
1 & 1 & 1
\end{bmatrix}
\]
is
(A) 0  (B) 1  (C) 2  (D) 3

2. \( \nabla \times \nabla \times \mathbf{P} \) where \( \mathbf{P} \) is a vector, is equal to
(A) \( \mathbf{P} \times \nabla \times \mathbf{P} - \nabla^2 \mathbf{P} \)
(B) \( \nabla^2 \mathbf{P} + \nabla(\nabla \cdot \mathbf{P}) \)
(C) \( \nabla^2 \mathbf{P} + \nabla \times \mathbf{P} \)
(D) \( \nabla(\nabla \cdot \mathbf{P}) - \nabla^2 \mathbf{P} \)

3. \( \int (\nabla \times \mathbf{P}) \cdot ds \) where \( \mathbf{P} \) is a vector, is equal to
(A) \( \int \mathbf{P} \cdot dl \)
(B) \( \int \nabla \times \mathbf{P} \cdot dl \)
(C) \( \int \nabla \times \mathbf{P} \cdot dl \)
(D) \( \int \nabla \cdot \mathbf{P} dv \)

4. A probability density function is of the form
\( p(x) = Ke^{-|x|}, x \in (-\infty, \infty) \)

The value of \( K \) is
(A) 0.5  (B) 1  (C) 0.5  (D) \( \alpha \)

5. A solution for the differential equation
\[ x(t) + 2x(t) = \delta(t) \]

With initial condition \( x(0-) = 0 \)

(A) \( e^{-2t}u(t) \)  (B) \( e^{2t}u(t) \)
(C) \( e^{-t}u(t) \)  (D) \( e^t u(t) \)

6. A low-pass filter having a frequency response \( H(\omega) = A(\omega) e^{i\phi(\omega)} \) does not produce any phase distortions if
(A) \( A(\omega) = C_1e^{\omega^2}, \phi(\omega) = k\omega \)
(B) \( A(\omega) = C_2e^{\omega^2}, \phi(\omega) = k\omega \)
(C) \( A(\omega) = C_3e^{\omega^2}, \phi(\omega) = k\omega^2 \)
(D) \( A(\omega) = C_4, \phi(\omega) = k\omega^{-1} \)

7. The values of voltage \( (V_D) \) across a tunnel-diode corresponding to peak and valley currents are \( V_p, V_D \).

The range of tunnel-diode voltage for \( V_p \) which the slope of its \( I-V_D \) characteristics is negative would be
(A) \( V_D < 0 \)  (B) \( 0 \leq V_D < V_p \)
(C) \( V_p < V_D < V_c \)  (D) \( V_D > V_c \)

8. The concentration of minority carriers in an extrinsic semiconductor under equilibrium is
(A) Directly proportional to the doping concentration
(B) Inversely proportional to the doping concentration
(C) Directly proportional to the intrinsic concentration
(D) Inversely proportional to the intrinsic concentration

9. Under low level injection assumption, the injected minority carrier current for an extrinsic semiconductor is essentially the
(A) Diffusion current  (B) Drift current
(C) Recombination current  (D) Induced current
10. The phenomenon known as “Early Effect” in a bipolar transistor refers to a reduction of the effective base-width caused by
(A) Electron – Hole recombination at the base 
(B) The reverse biasing of the base – collector junction 
(C) The forward biasing of emitter-base junction 
(D) The early removal of stored base charge during saturation-to-cut off switching

11. The input impedance \(Z_i\) and the output impedance \(Z_o\) of an ideal trans-conductance (voltage controlled current source) amplifier are
(A) \(Z_i = 0, Z_o = 0\) 
(B) \(Z_i = 0, Z_o = \infty\) 
(C) \(Z_i = \infty, Z_o = 0\) 
(D) \(Z_i = \infty, Z_o = \infty\)

12. An n-channel depletion MOSFET has following two points on its \(I_D-V_{GS}\) curve :
(i) \(V_{GS} = 0\) at \(I_P = 12\) mA and
(ii) \(V_{GS} = -6\) Volts at \(I_P = 0\) mA
Which of the following Q – point will give the highest trans – conductance gain for small signals?
(A) \(V_{GS} = -6\) Volts 
(B) \(V_{GS} = -3\) Volts 
(C) \(V_{GS} = 0\) Volts 
(D) \(V_{GS} = 3\) Volts

13. The number of product terms in the minimized sum-of-product expression obtained through the following K – map is (where, “d” denotes don’t care states)

<table>
<thead>
<tr>
<th>1</th>
<th>0</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>d</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>d</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

(A) 2 
(B) 3 
(C) 4 
(D) 5

14. Let \(x(t) \leftrightarrow X(j\omega)\) be Fourier Transform pair. The Fourier Transform of the signal \(x(5t-3)\) in terms of \(X(j\omega)\) is given as
(A) \(\frac{1}{5}e^{-j\omega \frac{3}{5}}X\left(j\omega \frac{5}{5}\right)\) 
(B) \(\frac{1}{5}e^{j\omega \frac{3}{5}}X\left(j\omega \frac{5}{5}\right)\) 
(C) \(\frac{1}{5}e^{-j\omega \frac{3}{5}}X\left(j\omega \frac{5}{5}\right)\) 
(D) \(\frac{1}{5}e^{j\omega \frac{3}{5}}X\left(j\omega \frac{5}{5}\right)\)

15. The Dirac delta function is defined as
(A) \(\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}\) 
(B) \(\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}\) 
(C) \(\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}\) and \(\int_{-\infty}^{\infty} \delta(t) dt = 1\)
(D) \(\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}\) and \(\int_{-\infty}^{\infty} \delta(t) dt = 1\)

16. If the region of convergence of \(x_1[n] + x_2[n]\) is \(\frac{1}{3} < |\sigma| < \frac{2}{3}\) then the region of convergence of \(x_1[n] - x_2[n]\) includes
(A) \(\frac{1}{3} < |\sigma| < 3\) 
(B) \(\frac{2}{3} < |\sigma| < 3\) 
(C) \(\frac{3}{2} < |\sigma| < 3\) 
(D) \(\frac{1}{3} < |\sigma| < \frac{2}{3}\)

17. The open-loop function of a unity-gain feedback control system is given by
\[G(s) = \frac{K}{(s + 1)(s + 2)}\]
The gain margin of the system in dB is given by
(A) 0 
(B) 1 
(C) 20 
(D) 18.

18. In the system shown below, \(x(t) = (\sin t)u(t)\) In steady-state, the response \(y(t)\) will be
(A) \(\frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)\) 
(B) \(\frac{1}{\sqrt{2}} \sin\left(t + \frac{\pi}{4}\right)\) 
(C) \(\frac{1}{\sqrt{2}} e^{-t} \sin t\) 
(D) \(\sin t - \cos t\)

19. The electric field of an electromagnetic wave propagation in the positive direction is given by
\[E = \hat{\alpha}_x \sin(\omega t - \beta z) + \hat{\alpha}_y \sin(\omega t - \beta z + \pi/2)\]
The wave is
(A) Linearly polarized in the z–direction 
(B) Elliptically polarized 
(C) Left-hand circularly polarized 
(D) Right-hand circularly polarized
30. A two-port network is represented by ABCD, parameters given by
\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix}
\]
If port – 2 is terminated by the input impedance seen at port – 1 is given by
(A) \(A + BR_{L} \over C + DR_{L}\) 
(B) \(AR_{L} + C \over BR_{L} + D\) 
(C) \(DR_{L} + A \over C + BR_{L}\) 
(D) \(AR_{L} + B \over D + CR_{L}\)

31. In the two port network shown in the figure below \(z_{12}\) and \(z_{21}\) are respectively

(A) \(r_s\) and \(\beta r_o\) 
(B) 0 and \(-\beta r_o\) 
(C) 0 and \(\beta r_o\) 
(D) \(r_s\) and \(-\beta r_o\)

32. The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements, are a pole and a zero respectively. The above property will be satisfied by
(A) RL network only 
(B) RC network only 
(C) LC network only 
(D) RC as well as RL networks

33. A 2 mH inductor with some initial current can be represented as shown below, where \(s\) is the Laplace Transform variable. The value of initial current is

(A) 0.5A 
(B) 2.0A 
(C) 1.0 A 
(D) 0.0 A

34. In the figures shown below, assume that all the capacitors are initially uncharged. If \(v_i(t) = 10u(t)\) Volts, \(v_o(t)\) is given by

(A) 8\(e^{-0.004t}\) Volts 
(B) 8\(1 - e^{-0.004t}\) Volts 
(C) 8u(t) Volts 
(D) 8 Volts

35. Consider two transfer functions
\[
G_1(s) = \frac{1}{s^2 + as + b} \quad \text{And} \quad G_2(s) = \frac{s}{s^2 + as + b}
\]
The 3–dB bandwidths of their frequency responses are, respectively
(A) \(\sqrt{a^2 - 4b}, \sqrt{a^2 + 4b}\) 
(B) \(\sqrt{a^2 + 4b}, \sqrt{a^2 - 4b}\) 
(C) \(\sqrt{a^2 - 4b}, \sqrt{a^2 - 4b}\) 
(D) \(\sqrt{a^2 + 4b}, \sqrt{a^2 + 4b}\)

36. A negative resistance \(R_{neg}\) is connected to a passive network \(N\) having driving point impedance \(Z_1(s)\) as shown below. For \(Z_2(s)\) to be positive real,

(A) \(|R_{neg}| \leq \text{Re} Z_1(j\omega), \forall \omega\) 
(B) \(|R_{neg}| \leq |Z_1(j\omega)|, \forall \omega\) 
(C) \(|R_{neg}| \leq \text{Im} Z_1(j\omega), \forall \omega\) 
(D) \(|R_{neg}| \leq \angle Z_1(j\omega), \forall \omega\)

37. In the circuit shown below, the switch was connected to position 1 at \(t < 0\) and at \(t = 0\), it is changed to position 2. Assume that the diode has zero voltage drop and a storage time \(t_s\). For \(0 < t \leq t_s\), \(v_R\) is given by

(A) \(v_i(t)\) 
(B) \(v_i(t)\) 
(C) \(v_i(t)\) 
(D) \(v_i(t)\)
38. The majority carriers in an n–type semiconductor have an average drift velocity \( v \) in a direction perpendicular to a uniform magnetic field \( B \). The electric field \( E \) induced due to Hall effect acts in the direction.

(A) \( v \times B \)  
(B) \( B \times v \)  
(C) along \( v \)  
(D) opposite to \( v \)

39. Find the correct match between Group 1 and Group 2.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Varactor diode</td>
<td>1-Voltage reference</td>
</tr>
<tr>
<td>F-PIN diode</td>
<td>2-High frequency switch</td>
</tr>
<tr>
<td>G-Zener diode</td>
<td>3-Tuned circuits</td>
</tr>
<tr>
<td>H-Schottky diode</td>
<td>4-Current controlled attenuator</td>
</tr>
</tbody>
</table>

(A) E-4, F-2, G-1, H-3  
(B) E-2, F-4, G-1, H-3  
(C) E-3, F-4, G-1, H-2  
(D) E-1, F-3, G-2, H-4

40. A heavily doped n- type semiconductor has the following data:

- Hole-electron ratio :0.4  
- Doping concentration :4.2 \times 10^8 \text{ atoms/m}^3  
- Intrinsic concentration :1.5 \times 10^4 \text{ atoms/m}^3

The ratio of conductance of the n-type semiconductor to that of the intrinsic semiconductor of same material and at same temperature is given by

(A) 0.00005  
(B) 2,000  
(C) 10,000  
(D) 20,000

41. For the circuit shown in the following figure, the capacitor \( C \) is initially uncharged. At \( t = 0 \) the switch \( S \) is closed. In the figures shown the OP AMP is supplied with and the ground has been shown by the symbol

The voltage \( V_c \) across the capacitor at \( t = 1 \) is

(A) 0 Volt  
(B) 6.3 Volts  
(C) 9.45 Volts  
(D) 10 Volts

42. For the circuit shown below, assume that the zener diode is ideal with a breakdown voltage of 6 volts. The waveform observed across \( R \) is

43. A new Binary Coded Pentary (BCP) number system is proposed in which every digit of a base–5 number is represented by its corresponding 3–bit binary code. For example, the base–5 number 24 will be represented by its BCP code 010100. In this numbering system, the BCP code 10001001101 corresponds of the following number is base–5 system

(A) 423  
(B) 1324  
(C) 2201  
(D) 4231
44. An I/O peripheral device shown in Fig.(b) below is to be interfaced to an 8085 microprocessor. To select the I/O device in the I/O address range D4 H – D7 H, its chip-select \( CS \) should be connected to the output of the decoder shown in as below:

(A) output 7 (B) output 5 (C) output 2 (D) output 0

45. For the circuit shown in figures below, two 4-bit parallel – in serial – out shift registers loaded with the data shown are used to feed the data to a full adder. Initially, all the flip-flops are in clear state. After applying two clock pulses, the outputs of the full-adder should be

(A) \( S = 0, C_0 = 0 \) (B) \( S = 0, C_0 = 1 \) (C) \( S = 1, C_0 = 0 \) (D) \( S = 1, C_0 = 1 \)

46. A 4-bit D/A converter is connected to a free-running 3-bit UP counter, as shown in the following figure. Which of the following waveforms will be observed at \( V_o \) ?

(A) (B) (C) (D)

47. Two D – flip – flops, as shown below, are to be connected as a synchronous counter that goes through the following sequence

\[ 00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \rightarrow \ldots \]

The inputs \( D_0 \) and \( D_1 \) respectively should be connected as,

(A) \( \overline{Q}_0 \) and \( Q_0 \) (B) \( Q_0 \) and \( Q_1 \) (C) \( \overline{Q}_0 \overline{Q}_1 \) and \( Q_0 \) (D) \( Q_1 \) \( \overline{Q}_0 \) and \( Q_1 \) \( Q_0 \)

48. Following is the segment of a 8085 assembly language program

```
LXI SP, EFFF H
CALL 3000 H
:
:
3000 H LXI H, 3CF4
PUSH PSW
SPHL
POP PSW
RET
```

On completion of RET execution, the contents of SP is

(A) 3CF0 H (B) 3CF8 H (C) EFFF H (D) EFFF H

49. The point P in the following figure is stuck at 1. The output \( f \) will be
59. A linear system is described by the following state equation
\[ \dot{X}(t) = AX(t) + BU(t), \quad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \]

The state transition matrix of the system is
(A) \( \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \)
(B) \( \begin{bmatrix} -\cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix} \)
(C) \( \begin{bmatrix} -\cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix} \)
(D) \( \begin{bmatrix} -\cos t & -\sin t \\ \cos t & \sin t \end{bmatrix} \)

60. The minimum step-size required for a Delta–Modulator operating at 32 K samples/sec to track the signal (here \( u(t) \) is the unit function)
\[ x(t) = 125\left( u(t) - u(t-1)(250 - 125t) - u(t-1) - u(t-2) \right) \]
So that slope overload is avoided, would be
(A) \( 2^{-10} \)
(B) \( 2^{-8} \)
(C) \( 2^{-6} \)
(D) \( 2^{-4} \)

61. A zero mean white Gaussian noise is passed through an ideal lowpass filter of bandwidth 10 kHz. The output is then uniformly sampled with sampling period \( t_s = 0.03 \) msec. The samples so obtained would be
(A) correlated
(B) statistically independent
(C) uncorrelated
(E) orthogonal

62. A source generates three symbols with probabilities 0.25, 0.25, 0.50 at a rate of 3000 symbols per second. Assuming independent generation of symbols, the most efficient source encoder would have average bit rate as
(A) 6000 bits/sec
(B) 4500 bits/sec
(C) 3000 bits/sec
(D) 1500 bits/sec

63. The diagonal clipping in Amplitude Demodulation (using envelope detector) can be avoided if \( RC \) time constant of the envelope detector satisfies the following condition, (here \( W \) is message bandwidth and \( \omega_c \) is carrier frequency both in rad /sec)
\[ (A) \quad RC < \frac{1}{\omega_c} \quad \text{or} \quad \frac{1}{W} \]
\[ (B) \quad RC > \frac{1}{\omega_c} \quad \text{or} \quad \frac{1}{W} \]

64. In the following figure the minimum value of the constant “C”, which is to be added to \( y_1(t) \) and \( y_2(t) \) such that \( y_1(t) \) and \( y_2(t) \) and are different, is
\[ \text{Q is quantizer with L levels, stepwise \Delta allowable signal dynamic range [-V, V]} \]
\[ \Delta \]
\[ \frac{\Delta}{L} \]

65. A message signal with 10 kHz bandwidth is lower side Band SSB modulated with carrier \( f_c = 10^6 \) Hz frequency the resulting signal is then passed through a Narrow Band Frequency Modulator with carrier frequency \( f_{c2} = 10^8 \) Hz. The bandwidth of the output would be
(A) \( 4 \times 10^4 \) Hz
(B) \( 2 \times 10^6 \) Hz
(C) \( 2 \times 10^8 \) Hz
(D) \( 2 \times 10^{10} \) Hz

66. A medium of relative permittivity \( \varepsilon_r = 2 \) forms an interface with free – space. A point source of electromagnetic energy is located in the medium at a depth of 1 meter from the interface. Due to the total internal reflection, the transmitted beam has a circular cross-section over the interface. The area of the beam cross-section at the interface is given by
(A) \( \frac{\pi}{2} \) m²
(B) \( \pi^2 \) m²
(C) \( \frac{\pi}{2} \) m²
(D) \( \pi \) m²

67. A medium is divide into regions I and II about \( x=0 \) plane, as shown in the figure below. An electromagnetic wave with electric field \( E_1 = 4\delta_x + 3\delta_y + 5\delta_z \) is incident normally on the interface from region I. The electric field \( E_2 \) in region II at the interface is
\[ \text{Region I} \quad \mu_r = \mu_0, \quad \varepsilon_r = 4, \quad \sigma_r = 0 \]
\[ \text{Region II} \quad \mu_r = \mu_0, \quad \varepsilon_r = 4, \quad \sigma_r = 0 \]
58. When a plane wave traveling in free-space is incident normally on a medium having the fraction of power transmitted into the medium is given by
(A) 8/9  
(B) 1/2  
(C) 1/3  
(D) 5/6

59. A rectangular wave guide having TE_{10} mode as dominant mode is having a cut off frequency 18-GHz for the mode TE_{30}. The inner broad – wall dimension of the rectangular wave guide is
(A) 5/3 cms  
(B) 5 cms  
(C) 5/2 cms  
(D) 10 cms

70. A mast antenna consisting of a 50 meter long vertical conductor operates over a perfectly conducting ground plane. It is base-fed at a frequency of 600 kHz. The radiation resistance of the antenna in Ohms is
(A) \( \frac{2\pi^2}{5} \)  
(B) \( \frac{\pi^2}{5} \)  
(C) \( \frac{4\pi^2}{5} \)  
(D) \( 20\pi^2 \)

Common Data for Question 71, 72, 73:
In the transistor amplifier circuit show in the figure below, the transistor has the following parameters:
\[ \beta_{DC} = 60, \ V_{BE} = 0.7 \text{ V}, \ h_{ie} \to \infty, \ h_{fe} \to \infty \]
The capacitance can be assumed to be infinite.

71. Under the DC conditions, the collector – to- emitter voltage drop is
(A) 4.8 Volts  
(B) 5.3 Volts  
(C) 6.0 Volts  
(D) 6.6 Volts

72. If \( \beta_{DC} \) is increase by 10%, the collector – to- emitter voltage drop
(A) increases by less than or equal to 10%  
(B) decreases by less than or equal to 10%  
(C) increases by more than 10%  
(D) decreases by more than 10%

73. The small signal gain of the amplifier \( v_c/v_s \) is
(A) 10  
(B) \(-5.3\)  
(C) 5.3  
(D) 10

Common Data for Question 74, 75:
Let \( g(t) = p(t) \* p(t) \) where \( \* \) denotes convolution and \( p(t) = u(t) - u(t - 1) \) with \( u(t) \) being the unit step function.

74. The impulse response of filter matched to the signal \( s(t) = g(t) - \delta(t - 2) \* g(t) \) is given as:
(A) \( s(1-t) \)  
(B) \( -s(1-t) \)  
(C) \( -s(t) \)  
(D) \( s(t) \)

75. An Amplitude Modulated signal is given as
\[ x_{AM} = 100(p(t) + 0.5g(t))\cos \omega_c t \]
In the interval. One set of possible values of the modulating signal and modulation index would be
(A) \( t, 0.5 \)  
(B) \( t, 10 \)  
(C) \( t, 2.0 \)  
(D) \( t^2, 0.5 \)

Linked Answer Question : Q.75 to Q.85 carry two marks each.

Statement of Linked Answer Question 76 & 77:
A regulated power supply, shown in figure below, has an unregulated input (UR) of 15 volts and generates a regulated output. Use the component values shown in the figure.
take 400 μs for an electromagnetic wave to travel from source end to load end and vice – versa. At \( t = 400 \mu s \), the voltage at the load end is found to be 40 volts.

84. The load resistance is
(A) 25 Ohms  (B) 50 Ohms  
(C) 75 Ohms  (D) 100 Ohms

85. The steady state current through the load resistance is
(A) 1.2 Amps  (B) 0.3 Amps  
(C) 0.6 Amps  (D) 0.4 Amps

***********

ANSWER

41. D  42. B  43. D  44. B  45. D
71. C  72. B  73. A  74. D  75. A
76. C  77. B  78. A  79.  80. C
Q.1 - Q. 20 carry one mark each.

1. The order of the differential equation
   \[ \frac{d^2y}{dr^2} + \left( \frac{dy}{dt} \right)^3 + y^4 = e^{-1} \] is
   (A) 1           (B) 2
   (C) 3           (D) 4

2. The Fourier series of a real periodic function has only
   P. Cosine terms if it is even
   Q. sine terms if it is even
   R. cosine terms if it is odd
   S. sine terms if it is odd

Which of the above statements are correct?
   (A) P and S       (B) P and R
   (C) Q and S       (D) Q and R

3. A function is given by \( f(t) = \sin^2 t + \cos 2t \). Which of the following is true?
   (A) \( f \) has frequency components at 0 and 1/2\( \pi \)Hz
   (B) \( f \) has frequency components at 0 and 1/\( \pi \)Hz
   (C) \( f \) has frequency components at 1/2\( \pi \) and 1/\( \pi \)Hz
   (D) \( f \) has frequency components at 0.1/2\( \pi \) and 1/\( \pi \)Hz

4. A fully charged mobile phone with a 12 V battery is good for a 10 minute talk-time. Assume that, during the talk-time, the battery delivers a constant current of 2 A and its voltage drops linearly from 12 V to 10 V as shown in the figure. How much energy does the battery deliver during this talk-time?
   (A) 220 J   (B) 12 kJ
   (C) 13.2 kJ (D) 14.4 kJ

5. In an n-type silicon crystal at room temperature, which of the following can have a concentration of \( 4 \times 10^{19} \) cm\(^{-3} \)?
   (A) Silicon atoms   (B) Holes
   (C) Dopant atoms    (D) Valence electrons

6. The full forms of the abbreviations TTL and CMOS in reference to logic families are
   (A) Triple Transistor Logic and Chip Metal Oxide Semiconductor
   (B) Tristate Transistor Logic and Chip Metal Oxide Semiconductor
   (C) Transistor Transistor Logic and Complementary Metal Oxide Semiconductor
   (D) Tristate Transistor Logic and Complementary Metal Oxide Silicon

7. The ROC of Z-transform of the discrete time sequence
   \( x(n) = \left( \frac{1}{3} \right)^n u(n) - \left( \frac{1}{2} \right)^n u(-n - 1) \) is
   (A) \( \frac{1}{3} < |z| < \frac{1}{2} \)   (B) \( |z| > \frac{1}{2} \)
3. The magnitude plot of a rational transfer function \( G(s) \) with real coefficients is shown below. Which of the following compensators has such a magnitude plot?

- (A) Lead compensator
- (B) Lag compensator
- (C) PID compensator
- (D) Lead-lag compensator

![Magnitude plot](image)

11. A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads?

- (A) \( \left( \frac{1}{2} \right)^2 \)
- (B) \( 10 \left( \frac{1}{2} \right)^2 \)
- (C) \( \left( \frac{1}{2} \right)^{10} \)
- (D) \( 10 \left( \frac{1}{2} \right)^{10} \)

12. If the power spectral density of stationary random process is a sinc-squared function of frequency, the shape of its autocorrelation is

- (A)
- (B)
- (C)
- (D)

13. If \( f(z) = c_0 + c_1 z^{-1} \), then \( \int_{\text{unit circle}} \frac{1 + f(z)}{z} \, dz \) is given by

- (A) \( 2\pi c_1 \)
- (B) \( 2\pi(1 + c_0) \)
- (C) \( 2\pi c_1 \)
- (D) \( 2\pi(1 + c_0) \)

14. In the interconnection of ideal sources shown in the figure, it is known that the 60 V source is absorbing power.

Which of the following can be the value of the current source \( I \)?

- (A) 10 A
- (B) 13 A
- (C) 15 A
- (D) 18 A

15. The ratio of the mobility to the diffusion coefficient in a semiconductor has the units
16. In a microprocessor, the service routine for a certain interrupt starts from a fixed location of memory which cannot be externally set, but the interrupt can be delayed or rejected. Such an interrupt is
(A) non-maskable and non-vectored
(B) maskable and non-vectored
(C) non-maskable and vectored
(D) maskable and vectored

17. If the transfer function of the following network is
\[ V_o(s) = \frac{1}{2 + sCR} \]
the value of the load resistance \( R_L \) is
(A) \( R/4 \)
(B) \( R/2 \)
(C) \( R \)
(D) \( 2R \)

18. Consider the system
\[ \frac{dx}{dt} = Ax + Bu \]
where \( A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)
and \( B = \begin{bmatrix} p \\ q \end{bmatrix} \)
where \( p \) and \( q \) are arbitrary real numbers.

Which of the following statements about the controllability of the system is true?
(A) The system is completely state controllable for any nonzero values of \( p \) and \( q \)
(B) Only \( p = 0 \) and \( q = 0 \) result in controllability
(C) The system is uncontrollable for all values of \( p \) and \( q \)
(D) We cannot conclude about controllability from the given data

19. For a message signal \( m(t) = \cos(2\pi f_m t) \) and carrier of frequency \( f_c \), which of the following represents a single side-band (SSB) signal?
(A) \( \cos(2\pi f_m t) \cos(2\pi f_c t) \)
(B) \( \cos(2\pi f_c t) \)
(C) \( \cos(2\pi f_c t + f_m t) \)
(D) \( 1 + \cos(2\pi f_m t) \cos(2\pi f_c t) \)

20. Two infinitely long wires carrying current are as shown in the figure below. One wire is in the \( y-z \) plane and parallel to the \( y \)-axis. The other wire is in the \( x-y \) plane and parallel to the \( x \)-axis. Which components of the resulting magnetic field are non-zero at the origin?
(A) \( x, y, z \) components
(B) \( x, y \) components
(C) \( y, z \) components
(D) \( x, z \) components

Q. 21 to Q. 60 carry two marks each.

21. Consider two independent random variables \( X \) and \( Y \) with identical distributions. The variables \( X \) and \( Y \) take values 0, 1 and 2 with probabilities \( \frac{1}{2} \), \( \frac{1}{4} \) and \( \frac{1}{4} \) respectively. What is the conditional probability \( P(X + Y = 2 | X - Y = 0) \)?
(A) 0
(B) \( \frac{1}{16} \)
(C) \( \frac{1}{6} \)
(D) 1

22. The Taylor series expansion of \( \frac{\sin x}{x - \pi} \) at \( x = \pi \) is given by
(A) \( 1 + \frac{(x - \pi)^2}{3!} + \ldots \)
(B) \( -1 + \frac{(x - \pi)^2}{3!} + \ldots \)
(C) \( 1 - \frac{(x - \pi)^2}{3!} + \ldots \)
(D) \( -1 - \frac{(x - \pi)^2}{3!} + \ldots \)

23. If a vector field \( \vec{V} \) is related to another vector field \( \vec{A} \) through \( \vec{V} = \vec{\nabla} \times \vec{A} \), which of the following is true? Note: \( C \) and \( S_c \) refer to any closed contour and any surface whose boundary is \( C \).
(A) \( \oint_C \vec{V} \cdot d\vec{l} = \iint_{S_c} \vec{A} \cdot d\vec{S} \)
(B) \( \oint_C \vec{A} \cdot d\vec{l} = \iiint_{V_c} \vec{V} \cdot d\vec{S} \)
(C) \( \oint_C \vec{V} \cdot d\vec{l} = \iint_{S_c} \vec{V} \times \vec{A} \cdot d\vec{S} \)
(D) \( \iint_{S_c} \vec{V} \times d\vec{S} = \oint_C \vec{V} \cdot d\vec{l} = \iint_{V_c} \vec{A} \cdot d\vec{S} \)
24. Given that \( F(s) \) is the one-sided Laplace transform of \( f(t) \), the Laplace transform of \( \int_0^t f(\tau) d\tau \) is

(A) \( sF(s) - f(0) \)  
(B) \( \frac{1}{s} F(s) \)  
(C) \( \int_0^t F(\tau) d\tau \)  
(D) \( \frac{1}{s} [F(s) - f(0)] \)

25. Match each differential equation in Group I to its family of solution curves from Group II.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group II</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. ( \frac{dy}{dx} = \frac{y}{x} )</td>
<td>1. Circles</td>
</tr>
<tr>
<td>Q. ( \frac{dy}{dx} = -\frac{y}{x} )</td>
<td>2. Straight lines</td>
</tr>
<tr>
<td>R. ( \frac{dy}{dx} = \frac{x}{y} )</td>
<td>3. Hyperbolas</td>
</tr>
<tr>
<td>S. ( \frac{dy}{dx} = -\frac{x}{y} )</td>
<td></td>
</tr>
</tbody>
</table>

(A) \( P - 2, Q - 3, R - 3, S - 1 \)  
(B) \( P - 1, Q - 3, R - 2, S - 1 \)  
(C) \( P - 2, Q - 1, R - 3, S - 3 \)  
(D) \( P - 3, Q - 2, R - 1, S - 2 \)

26. The eigenvalues of the following matrix are

\[
\begin{bmatrix}
-1 & 3 & 5 \\
3 & -1 & 6 \\
0 & 0 & 3
\end{bmatrix}
\]

(A) 3, 3 + 5j, 6 - j  
(B) -6 + 5j, 3 + j, 3 - j  
(C) 3 + j, 3 - j, 5 + j  
(D) 3, -1 + 3j, -1 - 3j

27. An AC source of RMS voltage 20 V with internal impedance \( Z_L = (1 + 2j)\Omega \) feeds a load of impedance \( Z_L = (7 + 4j)\Omega \) in the figure below. The reactive power consumed by the load is

(A) 8 VAR  
(B) 16 VAR  
(C) 28 VAR  
(D) 32 VAR

28. The switch in the circuit shown was on position \( a \) for a long time, and is move to position \( b \) at time \( t = 0 \). The current \( i(t) \) for \( t > 0 \) is given by

\[ i(t) = \begin{cases} 
0 & \text{for } t < 0 \\
100 V & \text{for } t \geq 0
\end{cases} \]

(A) \( 0.2e^{-20t}u(t) \) mA  
(B) \( 20e^{-100t}u(t) \) mA  
(C) \( 0.2e^{-1250t}u(t) \) mA  
(D) \( 20e^{-1000t}u(t) \) mA

29. In the circuit shown, what value of \( R_L \) maximizes the power delivered to \( R_L \)?

(A) 2.4 \( \Omega \)  
(B) \( \frac{8}{3} \) \( \Omega \)  
(C) 4 \( \Omega \)  
(D) 6 \( \Omega \)

30. The time domain behavior of an \( RL \) circuit is represented by

\[ L \frac{di}{dt} + Ri = V_0 (1 + Be^{-Rt/L} \sin t)u(t). \]

For an initial current of \( i(0) = \frac{V_0}{R} \), the steady state value of the current is given by

(A) \( i(t) \rightarrow \frac{V_0}{R} \)  
(B) \( i(t) \rightarrow \frac{2V_0}{R} \)  
(C) \( i(t) \rightarrow \frac{V_0}{R} (1 + B) \)  
(D) \( i(t) \rightarrow \frac{2V_0}{R} (1 + B) \)

31. In the circuit below, the diode is ideal. The voltage \( V \) is given by

(A) \( \min(V_t, 1) \)  
(B) \( \max(V_t, 1) \)  
(C) \( \min(-V_t, 1) \)  
(D) \( \max(-V_t, 1) \)

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(A) NAND: first (0,1) then (0,1) NOR: first (1,0) then (0,0)
(B) NAND: first (1,0) then (1,0) NOR: first (1,0) then (1,0)
(C) NAND: first (1,0) then (1,0) NOR: first (1,0) then (0,0)
(D) NAND: first (1,0) then (1,1) NOR: first (0,1) then (0,1)

39. What are the counting states \((Q_1, Q_2)\) for the counter shown in the figure below?

\[
\begin{array}{c}
\text{Clock} \\
\begin{array}{c}
\text{J}_1 \\
\text{K}_1 \\
\text{J}_2 \\
\text{K}_2 \\
\text{Q}_1 \\
\text{Q}_2
\end{array}
\end{array}
\]

(A) 11, 10, 00, 11, 10,... (B) 01, 10, 11, 00, 01,...
(C) 00, 11, 01, 10, 00,... (D) 01, 10, 00, 01, 10,...

40. A system with transfer function \(H(z)\) has impulse response \(h(.)\) defined as \(h(2) = 1\), \(h(3) = -1\) and \(h(k) = 0\) otherwise. Consider the following statements.

S1: \(H(z)\) is a low-pass filter.
S2: \(H(z)\) is an FIR filter.

Which of the following is correct?
(A) Only S2 is true
(B) Both S1 and S2 are false
(C) Both S1 and S2 are true, and S2 is a reason for S1
(D) Both S1 and S2 are true, but S2 is not a reason for S1

41. Consider a system whose input \(x\) and output \(y\) are related by the equation

\[
y(t) = \int_{-\infty}^{t} x(t - \tau) h(2 \tau) d\tau
\]

where \(h(t)\) is shown in the graph.

Which of the following four properties are possessed by the system?

BIBO: Bounded input gives a bounded output.
Causal: The system is causal,
LP: The system is low pass.
LTI: The system is linear and time-invariant.

42. The 4-point Discrete Fourier Transform (DFT) of a discrete time sequence \([1,0,2,3]\) is

(A) \([0,-2+2j,2,-2-2j]\) (B) \([2,2+2j,6,2-2j]\)
(C) \([6,-1+3j,2,1+3j]\) (D) \([6-1+3j,0,-1,-3j]\)

43. The feedback configuration and the pole-zero locations of \(G(s) = \frac{s^2 - 2s + 2}{s^2 + 2s + 2}\) are shown below. The root locus for negative values of \(k\), i.e. for \(-\infty < k < 0\), has breakaway/break-in points and angle of departure at pole P (with respect to the positive real axis) equal to

(A) \(\pm \sqrt{2}\) and \(0^\circ\) (B) \(\pm \sqrt{2}\) and \(45^\circ\)
(C) \(\pm \sqrt{3}\) and \(0^\circ\) (D) \(\pm \sqrt{3}\) and \(45^\circ\)

44. An LTI system having transfer function \(\frac{s^2 + 1}{s^2 + 2s + 1}\) and input \(x(t) = \sin(t) + 1\) is in steady state. The output is sampled at a rate \(\omega_r\) rad/s to obtain the final output \(\{x(k)\}\). Which of the following is true?

(A) \(y(.)\) is zero for all sampling frequencies \(\omega_r\)
(B) \(y(.)\) is nonzero for all sampling frequencies \(\omega_r\)
(C) \(y(.)\) is nonzero for \(\omega_r > 2\), but zero for \(\omega_r < 2\)
(D) \(y(.)\) is zero for \(\omega_r > 2\), but nonzero for \(\omega_r < 2\)

45. The unit step response of an under-damped second order system has steady state value of -2. Which one of the following transfer functions has these properties?

(A) \(-\frac{2.24}{s^2 + 259s + 112}\) (B) \(-\frac{3.82}{s^2 + 191s + 191}\)
(C) \(-\frac{2.24}{s^2 - 259s + 112}\) (D) \(-\frac{3.82}{s^2 - 191s + 191}\)
46. A discrete random variable $X$ takes values from 1 to 5 with probabilities as shown in the table. A student calculates the mean of $X$ as 3.5 and her teacher calculates the variance of $X$ as 1.5. Which of the following statements is true?

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=k)$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(A) Both the student and the teacher are right
(B) Both the student and the teacher are wrong
(C) The student is wrong but the teacher is right
(D) The student is right but the teacher is wrong

47. A message signal given by

$$1n(t)=\left(\frac{1}{2}\right)\cos \omega_0 t - \left(\frac{1}{2}\right)\sin \omega_0 t$$

is amplitude-modulated with a carrier of frequency $\omega_c$ to generate

$$s(t) = [1 + m(t)]\cos \omega_0 t$$

What is the power efficiency achieved by this modulation scheme?

(A) 8.33%  
(B) 11.11%  
(C) 20%  
(D) 25%

48. A communication channel with AWGN operating at a signal to noise ratio $\text{SNR} \gg 1$ and bandwidth $B$ has capacity $C_1$. If the SNR is doubled keeping $B$ constant, the resulting capacity $C_2$ is given by

(A) $C_2 = 2C_1$  
(B) $C_2 = C_1 + B$  
(C) $C_2 = C_1 + 2B$  
(D) $C_2 = C_1 + 0.3B$

49. A magnetic field in air is measured to be

$$\vec{B} = B_0 \left( \frac{x}{x^2 + y^2} \hat{x} - \frac{y}{x^2 + y^2} \hat{y} \right)$$

What current distribution leads to this field?

[Hint: The algebra is trivial in cylindrical coordinates.]

(A) $j = \frac{B_0 \hat{\phi}}{\mu_0} \left( \frac{1}{x^2 + y^2} \right) r \neq 0$

(B) $j = -\frac{B_0 \hat{\phi}}{\mu_0} \left( \frac{2}{x^2 + y^2} \right) r \neq 0$

(C) $j = 0, r \neq 0$

50. A transmission line terminates in two branches, each of length $\lambda/4$, as shown. The branches are terminated by 50 Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance $Z_i$ as seen by the source.

(A) $200\Omega$  
(B) $100\Omega$  
(C) $50\Omega$  
(D) $25\Omega$

**Common Date Questions**

**Common Date for Questions 51 and 52:**

Consider a silicon p-n junction at room temperature having the following parameters:

- Doping on the n-side = $1 \times 10^{17}$ cm$^{-3}$
- Depletion width on the n-side = $0.1\mu m$
- Depletion width on the p-side = $1.0\mu m$
- Intrinsic carrier concentration = $1.4 \times 10^{16}$ cm$^{-3}$
- Thermal voltage = $26\text{ mV}$
- Permittivity of free space = $8.85 \times 10^{-12}$ F cm$^{-1}$
- Dielectric constant of silicon = 12

51. The built-in potential of the junction

(A) is 0.70 V  
(B) is 0.76 V  
(C) is 0.82  
(D) cannot be estimated from the data given

52. The peak electric field in the device is

(A) $0.15\text{ MV} \cdot \text{cm}^{-1}$, directed from p-region to n-region  
(B) $0.15\text{ MV} \cdot \text{cm}^{-1}$, directed from n-region to p-region  
(C) $1.80\text{ MV} \cdot \text{cm}^{-1}$, directed from p-region to n-region  
(D) $1.80\text{ MV} \cdot \text{cm}^{-1}$, directed from n-region to p-region

**Common Data for Questions 53 and 54:**

$$\vec{J} = \frac{B_0 \hat{\phi}}{\mu_0} \left( \frac{1}{x^2 + y^2} \right) r \neq 0$$
The Nyquist plot of a stable transfer function \( G(s) \) is shown in the figure. We are interested in the stability of the closed loop system in the feedback configuration shown.

53. Which of the following statements is true?
(A) \( G(s) \) is an all-pass filter
(B) \( G(s) \) has a zero in the right-half plane
(C) \( G(s) \) is the impedance of a passive network
(D) \( G(s) \) is marginally stable

54. The gain and phase margins of \( G(s) \) for closed loop stability are
(A) 6 dB and 180°
(B) 3 dB and 180°
(C) 6 dB and 90°
(D) 3 dB and 90°

Common Data for Questions 55 and 56:

The amplitude of a random signal is uniformly distributed between -5 V and 5 V.

55. If the signal to quantization noise ratio required in uniformly quantizing the signal is 43.5 dB, the step size of the quantization is approximately
(A) 0.0333 V
(B) 0.05 V
(C) 0.0667 V
(D) 0.10 V

56. If the positive values of the signal are uniformly quantized with a step size of 0.05 V, and the negative values are uniformly quantized with a step size of 0.1 V, the resulting signal to quantization noise ratio is approximately
(A) 46 dB
(B) 43.8 dB
(C) 42 dB
(D) 40 dB

Linked Answer Questions

Statement for Linked Answer Questions 57 and 58:

Consider the CMOS circuit shown, where the gate voltage \( V_g \) of the n-MOSFET is increased from zero, while the gate voltage of the p-MOSFET is kept constant at 3 V. Assume that, for both transistors, the magnitude of the threshold voltage is 1 V and the product of the trans-conductance parameter and the (W/L) ratio, i.e., the quantity \( \mu C_{ox} (W / L) \), is 1 mA V\(^{-2}\).

57. For small increase in \( V_g \) beyond 1 V, which of the following gives the correct description of the region of operation of each MOSFET?
(A) Both the MOSFETs are in saturation region
(B) Both the MOSFETs are in triode region
(C) n-MOSFET is in triode and p-MOSFET is in saturation region
(D) n-MOSFET is in saturation and p-MOSFET is in triode region.

58. Estimate the output voltage \( V_o \) for \( V_g = 15 \) V. [Hint: Use the appropriate current-voltage equation for each MOSFET, based on the answer to Q. 57.]

Statement for Linked Answer Questions 59 and 60:

Two products are sold from a vending machine, which has two push buttons \( P_1 \) and \( P_2 \). When a button is pressed, the price of the corresponding product is displayed in a 7-segment display.

If no buttons are pressed, '0' is displayed, signifying 'Rs. 0'.

If only \( P_1 \) is pressed, '2' is displayed, signifying 'Rs. 2'.

If only \( P_2 \) is pressed, '5' is displayed, signifying 'Rs. 5'.

If both \( P_1 \) and \( P_2 \) are pressed, 'E' is displayed, signifying 'Error'.